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Zhang Li; Gang Yu; Yanjun Shen

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# EXPONENTIAL STABILITY OF NONLINEAR SYSTEMS WITH EVENT-TRIGGERED SCHEMES AND ITS APPLICATION

Li Zhang, Gang Yu, and Yanjun Shen

In this paper, we discuss exponential stability for nonlinear systems with sampled-data-based event-triggered schemes. First, a framework is proposed to analyze exponential stability for nonlinear systems under some different triggering conditions. Based on these results, output feedback exponential stabilization is investigated for a class of inherently nonlinear systems under a kind of event-triggered strategies. Finally, the rationality of the theoretical work is verified by numerical simulations.

*Keywords:* sampled-data, event-triggered, exponential stability, output feedback, inherently nonlinear systems

*Classification:* 93C57, 93D23

## 1. INTRODUCTION

The extensive applications of the network not only produces a large number of cyber-physical systems, but also introduces new problems such as limited bandwidth, transmission delay, and data packet loss to the control system [30,31]. Recently, the sampled-data-based event-triggered strategies were proposed and discussed widely for networked control systems. In this scheme, the systems are monitored only at sampling instants. Thus, it is absolute to avoid Zeno behavior and more suitable for application to large-scale digital communication networks such as microgrid systems [3]. Until now, researchers have been in great enthusiasm in sampled-data-based event-triggered stability analysis and controller design, for example, feedback control, consensus, $H\infty$ filtering [9,16,17,24,25,29,34].

Although research on event-triggered strategies is in the ascendant, existing works still have some shortcomings. Consider the sampled-data-based event-driven system expressed as

$$\dot{x}(t) = \Psi(x(t), x(h_m)), t \in [h_m,\ h_{m+1}), \tag{1}$$

where $x(t) \in \mathcal{R}^n$ is the state, $\Psi(\cdot)$ is a continuous nonlinear function, $\{h_m | m = 0, 1, \ldots\}$ and $\{t_k | k = 0, 1, \ldots\}$ are two sets of triggering instants and sampling instants, respec-

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tively. In the published reports, the event-triggered system is often transformed into Model 1 through an error function [2, 8, 28], i. e.,

**Model 1**

$$\dot{x}(t) = \Phi_1(x(t), x(t_k)), t \in [t_k,\ t_{k+1}),$$

where $\Phi_1(\cdot)$ is a continuous nonlinear function. Then, stability analysis for (1) is transformed into stability analysis for Model 1. Obviously, Model 1 is a standard sampled-data system. It is a hot topic to analyze its stability. Researchers have proposed some stability analysis methods, for example, input delay method [6], continuous and discrete analysis [4, 33] and so on. Unfortunately, not all event-triggered systems can be converted to Model 1. This is because that Model 1 requires that all event-triggered design must be related to the sampling period, which sets a limit on the design process. It is worth mentioning that exponential stability of this model and its applications have discussed in [4, 12, 32, 33].

Another widely used model is proposed in [34]. By introducing a piecewise continuous linear function $\eta(t)$, the event-triggered system will be modeled as the following time delay system finally.

**Model 2**

$$\dot{x}(t) = \Phi_2(x(t), x(t - \eta(t))), t \in [h_m,\ h_{m+1}),$$

where $\Phi_2(\cdot)$ is a continuous nonlinear function. Then, by combing with LMI approach, it is easy to obtain asymptotical stable results. We think that Model 2 still has the characteristics of sampled-data system. By an abuse of terminology, there is only one discrete variable in interval $[h_m,\ h_{m+1})$. It can be regarded as an extension of the time-delay method for sampled-data system [6]. However, this method is generally used for linear systems. It is not flexible enough for complex nonlinear systems.

In this paper, we give a new Model 3, which can be described as

**Model 3**

$$\dot{x}(t) = \Phi_3(x(t), x(t_k), x(h_m)), t \in [t_k,\ t_{k+1}),$$

where $h_m$ is the last triggering instant before the sampling instant $t_k$, $\Phi_3(\cdot)$ is a continuous nonlinear function. We call it as a sampled-data-like system, which can be regarded as a promotion of Model 1. It can be seen that in each sampling period $[t_k,\ t_{k+1})$, the system is not only related to the current sampling state $x(t_k)$, but also to the most recent event triggering state $x(h_m)$. It must be pointed out that Model 3 is a general case which can bring more flexibility to system analysis and synthesis. For instance, in this case, an event-triggered system is allowed to have discrete states that are not directly involved in triggering conditions, but are driven by the triggered mechanism. In other words, these discrete states are passively triggered. In this article, we try to build a framework of stability analysis for this system as described above.

