Vera Nikolaevna Maslennikova

The asymptotic behavior at the large time of solutions of some mathematical physics problems

In: Miloš Ráb and Jaromír Vosmanský (eds.): Proceedings of Equadiff III, 3rd Czechoslovak Conference on Differential Equations and Their Applications. Brno, Czechoslovakia, August 28 - September 1, 1972. Univ. J. E. Purkyně - Přírodovědecká fakulta, Brno, 1973. Folia Facultatis Scientiarum Naturalium Universitatis Purkynianae Brunensis. Seria Monographia, Tomus I. pp. 189--197.

Persistent URL: http://dml.cz/dmlcz/700057

THE ASYMPTOTIC BEHAVIOR AT THE LARGE TIME OF SOLUTIONS OF SOME MATHEMATICAL PHYSICS PROBLEMS

by V. N. MASLENNIKOVA

In this lecture I am going to treat some results concerning asymptotic behavior of solutions for three types of equations and systems, recently obtained at the Mathematical Institute of the Academy of Sciences of the USSR in Moscow.
They are

1. the hydrodynamics systems of a rotating fluid and magnetohydrodynamics.
2. the parabolic equations of the second order.
3. the external boundary value problems for the wave equation.

The number of questions' we are going to consider in our report, will concern the decrease of a solution with this or that velocity when $t \to \infty$ or the stabilization of a solution.

Here we shall not touch upon the problems of the periodicity of solutions with respect to t, almost-periodicity and so on.

1. THE HYDRODYNAMICS SYSTEMS OF A ROTATING FLUID AND MAGNETOHYDRODYNAMICS

We shall give a series of theorems concerning the velocity of a curl decay in different continuous media, the movement in which is described by the systems mentioned below. In the first place we consider Sobolev system [1] for linearized equations of a rotating fluid

$$\left.\begin{aligned} &\frac{\partial \mathbf{v}}{\partial t} - [\mathbf{v}, \boldsymbol{\omega}] + \operatorname{grad} P = 0, \\ &\operatorname{div} \mathbf{v} = 0, \end{aligned}\right\} \tag{1}$$

in the domain $\{x \in E_3,\ t > 0\}$, where $\mathbf{v}(x, t) = (v_1, v_2, v_3)$, $[.,.]$ denotes the vector product, $\boldsymbol{\omega} = (0, 0, \omega)$ is a constant vector of angular velocity.
For a solution of the Cauchy problem with initial conditions

$$\mathbf{v}(x, t)\,\big|_{t=0} = \mathbf{v}^0(x), \quad \operatorname{div} \mathbf{v} = 0 \tag{2}$$

we obtained in [2] the explicit representation of the solution of (1), by means of which we have proved the following

Theorem 1. *If the initial data* (2) $\mathbf{v}^0(x) \in \mathring{C}^\infty(E_3)$, *then on any compact* $\Omega \subset E_3$, $x \in \Omega$ *the solution* $\mathbf{v}(x, t)$, $P(x, t)$ *of the Cauchy problem for the homogeneous system* (1) *decreases for* $t \to \infty$ *as* $O(1/t)$ *uniformly over* $x \in \Omega$.

If we introduce effect of compressibility in system, i.e. if we consider the system

$$\left.\begin{aligned} &\frac{\partial \mathbf{v}}{\partial t} - [\mathbf{v}, \omega] + \operatorname{grad} P = 0, \\ &\alpha^q \frac{\partial P}{\partial t} + \operatorname{div} \mathbf{v} = 0, \end{aligned}\right\} \tag{3}$$

(which describes, in particular, the propagation of the acoustic waves) with initial conditions

$$\mathbf{v}(x, t)\,|_{t=0} = \mathbf{v}^0(x), \quad P(x, t)\,|_{t=0} = P^0(x) \tag{4}$$

then we have the following theorem (see [3], [5], [6]) for a solution of the problem (3), (4)

Theorem 2. *If the initial data* $\mathbf{v}^0(x), P^0(x) \in \mathring{C}^\infty(E_3)$, *then the solution of the Cauchy problem for the homogeneous system* (3) *decreases as* $O(1/t)$ *for* $t \to \infty$ *uniformly over* $x \in \Omega$, *where* Ω *is any compact, belonging* E_3.

