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Numerical Solution of Compressible Flow

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Abstract. The main feature of the equations describing the motion of the viscous compressible flows, i.e. the Navier-Stokes equations, is the combination of dominating convective parts with the diffusive effects. These equations will be numerically solved by the combined finite volume — finite element method via operator inviscid-viscous splitting. The main idea of the method is to discretize nonlinear convective terms with the aid of the finite volume scheme, whereas the diffusion terms are discretized by piecewise linear conforming triangular finite elements. The nonlinear convective terms can also be solved by the method of characteristics. Numerical solution obtained by latter method is truly multidimensional and independent of the mesh character. We will present results of numerical experiments for some well-known test problems.

AMS Subject Classification. 65M12, 65M60, 35K60, 76M10, 76M25

Keywords. Compressible Navier-Stokes equations, nonlinear convection-diffusion equation, finite volume schemes, finite element method, numerical integration, truly multidimensional schemes, evolution-Galerkin schemes

1 Formulation of the problem

We consider gas flow in a space-time cylinder $Q_T = \Omega \times (0, T)$, where $\Omega \subset \mathbb{R}^2$ is a bounded domain representing the region occupied by the fluid and $T > 0$. By $\overline{\Omega}$ and $\partial\Omega$ we denote the closure and boundary of Ω, respectively.

The complete system of viscous compressible flow consisting of the continuity equation, Navier-Stokes equations and energy equation can be written in the form

$$\frac{\partial w}{\partial t} + \sum_{i=1}^{2} \frac{\partial f_i(w)}{\partial x_i} = \sum_{i=1}^{2} \frac{\partial R_i(w, \nabla w)}{\partial x_i} \quad \text{in } Q_T. \tag{1}$$

This is the final form of the paper.

Here

$$
\begin{aligned}
&w = (w_1, w_2, w_3, w_4)^{\mathrm{T}} = (\rho,\ \rho v_1,\ \rho v_2, e)^{\mathrm{T}}, \qquad (2)\\
&w = w(x,t), \quad x \in \Omega,\ t \in (0,T),\\
&f_i(w) = (\rho v_i,\ \rho v_i v_1 + \delta_{i1} p,\ \rho v_i v_2 + \delta_{i2} p,\ (e+p)\, v_i)^{\mathrm{T}},\\
&R_i(w, \nabla w) = (0,\ \tau_{i1},\ \tau_{i2},\ \tau_{i1} v_1 + \tau_{i2} v_2 + k \partial\theta/\partial x_i)^{\mathrm{T}},\\
&\tau_{ij} = \lambda \operatorname{div} \boldsymbol{v}\, \delta_{ij} + \mu \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right), \quad i,\ j = 1,\ 2.
\end{aligned}
$$

From thermodynamics we have

$$p = (\gamma - 1)\,(e - \rho |\boldsymbol{v}|^2/2), \quad e = \rho(c_v \theta + |\boldsymbol{v}|^2/2). \qquad (3)$$

We use the standard notation: t — time, x_1, x_2 — Cartesian coordinates in $\mathbb{R}^2$, ρ — density, $\boldsymbol{v} = (v_1, v_2)$ — velocity vector with components v_i in the directions x_i, $i = 1, 2$, p — pressure, θ — absolute temperature, e — total energy, τ_{ij} — components of the viscous part of the stress tensor, δ_{ij} — Kronecker delta, $\gamma > 1$ — Poisson adiabatic constant, c_v — specific heat at constant volume, k — heat conductivity, λ, μ — viscosity coefficients. We assume that c_v, k, μ are positive constants and $\lambda = -\frac{2}{3}\mu$. We neglect outer volume force. The functions f_i are called inviscid (Euler) fluxes and are defined in the set $D = \{(w_1, \ldots, w_4) \in \mathbb{R}^4;\ w_1 > 0\}$. The viscous terms R_i are obviously defined in $D \times \mathbb{R}^8$. (Due to physical reasons it is also suitable to require $p > 0$.)