Hereafter, we consider the event-driven output feedback control for a class of inherently nonlinear systems,

$$\begin{aligned} \dot{z}_i(t) &= \delta_i(t) z_{i+1}^{p_i}(t) + \varrho_i(\underline{z}_i(t)),\ i = 1,\ \ldots,\ n-1, \\ \dot{z}_n(t) &= \delta_n(t) u(t) + \varrho_n(\underline{z}_n(t)),\ y(t) = z_1(t), \end{aligned} \tag{2}$$

where $z_i(t)$ is the system's state, the nonlinear function $\varrho_i(\underline{z}_1)$ is in a lower-triangular from, i. e. $\varrho_i(\underline{z}_i(t)) = \varrho_i(z_1(t),\ \ldots,\ z_i(t))$, $\delta_i(t)$ is an unknown time-varying coefficient, $y(t)$ is the measurable output function, $u(t)$ is the control input. $p_i \geq 1$ is a positive odd integer and $p_0 = 1$. This system is also called $p$-normal, which has been widely studied in the past twenty years [18, 21, 22, 26]. Many physical systems such as underactuated mechanical systems [21] can be modeled in this form. Since there is an uncontrollable unstable linearization around the origin, the smooth feedback controllers is impossible to stabilize the system (2). In [21], the adding a power integrator was proposed to construct a continuous non-smooth control law for this system. The problem of output feedback control for a class of inherently nonlinear systems was investigated in [18]. Recently, sampled-data technique was introduced [5, 10, 12, 19]. However, sampled-data control will lead to unnecessary redundant calculations and bandwidth usage, especially in networked control systems. To the best knowledge of the authors, employing the event-triggered control is an efficient solution to these problems. But related work in this area is absent, which inspired our research.

The main contributions of this study are summarized as follows: 1) A new model of event-triggered system is proposed based on aperiodic sampled-data, which can bring more flexibility in system analysis and synthesis. Then, we investigate exponential stability of the event-triggered system. 2) A novel event-triggered condition with nonuniform sampled-data is presented. Under the proposed event-triggered condition, output feedback stabilization is discussed for a class of inherently nonlinear systems. The rest of this paper is organized as follows. Section 2 gives some preliminaries of this paper. Exponential stable analysis is presented for a class of event-triggered nonlinear systems in Section 3. Then, the problem of event-triggered stabilization is discussed for a class of inherently nonlinear systems in Section 4. In Section 5, simulation examples illustrate our algorithms. At last, this paper is concluded in Section 6.

## 2. PRELIMINARIES

We firstly introduce the following assumptions and lemmas which will be used later.

**Assumption 2.1.** (Li et al. [12], Qian and Lin [22], Polendo and Qian [18]) For the nonlinear function $\varrho_i(\underline{z}_i(t))$, there exists $l \geq 0$ such that

$$|\varrho_i(\underline{z}_i(t))| \leq l(|z_1|^{\frac{1}{p_1 \cdots p_{i-1}}} + |z_2|^{\frac{1}{p_2 \cdots p_{i-1}}} + \ldots + |z_i|).$$

**Assumption 2.2.** (Li and Zhao [14]) The unknown time-varying control coefficients $\delta_i(t)(i = 1, \ldots, n)$ are derivable, and there exist constants $\underline{\delta} > 0$, $\bar{\delta} > 0$, $\breve{\delta}$ and $\tilde{\delta}$, such that

$$\underline{\delta} \leq \delta_i(t) \leq \bar{\delta},\ \breve{\delta} \leq \dot{\delta}_i(t) \leq \tilde{\delta}.$$

**Lemma 2.3.** (Qian and Lin [20, 21]) For any $p \geq 1$ and any $g, h \in \mathcal{R}$, we have the following inequalities

$$
\begin{aligned}
&|g-h|^p \leq 2^{p-1}\left|g^p-h^p\right| \leq 2^{p-1} p\,|g-h|\left|g^{p-1}+h^{p-1}\right|, \\
&\leq c\,|g-h|^p+c\,|g-h|\,|h|^{p-1},
\end{aligned}
$$

$$
\left|g^{\frac{1}{p}}-h^{\frac{1}{p}}\right| \leq 2^{1-\frac{1}{p}}\,|g-h|^{\frac{1}{p}},
$$

where $c$ is a positive constant.