It is possible for the conditions of the finite support and infinite differentiability of initial conditions in the theorems 1 and 2 to be replaced by the sufficiently quick decrease for $|x| \to \infty$ and by some finite smoothness.

The fundamental solutions of the systems (1) and (3) are expressed by the Bessel function with respect to argument $\frac{\varrho\sqrt{t^2 - \alpha^2 r^2}}{r}$ (for the system (1), i.e. as $\alpha = 0$, this argument passes to $\varrho t/r$), where $r^2 = \varrho^2 + x_3^2$, $\varrho^2 = x_1^2 + x_2^2$; therefore the solutions of the systems (1) and (3) oscillate strongly, and the principal members of a asymptotic expansion of solution for a system (1) depend on t as $\frac{\sin t + \cos t}{t}$ with the corresponding coefficients, depending on the initial data; the corresponding expansion of solution for a system (3) contains retarded argument $\sqrt{t^2 - \alpha^2 r^2}$ under symbols sin and cos. We have also investigated the asymptotic behavior of a solution at the large time for system (1) in unbounded domain—half space $\{x_3 \geqq 0\}$ for two boundary value problems

$$P(x, t)\,|_{x_3=0} = 0 \tag{5}$$

or

$$v_3(x, t)\,|_{x_3=0} = 0 \tag{6}$$

with initial date (2).

It has been proved, that function $P(x, t)$ in the problem (2), (5) damps with the same velocity as in the case of the Cauchy problem, i.e. as $O(1/t)$; however vector $\mathbf{v}(x, t)$, which is not imposed upon by the condition on the boundary, may not damp for $t \to \infty$ (an example of periodic solution with respect to t has been constructed). The boundary condition on $v_3(x, t)$ in the problem (2) (6) induces the homogeneous boundary condition on the function $P(x, t)$:

$$\frac{\partial^3 P}{\partial t^2 \partial x_3} + \frac{\partial P}{\partial x_3}\bigg|_{x_3=0} = 0,$$

therefore it has been proved, that functions $P(x, t)$ and $v_3(x, t)$ in the problem (2), (6) decrease as $O(1/t)$ for $t \to \infty$.

It is of great interest to study the asymptotic behavior of a solution as $t \to \infty$ for the hydrodynamics systems of a rotatig viscous fluid

$$\left.\begin{aligned} &\frac{\partial \mathbf{v}}{\partial t} - [\mathbf{v}, \boldsymbol{\omega}] - \nu \Delta \mathbf{v} + \operatorname{grad} P = 0, \\ &\operatorname{div} \mathbf{v} = 0, \end{aligned}\right\} \tag{7}$$

with initial conditions (2), where ν-viscosity coefficient. In the papers [4], [7] for the problem (7), (2) the following has been proved

Theorem 3. *Let the initial data* (2) *be* $\mathbf{v}^0(x) \in L_1(E_3)$. *Then the solution of the Cauchy problem for the homogeneous system* (7) *decreases uniformly over* $x \in E_3$ *for* $t \to \infty$ *in the following way: vector* $\mathbf{v}(x, t)$ *decreases as* $O(1/t^{5/2})$, *and function* $P(x, t)$ *—as* $O(1/t^2)$; *at the large t and* ν, $\omega > 0$ *we have obtained the following estimates*:

$$|\mathbf{v}(x, t)| \leqq \frac{C_1}{(\nu t)^{3/2} \omega t} \| \mathbf{v}^0 \|_{L_1(E_3)}, \tag{8}$$

$$|P(x, t)| \leqq \frac{C_2}{(\nu t)(\omega t)} \| \mathbf{v}^0 \|_{L_1(E_3)}, \tag{9}$$

where C_i*—constants, independent on* t, x, $\mathbf{v}^0$.

