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EDUARD ČECH

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The outstanding Czechoslovak geometer and topologist Eduard Čech was born in 1893 in Strachov in northwestern Bohemia. In 1912, while still a student at the Charles University, he began a systematic study of mathematical literature of his own choice, giving preference to the various branches of geometry. In 1915 he was drafted into the army and took part in the First World War. In 1920 he defended his thesis and began to study differential projective geometry. After becoming acquainted with the work of Fubini he went to him in Turin and became his best student. The result of this trip was two books written by Fubini and Čech in which the foundations of differential projective geometry were laid. Čech worked in this field until 1930 after which he left differential geometry for a long time and began to study topology. His first publication in topology was in 1930, the last (not counting a note after the war) was in 1938. During this eight year period Čech obtained brilliant results in the most urgent problems of topology of that time.

Thirty years old, Čech became an Extraordinary Professor at Brno University. During the occupation, when the Czechoslovak universities were closed, he continued to conduct, together with his students B. Pospišil and Zh. Novák at Pospišil's apartment (until Pospišil was arrested by the Gestapo), his seminar, which had a decisive influence on the development of post-war mathematics in Czechoslovakia. After the war Čech's tireless organizing activities expanded: he became, consecutively, the director and organizer of three mathematics institutes of the Academy and the Charles University, was elected a member of the Academy (in 1952), became very occupied with the problems of university and school teaching, conducted, in particular, a seminar in elementary mathematics, wrote textbooks, and so on.

Čech worked hard for the development of mathematics in Czechoslovakia; he created a school which enjoyed world-wide fame and to which belonged such brilliant students of various generations as Novák, Katetov, Frolik, Bopenka and Pospišil who perished in the torture chambers of the Gestapo.

In 1949 a new period of creative activity began for Čech. He returned again to differential geometry and, by the time of his death in 1960, had written another 20 papers. The last of these papers were published by his students posthumously.

In the field of topology Čech was one of the leading representatives of the so-called Brouwer period of development in this science, of the period when a fusion took place of the homological methods of the topology of polyhedra, established by Poincaré, and the theory of general topological spaces, developed by Fréchet and Hausdorff. The beginning of this period goes back to 1912, to a paper of Brouwer "On the n-dimensional Jordan theorem", and to Brouwer's introduction of the simplicial approximation of arbitrary continuous mappings. Brouwer's seminar, in which P. S. Aleksandrov and L. Vietoris participated, decisively stimulated this process. In 1927–1928, together with S. Lefschetz, they solved by three different methods the fundamental problem: the determination of the homology for a fairly wide class of spaces, above all for compacta. After this there came a decade of an intense penetration of homological methods into the study of topological spaces when the fundamental concepts of topology such as dimension, or manifold, were translated into the language of homology, duality theorems were proved in great generality, and so on. The best topological talents of the time took part in this work, in the first place Aleksandrov, Lefschetz, Pontryagin and Čech. The fact that many results were obtained by two or three mathematicians almost simultaneously testifies to the intensiveness of this work. Čech joined in this work as a follower of the Moscow school. He wrote: "... I came to the idea of propagating homology theory on non-compact spaces after closely studying a memoir of P. S. Aleksandrov 'Untersuchungen über Gestalt und Lage ...'"[1]. The main merit of Čech's basic work on this problem: "Théorie générale de l'homologie dans un espace quelconque"[2] was the unusual breadth he gave to Aleksandrov' approach, which consists, as is well known, in the introduction of the concept of nerve and in the approximation of compacta by complexes, namely by the nerves of progressively finer coverings. This made it possible to obtain the homology of a compactum by a passage to the limit. Čech applied this method of Aleksandrov not only to topological spaces, but simply to sets with a selected system of "nets", finite collections of subsets with the corresponding refinement conditions, and so on. However, the finite coverings used by Čech do not give the required results here – even the homology of a straight line, as Dowker has shown, is isomorphic to the additive group of functions reduced modulo bounded functions. In general, in this way the homology of the maximal compactification is obtained (see S. Lefschetz, "Algebraic Topology", 1949, p. 331). The correct approach consists in applying

[1] "Topological papers of Eduard Čech", Prague 1968, p. 361.
[2] Fund. Math. 19 (1932), 149.

coverings with local finiteness conditions; it was not carried out until the end of the forties. However, the technique worked out by Čech, for example, the concept of an essential cycle and others, still preserved all its significance, and his article exerted a great influence on the subsequent development of algebraic topology. For Čech himself this article served as a starting point for all his later work in homology theory. His basic results in this field are: the creation (simultaneously with Lefschetz, Aleksandrov and Pontryagin) of the theory of generalized homological varieties, which subsequently turned out to be the most natural field for the application of homological methods in the theory of compact transformation groups, the analysis (almost simultaneously with Aleksandrov) of local homological properties of spaces, and the discovery (simultaneously with Steenrod) of the important universal coefficient formula in its initial form.