System (1.1), (1.3) is equipped with the initial conditions

$$w(x, 0) = w^0(x), \quad x \in \Omega \qquad (4)$$

(which means that at time $t = 0$ we prescribe, e. g., ρ, v_1, v_2 and θ) and boundary conditions: The boundary $\partial\Omega$ is divided into several disjoint parts. By Γ_I, Γ_O and Γ_W we denote inlet, outlet and impermeable walls, respectively, and assume that

$$
\begin{aligned}
&\text{(i)}\ \ \rho = \rho^*, \quad v_i = v_i^*, \quad i = 1,\ 2, \quad \theta = \theta^* \quad \text{on } \Gamma_I, \qquad (5)\\
&\text{(ii)}\ \ v_i = 0, \quad i = 1,\ 2, \qquad \frac{\partial\theta}{\partial n} = 0 \quad \text{on } \Gamma_W,\\
&\text{(iii)}\ \ \sum_{i=1}^{2} \tau_{ij} n_i = 0, \quad j = 1,\ 2, \quad \frac{\partial\theta}{\partial n} = 0 \quad \text{on } \Gamma_O.
\end{aligned}
$$

Here $\partial/\partial n$ denotes the derivative in the direction of unit outer normal $\boldsymbol{n} = (n_1, n_2)$ to $\partial\Omega$; w^0, ρ^*, v_i^* and θ^* are given functions.

Let us note that nothing is known about the existence and uniqueness of the solution of problem (1.1), (1.3)–(1.5). Some solvability results for system (1.1) & (1.3) were obtained either for small data or on a very small time interval under simple Dirichlet boundary conditions (for reference, see e. g., [3, Par. 8.10]).

We do not take care of the lack of theoretical results and deal with the numerical solution of the above problem. Since the viscosity μ and heat conductivity

k are small, we treat the diffusion terms on the right hand side of (1.1) as a perturbation of the inviscid Euler system and conclude that a good method for the solution of viscous flow should be based on a sufficiently robust scheme for inviscid flow simulation. Therefore, we will split the complete system (1.1) into inviscid and viscous part:

$$\frac{\partial w}{\partial t} + \sum_{i=1}^{2} \frac{\partial f_i(w)}{\partial x_i} = 0, \tag{6}$$

$$\frac{\partial w}{\partial t} = \sum_{i=1}^{2} \frac{\partial R_i(w, \nabla w)}{\partial x_i} \tag{7}$$

and discretize them separately. First we will pay attention to the inviscid flow problem.

2 Numerical solution of the Euler equations

In what follows we will describe some numerical methods for solving the Euler equations system (6). The first part of this section will be devoted to the finite volume methods, in the second part we will briefly describe truly multidimensional methods based on the method of characteristics, the so-called evolution Galerkin schemes. In the third part we present some numerical experiments for the Euler equations system.

It is easy to realize that $f_j \in C^1\left(D; \mathbb{R}^4\right)$ for $j = 1, 2$. Thus, we can apply the chain rule to the function $f_j\left(w\right)$ and obtain a first order quasilinear system of PDE's

$$\frac{\partial w}{\partial t} + \sum_{j=1}^{2} \mathbb{A}_j\left(w\right) \frac{\partial w}{\partial x_j} = 0, \tag{8}$$

where $\mathbb{A}_j\left(w\right) = \frac{D f_j(w)}{D w}$ are Jacobi matrices of $f_j\left(w\right), \; j = 1, 2.$

Definition 1. Let us consider general first order system of type (8). The system is said to be *hyperbolic*, if for arbitrary vectors $w \in D$ and $\boldsymbol{n} = (n_1, n_2) \in \mathbb{R}^2$ the matrix

$$\mathbb{P}\left(w, \boldsymbol{n}\right) = \sum_{j=1}^{2} n_j \, \mathbb{A}_j\left(w\right)$$

has four real eigenvalues $\lambda_i = \lambda_i\left(w, \boldsymbol{n}\right), \; i = 1, \ldots, 4$, and is diagonizable, i.e. there exists a nonsingular matrix $\mathbb{T} = \mathbb{T}\left(w, \boldsymbol{n}\right)$, s.t.

$$\mathbb{T}^{-1} \cdot \mathbb{P} \cdot \mathbb{T} = \mathbb{D}\left(w, \boldsymbol{n}\right) = \mathrm{diag}(\lambda_1, \lambda_2, \lambda_3, \lambda_4)$$

Theorem 2. *The system of Euler equations* (8) *is hyperbolic.*
The eigenvalues of the matrix $\mathbb{P}\left(w, \boldsymbol{n}\right)$ *are*

$$\lambda_1 = \lambda_2 = n_1 v_1 + n_2 v_2, \;\; \lambda_3 = \lambda_1 + a|\boldsymbol{n}|, \quad \lambda_4 = \lambda_1 - a|\boldsymbol{n}|.$$

Here a is a local speed of sound, i.e. $a = \sqrt{\frac{kp}{\rho}}$.