The formulas (8), (9) show, that the decrease of a solution of the order $1/\omega t$ takes place because of Coriolis term $[\mathbf{v}, \boldsymbol{\omega}]$ and the order $1/(\nu t)^{3/2}$ (for vector $\mathbf{v}(x, t)$)— owing to viscosity.

It was of great interest to investigate in what way the asymptotics might changed when changing the dimension of x-space.

In the case $x \in E_2$, if axis x_3 is a rotation axis and the functions contained in the systems (7) depend on x_1 and x_3 (or x_1 and x_3), then we have the following

Theorem 4. *Let the initial date* $\mathbf{v}^0(x_1, x_3) \in L_1(E_2)$ *and* $\operatorname{div} \mathbf{v}^0 = 0$. *Then a solution of the Cauchy problem for the homogeneous system* (7) *uniformly over* $x = (x_1, x_3) \in E_2$ *decreases for* $t \to \infty$ *in the following way: vector* $\mathbf{v}(x, t)$ *decreases as* $O(1/t^{3/2})$, *and function* $P(x, t)$ *as* $O(1/t)$; *at the large t and* ν, $\omega > 0$ *the following estimates*

$$\begin{aligned} &|\mathbf{v}(x, t)| \leqq \frac{C_3}{(\nu t)\sqrt{\omega t}} \| \mathbf{v}^0 \|_{L_1(E_2)}, \\ &|P(x, t)| \leqq \frac{C_4}{\sqrt{\nu t}\sqrt{\omega t}} \| \mathbf{v}^0 \|_{L_1(E_2)}, \end{aligned} \tag{10}$$

hold, where C_3, C_4*—constant, independent on* $\mathbf{v}^0$, t, x.

Thus in the case of two space variable the confluence of a curl originates slower on $\dfrac{1}{\sqrt{\nu t}\sqrt{\omega t}}$ in a comparison with the three-dimensional case.

Ju. N. Drožžinov has considered the solution of system (3) with the initial data independent on x_3 (at that time of axis x_3 remaining a rotation axis) i.e. $\mathbf{v}^0(x_1, x_2)$, $P^0(x_1, x_2)$. He has proved, that if initial data satisfy the additional condition $\operatorname{rot}\mathbf{v}^0 + \boldsymbol{\omega}P^0 = 0$ and $\mathbf{v}^0$, P^0 are finite over x_1, x_2 and sufficiently smooth, then a solution of the system (3) decreases on compactum as $O(1/t)$. Or, if $\operatorname{rot}\mathbf{v}^0 + \boldsymbol{\omega}P^0 \neq 0$, it approaches the stationary solution with the same velocity.

He has also considered a linearized system of magnetohydrodynamics in the three dimensional space on x (compare with the system (3))

$$\left.\begin{aligned} &\frac{\partial \mathbf{v}}{\partial t} - [\operatorname{rot}\mathbf{H}, \boldsymbol{\omega}] + \operatorname{grad} P = 0,\\ &\frac{\partial \mathbf{H}}{\partial t} - \operatorname{rot}[\mathbf{v}, \boldsymbol{\omega}] = 0,\\ &\alpha^2\frac{\partial P}{\partial t} + \operatorname{div}\mathbf{v} = 0, \end{aligned}\right\} \tag{11}$$

where all notations are previous and $\mathbf{H} = (H_1, H_2, H_3)$. He has proved

Theorem 5. *Let the initial data*

$$\mathbf{u}(x, t)\big|_{t=0} = \mathbf{u}^0(x) = (\mathbf{v}^0, \mathbf{H}^0, P^0) \in \mathring{C}^\infty(E_3),$$

and fulfil the agreement condition $\operatorname{div}\mathbf{H}^0 = 0$. *Then a solution of the Cauchy problem for the system* (11) *decreases as* $O(1/t)$ *for* $t \to \infty$ *on any compactum.*

Thus the movement of a curl described by systems (1), (3) or (11) has the same velocity of decay that equals $O(1/t)$ *for* $t \to \infty$.