The period of the preeminent development of homological topology quickly led to a clarification of the limitations of applicability of these methods. After Hopf's discovery in 1931 of the algebraically (that is, homologically) trivial, but all the same, essential mapping of a three-dimensional sphere onto a two-dimensional one, the creation of what later came to be called homotopic topology became an imperative necessity. The beginning of this new period goes back to 1935 when Kolmogorov and Alexander discovered the cohomology ring and Hurewicz introduced and studied the concept of the homotopy type.

Čech made two important contributions to the new direction. Simultaneously with Whitney, he defined in a new way the cohomological multiplication (just as it is done now), establishing in passing the isomorphism of the cohomology ring of a variety with the Lefschetz ring. Above all, he was the first to define homotopy groups! This was done already in 1932 and was communicated at the Zurich Congress of Mathematicians three years before the work of Hurewicz. Unfortunately, however, Čech's achievement did not receive its due recognition, because the groups turned out to be commutative and, consequently, did not give under commutation the homology, which could be expected, according to contemporary concepts, from a "correct" extension of the Poincaré group. Čech did not leave any account of his results except a small note of six lines in the Proceedings of the Congress, where not even the definition of the group operation is given. However, Hurewicz shows that his definition is equivalent to Čech's definition (but then, it could not be otherwise).

A characteristic feature of Čech's work was his striving for a unity of various methods. In set-theoretical topology he was particularly attracted by "its ability to become saturated with everything that is really essential and fruitful in other methods". He gave special significance "to the fact that the most advanced parts of combinatorial topology, namely homology theory could be put in a form based exclusively on the general theory of sets". In general, "the set-theoretical approach of the German geometers" occupies a fundamental place in Čech's topological research; in this he saw the difference of his point of view from that of Moscow, where the accent was on the application of combinatorial methods in general topology. It is natural that Čech's papers that are strictly concerned with set-theoretical topology occupy an equal if not more significant place in his work.

Čech worked a good deal in dimension theory. He greatly advanced the study of the so-called large inductive dimension Ind. The definition itself was already given, in a special case, by Brouwer, a fact that apparently was unknown to Čech. Actually, just this definition (between disjoint closed sets there is an $(n - 1)$-dimensional partition) is the adequate mathematical tool for a famous idea of Poincaré. But Čech gave the new concept the rights of citizenship, by proving for Ind the basic properties of dimension: the sum theorem, monotonicity, and so forth. We remark that recently a young Soviet topologist V. V. Filippov solved a problem that was raised then, by proving that in the case of bicompacta Ind need not coincide with the basic dimension function dim.

But a paper of 1937 "On bicompact spaces" brought Čech the greatest fame – the fame of being the first to understand the whole significance in topology of the concept of compactification of a topological space. Like every fundamental mathematical concept, the concept of the maximal compactification appeared implicitly, long before it was perceived by mathematicians. Čech himself came close to it already in his memoir on homology theory – as we have said, his method led to the homology not of the given space, but of its maximal extension. In 1930 Tikhonov examined the compactification of a space on the basis of the standard Uryson construction. But only in 1937 did Čech clarify the significance of this construction when he proved the maximality of the so obtained extension, that is, the fact that every continuous mapping of a given completely regular space in to a compact set can be extended to this compactification. Simultaneously with Čech, but from a dual algebraic point of view, the characteristics of the maximal compactification were studied by M. Stone. Nowadays the Čech – Stone compactification is one of the strongest threads connecting general topology with modern analysis and with other branches of mathematics. In the same paper Čech introduced and studied the important concept of topological completeness – the property of being a set of type G_δ in the Čech compactification, which for metrizable spaces is equivalent to the possibility of introducing a complete metric. We mention that an intrinsic criterion for this "completeness in Čech's sense" was given by Arkhangel'skii.[1]

Čech's work has established firm ties between the Czechoslovak and Soviet mathematical schools. In his homological research he started from Aleksandrov's work, and in his research on general topology from Urysohn's

[1] Vestník Moskov. Gos. Univ. 2 (1961), 37–40.

and Tikhonov's work, the seminar led by him began its work, as his students testify, from a study of Aleksandrov's and Uryson's articles, while in his work on differential geometry he was often close to the work of Finikov and his school; but in their turn Čech's ideas, concepts he introduced such as topological completeness, the large inductive dimension or the Cech's compactification, became a stimulus for fruitful work of many Soviet topologists of the post-war generation.

Soviet mathematicians pay their deep respect to the memory of an eminent Czechoslovak scholar.

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Translated by Diane Thompson