2.1 Finite volume schemes

The above properties of the Euler equations allow us to construct efficient numerical schemes for the solution of inviscid flow. We will carry out the discretization of system (6) with the use of the finite volume method (FVM) which is very popular because of its flexibility and applicability.

Let $\mathcal{T}_h$ be a triangulation of the domain Ω_h which is a polygonal approximation of the domain Ω . The so-called dual finite volume partition of Ω_h will be denoted by $\mathcal{D}_h = \{D_i\}_{i\in J}$, J is a suitable index set. Moreover, it holds

$$\overline{\Omega}_h = \bigcup_{i\in J} D_i. \tag{9}$$

The dual finite volumes will be constructed in the following way: Join the centre of gravity of every triangle $T \in \mathcal{T}_h$, containing the vertex P_i, with the centre of every side of T containing P_i. If $P_i \in \partial\Omega_h$, then we complete the obtained contour by the straight segments joining P_i with the centres of boundary sides that contain P_i. In this way we get the boundary ∂D_i of the finite volume D_i. (See Figure 1.) Dual finite volume meshes were successfully used in a number of works. See, e.g., [1], [8].

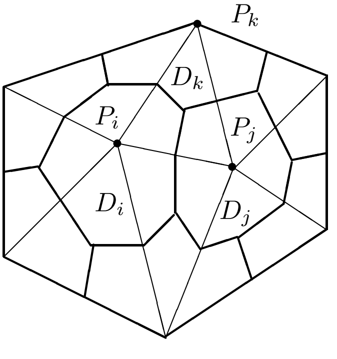

Figure 1

If for two different finite volumes D_i and D_j their boundaries contain a common straight segment, we call them *neighbours*. Then we write

$$\Gamma_{ij} = \partial D_i \cap \partial D_j = \Gamma_{ji}. \tag{10}$$

The index set of all neighbours for the dual volume D_i will be denoted by $S(i)$. Furthermore, we introduce the following notation: $|D_i|$ = area of $D_i, \boldsymbol{n}_{ij} = (n_{1ij},\, n_{2ij})$ = unit outer normal to ∂D_i on Γ_{ij}, ℓ_{ij} = length of Γ_{ij}, and consider a partition $0 = t_0 < t_1 < \ldots$ of the time interval $(0, T)$ and set $\tau_k = t_{t+1} - t_k$ for $k = 0, 1, \ldots$.

The *finite volume method* reads

$$w_i^{k+1} = w_i^k - \frac{\tau_k}{|D_i|} \sum_{j \in S(i)} g(w_i^k,\, w_j^k,\, \boldsymbol{n}_{ij})\, \ell_{ij}, \qquad (11)$$

$$D_i \in \mathcal{D}_h \text{ (i. e., } i \in J),\ k = 0,\, 1, \ldots .$$

To derive (11) we integrate (6) over every set $D_i \times (t_k,\, t_{k+1})$, use Green's theorem, the approximation of the exact solution by a piecewise constant function with values w_i^k on $D_i \times \{t_k\}$ and the approximation of the flux

$$\int_{\Gamma_{ij}} \sum_{r=1}^{2} f_r(w)\, n_r \,\mathrm{d}S$$

of the quantity w through the segment Γ_{ij} in the direction $\boldsymbol{n}_{ij}$ with the aid of the so-called *numerical flux* $g(w_i^k,\, w_j^k,\, \boldsymbol{n}_{ij})$ calculated from $w_i^k,\ w_j^k$ and $\boldsymbol{n}_{ij}$.

In order to ensure the stability of the scheme (11) the so-called CFL stability condition has to be fulfilled

$$\frac{\tau_k}{|D_i|} \max_{j \in S(j)} \max_{s=1,\ldots,4} \lambda_s(w^k, \boldsymbol{n}_{ij}) \le \mathrm{CFL} \qquad \forall j \in J, \qquad (12)$$

where CFL $\in (0, 1]$. In literature one can find a lot of numerical flux functions, e.g. Steger-Warming, Osher-Solomon, Van-Leer, Vijayasundaram numerical fluxes. For references, see e.g., [3].