2. THE PARABOLIC EQUATIONS OF THE SECOND ORDER

Consider a linear uniformly parabolic equation of the second order

$$\begin{aligned} u_t = \operatorname{div}_x(A(t, x)\operatorname{grad}_x u(t, x)) + (B(t, x), \operatorname{grad}_x u(t, x)) +\\ + c(t, x)\,u(t, x),\ t > 0, \quad x = (x_1, \ldots, x_n) \in \Omega, \end{aligned} \tag{12}$$

where $A(t, x)$-symmetric matrix, Ω an unbounded domain with Ljapunov boundary. The coefficients of a equation (12) are sufficiently smooth bounded functions, satisfying the conditions:

$$c(t, x) \leqq 0, \quad 0 \leqq \operatorname{div}_x B(t, x) - c(t, x) \leqq M,$$

where M a certain constant.

Below we shall assume the solutions to be bounded or belonging to the uniqueness class in the case of unbounded initial data. At first we shall consider the Cauchy problem (i.e. $\Omega = R_n$) for heat conduction equation $A(t, x) \equiv E$, $B(t, x) \equiv c(t, x) \equiv 0$ with the initial condition

$$u(t, x)\big|_{t=0} = \varphi(x)\,, \tag{130}$$

where $\varphi(x)$ continuous function of the uniqueness class. It follows from Tauberian theorem of Viener [8], that the existence limit

$$\lim_{R\to\infty} \frac{1}{\operatorname{mes}\{|x| < R\}} \int\limits_{|x-\xi|<R} \varphi(\xi)\,\mathrm{d}\xi = a(x) \tag{14}$$

is necessary and sufficient for stabilization of solution $u(t, x)$ for $t \to \infty$ in the class of bounded initial functions. In this case it should be emphasized that $a(x)$ in (14) may be only constant.

In this case

$$\lim_{t\to\infty} u(t, x) = a \tag{15}$$

In [9] the analogous result was established for semi-bounded initial functions. Naturally there appears a problem to find necessary and sufficient condition for stabilization of a solution in the case of arbitrary initial function belonging to the uniqueness class.

This problem was solved in [10], where it was proved, that the condition (14) was criterion of the uniform in R_n stabilization of a solution as well as in the case of unbounded initial functions; moreover a function $a(x)$ was found harmonic and $u(t, x) \to a(x)$ for $t \to \infty$.

We shall also give one of the results [10], relating to uniform stabilization $u(x, t)$: on any compact from E_n : if $|\varphi(x)| \leqq C(|x|^s + 1)$ for certain $C \geqq 0$ and $s \geqq 0$ then the fulfilment of relation

$$\frac{1}{\omega_n B(\alpha, n)\, R^{n+\alpha+1}} \int_0^R (R - \varrho)^{\alpha-1} \varrho^{n-1} \int\limits_{|\omega|=1} \varphi(x + \varrho\omega)\,\mathrm{d}\omega\,\mathrm{d}\varrho \to a(x),$$

for $\alpha > s$ is necessary and sufficient for the stabilization of solution $u(x, t)$.

Let us consider now the Cauchy problem for variable coefficients. We shall assume the initial function bounded. If the coefficients of equations (12) depend on t only, then it is easy to obtain explicit form of the fundamental solution by the method of Fourier transform; this gives a possibility to apply the Tauberian theorem of Viener. It is more difficult in the case when the coefficients of equations depend on x. For the equation (12) with the coefficients depending on x the problem was solved only in the simplest case $B(t, x) \equiv 0$, $c(t, x) \equiv 0$, $A(t, x) = \dfrac{1}{p(x)} E$. In [11], [12] it was proved: if there is a constant $b > 0$, satisfying uniformly at $x \in E_n$ the condition

$$\frac{1}{\operatorname{mes}\{|x| < R\}} \int\limits_{|x-\xi|<R} |P(\xi) - b| \, d\xi \to 0 \quad \text{as} \quad R \to \infty,$$