**Lemma 2.4.** (Qian and Lin [20]) For any real numbers $g_1, \ldots,\ g_n$ and $p>0$,

$$
(g_1+\ldots+g_n)^p \leq \max\{n^{p-1},\ 1\}(g_1^p+\ldots+g_n^p).
$$

**Lemma 2.5.** (Qian and Lin [20]) There exist positive constants $m, n, \phi$ and $g, h \in \mathcal{R}$ such that the following inequality

$$
|g|^m|h|^n \leq \frac{m}{m+n}\phi|g|^{m+n}+\frac{n}{m+n}\phi^{-\frac{m}{n}}|h|^{m+n}
$$

holds.

We also have the following results.

**Lemma 2.6.** Let $\{V(k)\}$ denote a positive sequence. If there exist three real numbers $0<\theta,\ \Upsilon<1,\ \varsigma \geq 0$, and positive integers $k \geq p$, $r(k-p) \geq k-p$ satisfy the following inequality

$$
V(k) \leq \theta V(k-p)+\varsigma \Upsilon^{r(k-p)}, \tag{3}
$$

then, $\lim_{k \rightarrow \infty} V(k)=0$, that is, the sequence $\{V(k)\}$ is convergent.

Proof. From (3), it follows that $V(k) \leq \theta V(k-p)+\varsigma \Upsilon^{r(k-p)} \leq \theta V(k-p)+\varsigma \Upsilon^{k-p} \leq$

$$
\theta^2(V(k-2p)-\frac{\varsigma \Upsilon^{k-2p}}{\Upsilon^p-\theta})+\frac{1}{\Upsilon^p-\theta}\varsigma \Upsilon^k \ldots \leq \theta^{[\frac{k}{p}]}(V(k-[\frac{k}{p}]p)-\frac{\varsigma \Upsilon^{k-[\frac{k}{p}]p}}{\Upsilon^p-\theta})+\frac{1}{\Upsilon^p-\theta}\varsigma \Upsilon^k,
$$

where $[\frac{k}{p}]$ is the integer part of $\frac{k}{p}$.

For any positive integer $k$, there exist two integers $q$, and $1 \leq k_0 \leq p-1$, such that $k=qp+k_0$. Then, we have $V(k) \leq \theta^q(V(k_0)-\frac{\varsigma \Upsilon^{k_0}}{\Upsilon^p-\theta})+\frac{1}{\Upsilon^p-\theta}\varsigma \Upsilon^k$. Since $0<\theta,\ \Upsilon<1$, then

$$
\lim_{q \rightarrow \infty} \theta^q(V(k_0)-\frac{\varsigma \Upsilon^{k_0}}{\Upsilon^p-\theta})+\frac{1}{\Upsilon^p-\theta}\varsigma \Upsilon^{qp+k_0}=0.
$$

Simultaneously, note that $V(qp+k_0)>0$. Therefore, $\lim_{k \rightarrow \infty} V(k)=0$. The proof is completed. □

## 3. EXPONENTIAL STABILITY FOR A CLASS OF EVENT-TRIGGERED NONLINEAR SYSTEMS

In this section, we discuss exponential stability for a class of event-triggered nonlinear systems, which is the basis of the next section. Consider the following nonlinear system

$$\dot{x}(t) = f(x(t), u(t)),\ y(t) = g(x(t)), \tag{4}$$

where $x(t) \in \mathcal{R}^n$ , $y(t) \in \mathcal{R}^m$ and $u(t) \in \mathcal{R}^p$ are the state, the output and the input, respectively. The nonlinear functions $f(\cdot)$ and $g(\cdot)$ are continuous and $f(\cdot)$ satisfies $f(0,0) = 0$. We establish an event-based dynamic output feedback controller described by

$$\begin{aligned} &\dot{\hat{x}}(t) = \Gamma(\hat{x}(t),\ \hat{x}(h_m),\ y(h_m)),\ \hat{x}(t) \in \mathcal{R}^n, \\ &u(t) = \Pi(\hat{x}(t),\ \hat{x}(h_m),\ y(h_m)),\ t \in [t_k,\ t_{k+1}), \end{aligned} \tag{5}$$