We do not discuss now the question of implementation of the boundary conditions. They have to be prescribed in such a way that the hyperbolic character of the equations is taking into account. For more details the reader is referred to [4]. In the approach described about only the piecewise constant approximation is considered. Nevertheless, also the higher order schemes, using for example discontinuous piecewise linear approximate functions, can be constructed. The details can be found, e.g., in [4].

2.2 Evolution Galerkin methods

Although in the recent years the most commonly used methods for hyperbolic problems are the finite volume methods, it turns out that in special cases this approach leads to structural deficiencies in the solution (see, e.g., [7], [13]). This is due to the fact that the finite volume methods are based on a quasi dimensional splitting using one-dimensional Riemann solvers.

The evolution Galerkin method, first considered by Morton *et al.* in [2] for scalar hyperbolic equation, combines the theory of characteristics for hyperbolic problems with the finite element ideas. The initial function is transported along the characteristic cone and then projected onto a finite element space.

Let $E(t)$ be the exact evolution operator for our hyperbolic problem (8), i.e.

$$w(\cdot, t_{k+1}) = E(\Delta t) w(\cdot, t_k), \qquad (13)$$

then the *evolution Galerkin scheme* reads:

$$w^{k+1} = P_h E_\Delta w^k, \tag{14}$$

where E_Δ is an approximate evolution operator and P_h is a projection onto a finite element space. It can be shown that the method is unconditionally stable and the accuracy can be increased by increasing the order of the approximate space and the accuracy of the approximate evolution operator. Using different approximate evolution operators E_Δ and projections P_h one obtains a class of the evolution Galerkin methods.

The approach described can be fully exploited for simple problems, e.g. the linear hyperbolic system of wave equation (see Lukáčová, Morton and Warnecke [9], [10], [11]). More details about the application of this method to the Euler equations can be found in the works of Fey [7] and Ostkamp [13].

2.3 Numerical experiments

(1) Flow through the GAMM channel (10 % circular arc in the channel of width 1 m) for air, i. e. $\gamma = 1.4$, and inlet Mach number $M := \frac{|\boldsymbol{v}|}{a} = 0.67$ was solved by the Vijayasundaram higher order scheme applied on the dual mesh over a triangular grid. In Figure 2 the basic grid and dual mesh, respectively, are shown. Our aim was to obtain a steady state solution with the aid of the time marching process for $t_k \to \infty$. After 10000 time iterations the stability of the solution up to 10^{-5} was achieved. Figure 3 shows Mach number isolines and entropy isolines. We can see a sharp shock wave which is resolved very well.

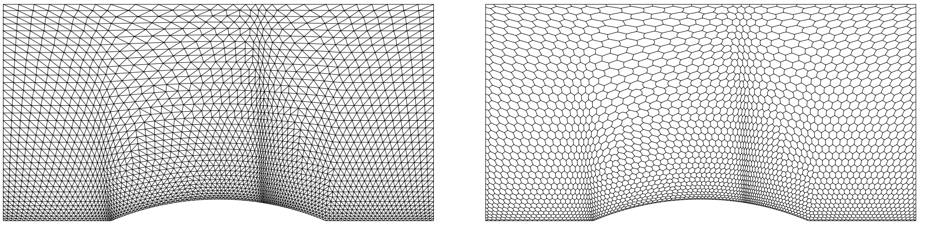

Figure 2: Triangular mesh in the GAMM channel and the dual mesh

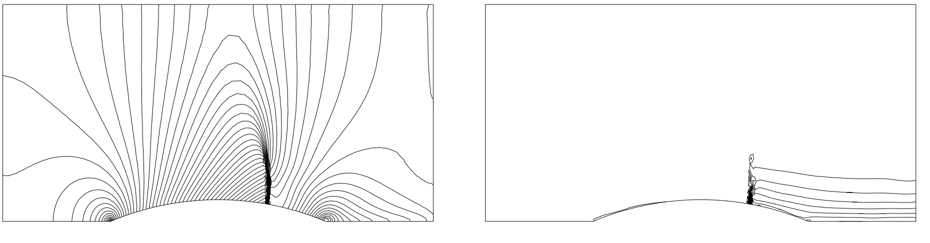

Figure 3: Mach number isolines and entropy isolines

(2) Two-dimensional Sod's problem. Now we will show a comparison of the solution obtained by the finite volume method and by the evolution Galerkin scheme. It will be showed that some symmetry structures are better preserved by the truly multidimensional evolution Galerkin method than by the finite volume scheme.