then (15) is satisfied for some $x \in E_n$ (uniformly on any compactum or in E_n) if and only if the limit (14) exists at this point x (uniformly on any copmactum or in E_n respectively). In case $n = 1$, i.e. if we consider the equation $p(x)\, u_t = u_{xx}$, in [13], [14] the analogous results were established for lesser assumption on function $p(x)$. In particular it was proved: if the limits

$$\lim_{T\to\infty} \frac{1}{T} \int\limits_0^T p(\xi)\, d\xi = b_1^2, \qquad \lim_{T\to\infty} \frac{1}{T} \int\limits_{-T}^0 p(\xi)\, d\xi = b_2^2$$

exist, then existence of the limit

$$\lim_{T\to\infty} \frac{1}{(b_1 + b_2)\, T} \int\limits_{-T/b_2}^{T/b_1} \varphi(\xi)\, p(\xi)\, d\xi = a,$$

is criterion of the existence of the limit (15) uniformly on any compact. Now we consider boundary value problem. Let $u(t, x)$ be solution of a equation (12) in an unbounded domain Ω, satisfying the initial condition (13) and boundary condition

$$\left. \frac{\partial u(t, x)}{\partial N} + q(t, x)\, u(t, x) \right|_{x \in \Gamma} = 0 \tag{16}$$

with the non-negative continuous bounded function $q(t, x)$, for which $-q(t, x) + (B(t, x), \nu(x))\,|_{x\in\Gamma} \leqq 0$ where $\nu(x)$-unit vector of the exterior normal to Γ, and N be conormal. In [15] in case non-compact boundary Γ it was established the estimates for Green function $G(t, x; \tau, \xi)$ and for solution of the problem (12), (13), (16) with the initial function from L_1. If $B(t, x) \equiv 0$, $c(t, x) \equiv q(t, x) \equiv 0$ the obtained estimates are exact with respect to the order of the tending of a solution to zero for $t \to \infty$. In particular under some condition on domain the following inequalities

$$\frac{c_1}{v_0(\sqrt{t})} \leqq \frac{c_0}{v_\xi(\sqrt{t})} \leqq \sup_{x\in\Omega} G(t, x; 0, \xi) \leqq \frac{C_0}{v_0(\sqrt{t})} \tag{17}$$

are true, where $v_\xi(R) = \operatorname{mes}\{(|x - \xi| < R) \cap \Omega\}$, constant C_0 depends on elliptic constant only, and c_0, c_1 depends on point ξ as well. For example, in case $\Omega = \{x_1, x_2; x_2 > f(x_1)\} \subset E_2$ where $f(x_1)$ is arbitrary smooth even function with the monotonically non-decreasing derivative, the estimates (17) have form

$$\text{const} \,.\, \max\{t^{-1}(\sqrt{t}\, f_{-1}(\sqrt{t}))^{-1}\} \leqq \sup_{x\in\Omega} G(t, x; 0, \xi) \leqq$$
$$\leqq \text{const} \,.\, \max\{t^{-1}(\sqrt{t}\, f_{-1}(\sqrt{t}))^{-1}\},$$

where $f_{-1}(R)$ is inverse for $f(x_1)$ function.

3. THE EXTERNAL BOUNDARY VALUE PROBLEMS FOR THE WAVE EQUATION

Let Ω be exterior of compact in E_3 or in E_2 with the boundary $\Gamma \subset C^2$ and $u(x,t)$ be solution in $\Omega \times (0 < t)$ of a wave equation

$$\partial^2 u/\partial t^2 = \Delta u \tag{18}$$

satisfying initial data

$$\left.\frac{\partial u(x,t)}{\partial t}\right|_{t=0} = f_0(x), \qquad u(x,t)\,|_{t=0} = f_1(x) \tag{19}$$

and one of the boundary conditions

$$u(x,t)\,|_{x\in\Gamma} = 0 \tag{20}$$

$$\partial u(x,t)/\partial n\,|_{x\in\Gamma} = 0 \tag{21}$$

$$\partial u(x,t)/\partial n + g(x)\,u(x,t)\,|_{x\in\Gamma} = 0 \tag{22}$$

where n is the exterior (relatively Ω) normal at Γ, function $g(x)$ is non-negative and continuously differentiable on Γ.