where $\Gamma(\cdot)$ and $\Pi(\cdot)$ are two continuous functions, and $h_m$ is the last event-triggering instant before the sampling instant $t_k$. The event-triggering condition is rewritten as

$$\|e(t_k)\|^2 \le E_1(y(t_k),\ y(h_m),\ \Upsilon^m), \tag{6}$$

where $\Upsilon \ge 0$, and $E_1(\cdot)$ is a nonlinear function. The condition $E_1(y(t_k), y(h_m), \Upsilon^m)$ includes two typical cases: state dependent condition $E_1(y(t_k),\ y(h_m, 0)$ and time dependent condition $E_1(0, 0,\ \Upsilon^m)$. We will discuss them separately later. The error function is defined as $e(t_k) = y(h_m) - y(t_k)$. Different from those works in [8,28], the event-triggered strategy is based on nonperiodic sampled-data. We define the sampling instants as an increasing sequence $\{t_k\}$. Let $T_k = t_{k+1} - t_k$ denote the time-varying sampling period which is bounded by $0 < T_{\min} \le T_k \le T_{\max}$. The positive constant $T_{\min}$ absolutely avoids Zeno behavior for the event-triggered system. We assume that the number of sampling intervals in each event-triggered interval $[h_i,\ h_{i+1})$ is $s_i$, $(i = 0,\ 1,\ \ldots)$, and set the initial time $t_0 = h_0$. The event-triggered scheme is shown in Figure 1.

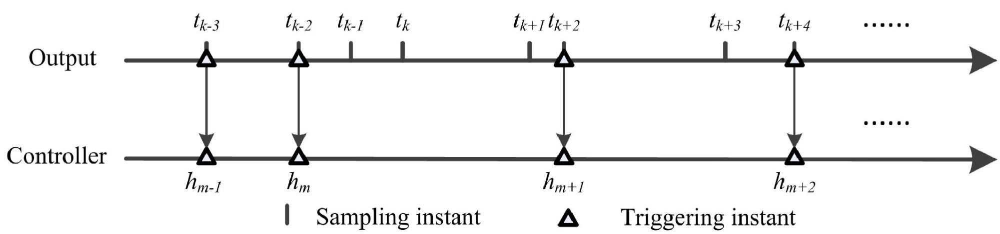


**Fig. 1.** The event-triggered scheme.

We make the following assumption on $s_i$.

**Assumption 3.1.** There exists a positive integer $s^*$ such that

$$s^* \geq s_i, \text{ for all } i = 0,\ 1,\ 2,\ \ldots.$$

In what follows, for the sake of simplicity, we will abbreviate the continuous state $x(t)$ as $x$, and still mark the sampling state and the event-triggering state as $x(t_k)$ and $x(h_m)$, respectively. Similar variables have the same meaning.

From (4), (5) and the triggering condition (6), the following closed-loop system can be constructed,

$$\begin{cases} \dot{x} = f(x, u(h_m)),\ t \in [t_k, t_{k+1}), \\ e(t_k) = y(t_k) - y(h_m), \\ \|e(t_k)\|^2 \leq E_1(y(t_k), y(h_m), \Upsilon^m), \\ h_m = \max\limits_{h_j}\{h_j \leq t_k\},\ x(h_{m+1}) = \lim\limits_{t \to h_{m+1}^-} x(t). \end{cases} \tag{7}$$

Obviously, the closed-loop system (7) can be expressed as Model 3 rather than Model 1 and 2 because there exists the passively triggered discrete variable $\hat{x}(h_m)$ in (5), which remains constant in the event interval $[h_m, h_{m+1})$. Next, the following sufficient conditions are proposed to ensure global exponential stability.

**Theorem 3.2.** The event-triggered nonlinear system (7) is globally exponentially stable, if there exist positive constants $\alpha,\ \beta,\ \Upsilon$, and $\gamma \geq 0,\ \omega \geq 0$, and a positive definite and radially unbounded function $V(t)$ defined on $[t_0, +\infty)$, satisfying the following conditions

$$\dot{V}(t) \leq -\alpha V(t) + \beta V(t_k) + \gamma V(h_m) + \omega \Upsilon^m, \tag{8}$$

$$\alpha > \beta + s^*\gamma,\ 0 < \Upsilon < 1,\ t \in [t_k,\ t_{k+1}), \tag{9}$$

and the sampling periods satisfy

$$\frac{1 - e^{-\alpha T_{\min}}}{1 - e^{-\alpha T_{\max}}} > \frac{\beta + s^*\gamma}{\alpha}. \tag{10}$$