The computational domain is the square [-1,1] × [-1,1]. To ensure the CFL stability condition, the CFL number is taken 0.8. We choose periodical boundary conditions and the following initial data

$$\begin{aligned} &\rho = 1, u = 0, v = 0, p = 1 && \text{if } |x| \leq 0.4 \\ &\rho = 0.125, u = 0, v = 0, p = 0.1 && \text{otherwise.} \end{aligned} \tag{15}$$

In Figure 4 the first picture on the left hand side shows the isolines of pressure for the solution computed by the evolution Galerkin scheme at time $T = 0.2$ for quadrilateral grid with 200 × 200 grid cells. The symmetry of the data can be observed very well. The resolution of the flow phenomena is the same in all directions and information is moving in infinitely many directions in a circular manner. However this is not the case for the finite volume method. In the next two pictures of Figure 4 the isolines of pressure for the solution computed by the Osher-Solemn finite volume scheme on the quadrilateral mesh (middle) and on the dual mesh (right hand side) are plotted.

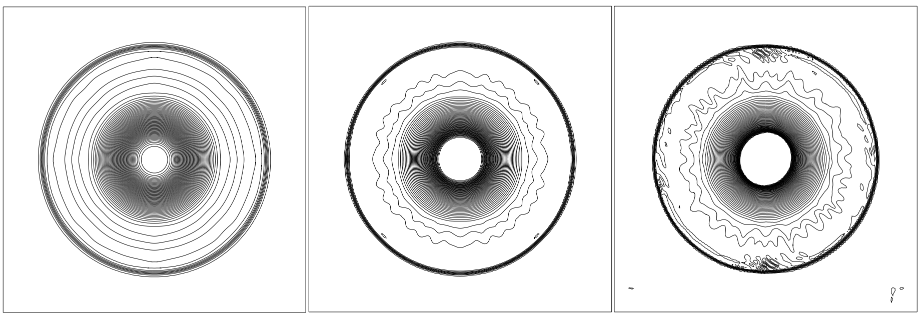

Figure 4: Evolution Galerkin scheme
and the Osher-Solomon FVM on the quadrilateral mesh and the dual mesh

3 Discretization of the complete system of the Navier-Stokes equations

In this section we will describe the combined finite volume – finite element method which is used for the discretization of the Navier-Stokes equations (1.1). Let us note that we now use only the finite volume method in order to discretize the Euler equations, however also other possibilities are open (cf. Section 2.2 Evolution Galerkin methods).

First of all we will describe the finite element discretization of the purely viscous system (7) equipped with initial conditions (1.4) and boundary conditions (1.5). We use conforming piecewise linear finite elements. This means that the components of the state vector are approximated by functions from the finite dimensional space

$$X_h = \left\{\varphi_h \in C(\overline{\Omega}_h);\ \varphi_h|_T \text{ is linear for each } T \in \mathcal{T}_h\right\}.$$

Further, we set $\boldsymbol{X}_h = [X_h]^4$ and

a) $\boldsymbol{V}_h = \{\varphi_h = (\varphi_1, \varphi_2, \varphi_3, \varphi_4) \in \boldsymbol{X}_h,\ \varphi_i = 0$ on the part of $\partial\Omega_h$ approximating the part of $\partial\Omega$ where w_i satisfies the Dirichlet condition$\}$

b) $\boldsymbol{W}_h = \{w_h \in \boldsymbol{X}_h;$ its components satisfy the Dirichlet boundary conditions following from (1.5)$\}$.

Multiplying (7) considered on time level t_k by any $\varphi_h \in \boldsymbol{V}_h$, integrating over Ω_h, using Green's theorem, taking into account the boundary conditions (1.5) and approximating the time derivative by a forward finite difference, we obtain the following *explicit scheme* for the calculation of an approximate solution w_h^{k+1} on the $(k+1)$-th time level

$$\text{a)}\ \ w_h^{k+1} \in \boldsymbol{W}_h, \tag{16}$$

$$\text{b)}\ \ \int_{\Omega_h} w_h^{k+1}\varphi_h \,\mathrm{d}x = \int_{\Omega_h} w_h^k\,\varphi_h \,\mathrm{d}x - -\tau_k \int_{\Omega_h} \sum_{s=1}^{2} R_s(w_h^k,\, \nabla w_h^k)\,\frac{\partial \varphi_h}{\partial x_s}\,\mathrm{d}x \quad \forall\,\varphi_h \in \boldsymbol{V}_h.$$

