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CATEGORIAL ASPECTS ARE USEFUL IN TOPOLOGY

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Under the same title a common lecture with V.Trnková was presented in this conference.The lecture was linked by using categorial methods to obtain topological results.Because results of this part were already published,it appears separately as an abstract only.

One of problems concerning proximally fine uniform spaces is the question whether they are productive or not [Poljakov$_{1,2}$], [Isbell$_2$].We shall present here results concerning that question in a more general setting which gives solutions of similar problems for more classes.For details in the case of proximally fine spaces see [Hušek$_{1,2}$];the proofs of more general results presented here are practically the same.In the sequel,C denotes a coreflective subcategory of Unif.

THEOREM 1. *If C contains all topologically fine spaces,then a product belongs to C iff all countable subproducts belong to C.*

Theorem 1 for cozero-fine and also for proximally fine spaces was proved in [Tashjian],for topologically fine and locally fine in [Isbell$_1$].For some C we can prove more:

THEOREM 2. *If C contains all topologically fine and metrizable spaces,then a product belongs to C iff all finite subproducts belong to C.*

Thus a product is proximally fine iff any finite subproduct is proximally fine;consequently,any uniform space is a subspace of a proximally fine space - a product of pseudo-metrizable spaces.The original question reduces now to product of two spaces.It follows from the next result that such product need not be proximally fine.

THEOREM 3. *Any uniform space is a quotient of a product of a uniformly discrete space and of a topologically fine space.*

COROLLARY. *If C contains all topologically fine spaces,then C is not finitely productive.*

For proximally fine spaces and some F-fine spaces we can prove more:

THEOREM 4. *For any nonprecompact proximally fine space X there is a topologically fine P such that $X\times P$ is not proximally fine.*

The assumption posed on X cannot be omitted:

THEOREM 5. *Product of a precompact proximally fine space and of a proximally fine space is proximally fine.*

This is a generalization of Kůrková's result: compact instead of precompact.

Theorem 3 is proved by means of special simple spaces inductively generating Unif ([Hušek$_3$] contains their further applications) and Theorem 4 is deduced from Theorem 3 by means of Cone functor.

Theorems 1,2 and 5 are proved by means of factorizations of mappings defined on subspaces of products. By investigation of such factorizations [Hušek$_4$] the following concept proved to be useful (Δ_X is the diagonal of $X\times X$):

DEFINITION. Topological space X is said to be a D-space if for any sequence $\{\langle x_n,y_n\rangle\}\subset X\times X-\Delta_X$ there is a neighborhood U of Δ_X in $X\times X$ and a subsequence $\{\langle x_{k_n},y_{k_n}\rangle\}$ disjoint with $\overline{U}$.

Clearly, D-spaces are hereditary and finitely productive.

THEOREM 6. *Any F-space is a D-space.*

THEOREM 7. *Any continuous mapping on a pseudocompact product into a D-space depends on a finitely many coordinates.*

THEOREM 8. *If P is a countably compact subspace of a countable product, then any continuous mapping on P into a D-space depends on finitely many coordinates.*

COROLLARIES. *(a) If X is a dyadic compact D-space, then X is finite.*

(b) If $\bigcup_1^{\infty} F_n \subset C(\beta N,X)$ distinguishes points of βN, then $\bigcup_1^{k} F_n$ distinguishes points of βN for a natural k.

(c) If A_n, B, C are Boolean algebras, B is σ-complete, $f:\sum_1^{\infty} A_n \longrightarrow C$ is an epimorphism, then for any homomorphism $g:B\longrightarrow C$ there is a natural k such that $g(B)\subset f(\sum_1^{k} A_n)$.

(d) If βN is a continuous image of a pseudocompact power P^{κ}, then $\operatorname{card} P \geq 2^{2^{\omega}}$.

REFERENCES

Hušek M.: 1. Factorizations and mappings on products of uniform spaces,Seminar Uniform Spaces 1973/74, Prague 1975,173-190.

2. Cones and proximally fine spaces,Seminar Uniform Spaces 1975/76,Prague 1976,135-138.

3. Lattices of reflections and coreflections in continuous structures,Lecture Notes Math.540 (1976),404-424.

4. Continuous mappings on subspaces of products, Symp.Math.17(1976),25-41.

Isbell J.R.: 1. Uniform spaces,Amer.Math.Soc.Providence 1964.

2. Spaces without large projective subspaces, Math.Scand.17(1965),89-105.

Kůrková V.: Concerning products of proximally fine uniform spaces,Seminar Uniform Spaces 1973/74,Prague 1975,159-171.

Poljakov V.Z.: 1. Regularity,spectra and products of proxinity spaces,Dokl.Akad.Nauk SSSR 154(1964),51-54.

2. Regularity and products of proximity spaces, Math.Sb.67(1965),428-439.

Tashjian G.: Cozero and Baire maps on products of uniform spaces,preprint of Wesleyan Univ.,1974.

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