P r o o f. First, from (8), we have

$$V(t) \leq \left(\frac{\beta}{\alpha}(1 - e^{\alpha(t_k - t)}) + e^{\alpha(t_k - t)}\right) V(t_k) + \frac{\gamma}{\alpha}(1 - e^{\alpha(t_k - t)})V(h_m) + \frac{\omega}{\alpha}\Upsilon^m,\ t \in [t_k,\ t_{k+1}).$$

Since $x(t)$ is continuous in the event interval $[h_m,\ h_{m+1})$, the continuity still holds at each sampling instant. Then, we have

$$V(t_{k+1}) = \lim_{t \to t_{k+1}} V(t) \leq \left(\frac{\beta}{\alpha}(1 - e^{-\alpha T_k}) + e^{-\alpha T_k}\right) V(t_k) + \frac{\gamma}{\alpha}(1 - e^{-\alpha T_k})V(h_m) + \frac{\omega}{\alpha}\Upsilon^m.$$

Next, dividing the event interval as $[h_m,\ h_{m+1}) = \cup_{i=j}^{i=j+s_m-1}[t_i, t_{i+1})$, we can obtain the following inequalities in different intervals

$$V(t_{i+1}) \leq \left(\frac{\beta}{\alpha}(1 - e^{-\alpha T_i}) + e^{-\alpha T_i}\right) V(t_i) + \frac{\gamma}{\alpha}(1 - e^{-\alpha T_i})V(h_m) + \frac{\omega}{\alpha}\Upsilon^m,\ t \in [t_i,\ t_{i+1}). \tag{11}$$

Since $T_i \leq T_{\max}$, we have $1 - e^{-\alpha T_i} \leq 1 - e^{-\alpha T_{\max}}$. Then, superimposing (11) from $i = j$ to $i = j + s_m - 1$, we have

$$V(t_{j+1}) + \ldots + V(t_{j+s_m}) \leq \varpi(V(t_j) + \ldots + V(t_{j+s_m-1})) + \frac{s_m\omega}{\alpha}\Upsilon^m,\ t \in [h_m,\ h_{m+1}),$$

where $\varpi = \max\{\frac{\beta}{\alpha}(1-e^{-\alpha T_j})+e^{-\alpha T_j} + \frac{s_m\gamma}{\alpha}(1-e^{-\alpha T_{\max}}), \frac{\beta}{\alpha}(1-e^{-\alpha T_{j+1}})+e^{-\alpha T_{j+1}}, \ldots, \frac{\beta}{\alpha}(1-e^{-\alpha T_{j+s_m-1}}) + e^{-\alpha T_{j+s_m-1}}\}$.

From (9) and (10), it is obvious that $\varpi < 1$. Since $V(h_m) = V(t_j)$, $V(h_{m+1}) = V(t_{j+s_m})$, then,

$$V(h_{m+1}) \leq \varpi V(h_m) + \frac{s_m\omega}{\alpha}\Upsilon^m \leq \varpi V(h_m) + \frac{s^*\omega}{\alpha}\Upsilon^m.$$

Finally, by using Lemma 2.6 with $k = m,\ p = 1,\ r(k-p) = m-1$, we have

$$V(h_m) \leq \varpi^m(V(h_0) - \frac{s^*\omega}{\alpha(\Upsilon - \varpi)}) + \frac{s^*\omega}{\alpha(\Upsilon - \varpi)}\Upsilon^m.$$

Then,

$$\begin{aligned} V(t) &\leq (s^*+1)V(h_m) + \frac{s^*\omega}{\alpha}\Upsilon^m \\ &\leq (s^*+1)e^{(\frac{t}{s^*T_{\max}}-1)\ln\varpi}\left(V(h_0) - \frac{s^*\omega}{\alpha(\Upsilon-\varpi)}\right) + e^{(\frac{t}{s^*T_{\max}}-1)\ln\Upsilon}\left(\frac{(s^*+1)s^*}{\Upsilon-\varpi}+1\right)\frac{\omega}{\alpha}, \end{aligned}$$

which indicates the event-triggered system (7) is globally exponentially stable.