In [16] in case three space variables it was established the exponential decrease uniformly on any compact $K \subset \bar{\Omega}$ for $t \to \infty$ of the solutions of the problems (18) to (22) in the assumption that initial functions f_0, f_1 are finite and the boundary has the positive gaussian curvarute. The asymptotic behavior for $t \to \infty$ of the solutions of the problem (18)–(21) and (18)–(22) in the case of two space variables was investigated in [17–19]. Let $u_\alpha(x,t)$, $\alpha = 0, 1$ be a solution of the problems (18), (19), (21) or (18), (19), (22) at $f_{1-\alpha} \equiv 0$. In the above writings it was proved the following

Theorem 6. *Let Γ be convex boundary and functions $f_\alpha(x) \in C^{1+\alpha}(\bar{\Omega})$ for $\alpha = 0, 1$ have the bounded support and satisfy the agreement conditions $\partial f_\alpha(x)/\partial n + g(x) f_\alpha(x)\,|_\Gamma = 0$, $\alpha = 0, 1$. Then for any $x \in \bar{\Omega}$ for $t \to \infty$ for the problem* (18), (19), (21) *there takes place the equality*

$$u_\alpha(x,t) = \frac{(-1)^\alpha}{2\pi t^{1+\alpha}} \int_\Omega f_\alpha(y)\,\mathrm{d}y + (-1)^{1+\alpha}(2+\alpha)!\,\frac{\mathrm{mes}\,(E_2\backslash\Omega)}{\pi} \times$$
$$\times \frac{\ln t}{t^{3+\alpha}} \int_\Omega f_\alpha(y)\,\mathrm{d}y + \sum_{n=1}^{N}\sum_{m=0}^{n} c_{m,n}^{(\alpha)}(x)\,\frac{\ln^m t}{t^{2n+1+\alpha}} + O\left(\frac{\ln^{N+1} t}{t^{2N+3+\alpha}}\right). \tag{23}$$

For the solution of the problem (18), (19), (22) *there holds the equality*

$$u_\alpha(x,t) = \frac{(-1)^\alpha \varphi_\alpha(t)}{\pi t^{1+\alpha}\ln^2 t}\,v(x) \iint_\Omega \mu(y) f_\alpha(y)\,\mathrm{d}y + \sum_{n=1}^{N}\sum_{m=0}^{n-1} c_{m,n}^{(\alpha)}(x) \times$$
$$\times \frac{\ln^m t}{t^{2n+1+\alpha}} + \sum_{n=1}^{N}\sum_{m=2}^{n+2} d_{m,n}^{(\alpha)}(x)\,\frac{\varphi_{m,n}^{(\alpha)}(t)}{t^{2n+1+\alpha}\ln^m t} + O\left(\frac{\ln^N t}{t^{2N+3+\alpha}}\right). \tag{24}$$

Here the functions $c^{(\alpha)}_{m,n}(x)$ and $d^{(\alpha)}_{m,n}(x)$ are continuous in $\bar{\Omega}$ (in the equality (23) $c^{(\alpha)}_{1,1} = 0$). The functions $v(x)$ and $\mu(x)$ are harmonic in Ω and are continuous in $\bar{\Omega}$.

The functions $\varphi_\alpha(t)$ and $\varphi^{(\alpha)}_{m,n}(t)$ have the finite limits different from zero for $t \to \infty$. The estimates of the remaining terms in the equalities (23), (24) for $x \in \bar{\Omega} \cap \{|x| \leqq R\}$ (where R is an arbitrary sufficiently big number) are uniform up to boundary Γ and depend on Γ, R, f_α and N, where N is any natural number.

REFERENCES

[1] S. L. Sobolev: *Ob odnoj novoj zadače matematičeskoj fiziki.* Izv. AN SSSR, serija matem., 18, No 1, 1954.