The integrals are approximated by a numerical quadrature, called *mass lumping*, using the vertices of triangles as integration points:

$$\int_T F\,\mathrm{d}x \approx \frac{1}{3}|T| \sum_{i=1}^{3} F(P_T^i) \tag{17}$$

for $F \in C(T)$ and a triangle $T = T(P_T^1,\, P_T^2,\, P_T^3) \in \mathcal{T}_h$ with vertices $P_T^i,\ i = 1,\, 2,\, 3$. The numerical integration yields

$$w_i^{k+1} = w_i^k - \frac{\tau_k}{|D_i|} \sum_{s=1}^{2} \sum_{T\in\mathcal{T}_h} |T| R_s^k\Big|_T \frac{\partial \varphi_i^m}{\partial x_s}\Big|_T, \tag{18}$$

where $i \in J,\ k = 0, 1, \dots,\ \mathrm{m} = 1,\ \dots,\ 4$, and φ_i^m is a basis function from $\boldsymbol{V}_h$ having the only non-zero component on the m-th position; namely $\varphi_i \in X_h$, which corresponds to the vertex P_i.

Now we combine the finite volume scheme (11) with the finite element scheme (18). The resulting finite volume – finite element operator splitting scheme has the following form:

$$
\begin{aligned}
&w_i^0 = \frac{1}{|D_i|}\int_{D_i} w(x,y,0),\\
&w_i^{k+1/2} = w_i^k - \frac{\tau_k}{|D_i|}\sum_{j\in S(i)} g(w_i^k,\, w_j^k,\, \boldsymbol{n}_{ij})\,\ell_{ij},\\
&w_i^{k+1} = w_i^{k+1/2} - \frac{\tau_k}{|D_i|}\sum_{s=1}^{2}\sum_{T\in\mathcal{T}_h} |T| R_s^{k+1/2}\Big|_T \frac{\partial\varphi_i^m}{\partial x_s}\Big|_T,\\
&i\in J,\ m=1,\dots,4,\ k=0,1,\dots.
\end{aligned}
\tag{19}
$$

The above scheme can be applied only under some stability conditions. In the case of explicit discretization of the viscous terms we have to consider not only (12) but also the additional stability condition in the form

$$
\frac{3}{4}\frac{h}{\rho}\frac{\tau_k}{|T|}\max(\mu,k) \le \mathrm{CFL}, \quad T\in\mathcal{T}_h, \tag{20}
$$

where h is the length of the maximal side in $\mathcal{T}_h$ and $\rho = \min_{T\in\mathcal{T}_h}\rho_T$, ρ_T = radius of the largest circle inscribed into T.

Concerning the theoretical results we are able to prove the convergence and the error estimates for the combined finite volume – finite element method. These results are obtained for one scalar nonlinear convection – diffusion equation. The convergence was proved by Feistauer, Felcman and Lukáčová in [5] and by Lukáčová in [12]. Using the piecewise constant approximate functions in the finite volume step and the piecewise linear approximation in the finite element step it is possible to show that the method is of first order, see Feistauer, Felcman, Lukáčová and Warnecke [6].

3.1 Computational Results

Viscous flow through the GAMM channel for $\gamma = 1.4$, $\mu = 1.72\cdot 10^{-5}\,\mathrm{kg\,m^{-1}\,s^{-1}}$, $\lambda = -1.15\cdot 10^{-5}\,\mathrm{kg\,m^{-1}\,s^{-1}}$, $k = 2.4\cdot 10^{-2}\,\mathrm{kg\,m\,s^{-3}}\,K^{-1}$, $c_v = 721.428\,J\cdot\mathrm{kg}\cdot K^{-1}$ and the inlet Mach number $M = 0.67$ was computed by scheme (19). In Figure 5 Mach number isolines are drawn. Here we can see boundary layer at the walls, shock wave, wake and interaction of the shock with boundary layer.

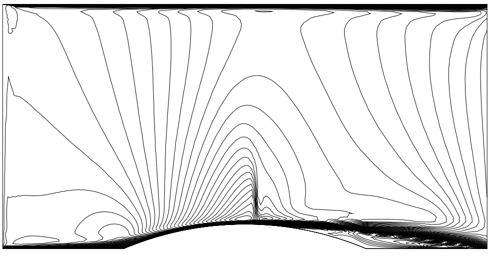

Figure 5: Mach number isolines of viscous flow

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