[2] V. N. Maslennikova, *Ocenki v L_p i asimptotika pri $t \to \infty$ rešenija zadači Koši dlja sistemy S. L. Soboleva.* Trudy MIAN SSSR, tom 103, 1968.

[3] V. N. Maslennikova: *Javnoe predstavlenie i asimptotika pri $t \to \infty$ rešenija zadači Koši dlja linearizovannoj sistemy vraščajuščejsja sžimaemoj židkosti.* DAN SSSR, tom 187, No 5, 1969.

[4] V. N. Maslennikova: *Rešenie zadači Koši i ego asimptotika pri $t \to \infty$ dlja linearizovannykh uravnenij vraščajuščejsja vjazkoj židkosti,* DAN SSSR, tom 189, No 6, 1969.

[5] V. N. Maslennikova: *Asimptotika pri $t \to \infty$ rešenija zadači Koši dlja odnoj giperboličeskoj sistemy, opisyvajuščej dviženie vraščajuščejsja židkosti.* Differenciáľnye uravnenija, tom VIII, No 1, 1972.

[6] *Ocenki v L_p dlja odnoj giperboličeskoj sistemy.* Sibirskij matematičeskij žurnal, tom 13, No 3, 1972.

[7] V. N. Maslennikova: *O skorosti zatukhanija vikhra v vjazkoj židkosti.* Trudy MIAN SSSR, tom 126, 1973.

[8] N. Viner: *Integral Fur'e i nekotorye ego primenenija,* M., 1963.

[9] Ju. N. Drožžinov: *Stabilizacija rešenij obobščennoj zadači Koši dlja uľtraparaboličeskogo uravnenija.* Izv. AN SSSR, ser. matem. 33, No 2, 1969.

[10] V. P. Mikhajlov: *O stabilizacii rešenij zadači Koši dlja uravnenija teploprovodnosti,* DAN SSSR, 190, No 1, 1970.

[11] A. K. Guščin, V. P. Mikhajlov: *O stabilizacii rešenija zadači Koši dlja paraboličeskogo uravnenija,* DAN SSSR, 194, No 3, 1970.

[12] A. K. Guščin, V. P. Mikhajlov: *O stabilizacii rešenija zadači Koši dlja paraboličeskogo uravnenija,* Differenciaľnye uravnenija, 7, No 2, 1971.

[13] A. K. Guščin, V. P. Mikhajlov: *O stabilizacii rešenija zadači Koši dlja odnomernogo paraboličeskogo uravnenija,* DAN SSSR, 197, No 2, 1971.

[14] A. K. Guščin, V. P. Mikhajlov: *O stabilizacii rešenija zadači Koši dlja paraboličeskogo uravnenija s odnoj prostranstvennoj peremennoj,* Trudy MIAN 112, 1971.

[15] A. K. Guščin: *Ob ocenkakh rešenij kraevykh zadač dlja paraboličeskogo uravnenija vtorogo porjadka.* Trudy MIAN, tom 126, 1973.

[16] V. P. Mikhajlov: *O principe predeľnoj amplitudy,* DAN SSSR, tom 159, No 4, 1964.

[17] L. A. Muravej: *Ubyvanie rešenij vtoroj vnešnej kraevoj zadači dlja volnovogo uravnenija s dvumja prostranstvennymi peremennymi,* DAN SSSR, tom 193, No 5, 1970.

[18] L. A. Muravej: *Asimptotičeskoe povedenie rešenij vtoroj vnešnej kraevoj zadači dlja dvumernogo volnovogo uravnenija.* Differenciaľnye uravnenija, tom 6, No 2, 1970.

[19] L. A. MURAVEJ: *Asimptotičeskoe povedenie pri boľšikh značenijakh vremeni rešenij vtoroj u treťej vnešnikh kraevykh zadač dlja volnovogo uravnenija s dvumja prostranstvennymi peremennymi.* Trudy MIAN SSSR, tom 126, 1973.

Author's address:
V. N. Maslennikova
Matematičeskij institut im. Steklova AN SSSR
Vavilova 42, Moskva V-333
USSR