□

**Remark 3.3.** Based on the continuous and discrete analysis, exponential stability was obtained for a kind of sampled-data systems [4, 12, 33] by proving $V(t_{k+1}) < V(t_k)$. In Theorem (3.4), we extend the continuous and discrete analysis from sampled-data systems to event-driven systems. By setting $\gamma = 0,\ \omega = 0$, Theorem 3.2 will be reduced to the results obtained in [4, 12, 33].

The results in Theorem 3.2 can be reduced to the following output feedback control

$$\begin{aligned} &\dot{\hat{x}} = \Gamma(\hat{x},\ y(h_m)),\ \hat{x} \in \mathcal{R}^n, \\ &u(h_m) = \Pi(\hat{x},\ y(h_m)),\ t \in [t_k,\ t_{k+1}), \end{aligned} \tag{12}$$

with the state-dependent triggering condition defined by

$$\|e(t)\|^2 \leq E_2(y(t_k),\ y(h_m)),\ t \in [t_k,\ t_{k+1}), \tag{13}$$

where $E_2(y(t_k),\ y(h_m)) = E_1(y(t_k),\ y(h_m), 0)$, which can be the one proposed in [9, 17, 29], or the triggering conditions designed in [16, 24, 34], or the mixed event-triggering conditions investigated recently in [7].

From (12), the following closed-loop system can be obtained,

$$\begin{cases} \dot{x} = f(x, u(h_m)),\ t \in [t_k, t_{k+1}), \\ e(t_k) = y(t_k) - y(h_m), \\ \|e(t_k)\|^2 \le E_2(y(t_k),\ y(h_m)), \\ h_m = \max_{h_j}\{h_j \le t_k\},\ x(h_{m+1}) = \lim_{t \to h_{m+1}^-} x(t). \end{cases} \tag{14}$$

**Corollary 3.4.** The event-triggered system (14) is globally exponentially stable, if there exist positive constants $\alpha,\ \beta,\ \gamma \ge 0$ and a positive definite and radially unbounded function $V(t)$ defined on $[t_0, +\infty)$ such that

$$\dot{V}(t) \le -\alpha V(t) + \beta V(t_k) + \gamma V(h_m),\ t \in [t_k,\ t_{k+1}), \tag{15}$$

$$\alpha > \beta + s^*\gamma, \tag{16}$$

and the sampling periods satisfy (10).

Due to simplicity of design and implementation, time-dependent (or state-independent) triggering conditions have been extensively studied in [2, 25, 28]. Now, we discuss exponential stability for the nonlinear system (4) with the time-varying triggering condition

$$\|e(t_k)\|^2 \le \mu \Upsilon^k, \tag{17}$$

where $\mu > 1,\ 0 < \Upsilon < 1$ [2], or

$$\|e(t_k)\|^2 \le \varepsilon^{-\alpha k T}, \tag{18}$$

where $\varepsilon > 1,\ 0 \le \alpha < 1$ [25, 28]. It is not difficult to verify that the conditions (17) and (18) are substantially the same. We denote it as $E_3(\Upsilon^k) = E_1(0,\ 0,\ \Upsilon^k)$. Under this scheme, the following closed-loop system can be established,

$$\begin{cases} \dot{x} = f(x, u(h_m)),\ t \in [t_k, t_{k+1}), \\ e(t_k) = y(t_k) - y(h_m),\ \|e(t_k)\|^2 \le E_3(\Upsilon^k), \\ h_m = \max_{h_j}\{h_j \le t_k\},\ \ x(h_{m+1}) = \lim_{t \to h_{m+1}^-} x(t). \end{cases} \tag{19}$$

**Corollary 3.5.** The event-triggered system (19) is globally exponentially stable, if there exist positive constants $\alpha,\ \beta,\ \omega,\ \Upsilon$ and a positive definite and radially unbounded function $V(t)$ defined on $[t_0, +\infty)$ such that

$$\dot{V}(t) \le -\alpha V(t) + \beta V(t_k) + \omega \Upsilon^k,\ t \in [t_k,\ t_{k+1}),$$
$$\alpha > \beta,\ 0 < \Upsilon < 1.$$

**Remark 3.6.** In practical applications, the triggering conditions (17) and (18) will not work if $\Upsilon$ is selected too small. Then, the event-triggered mechanism becomes almost a time-triggered mechanism. This is because (17) and (18) decay rapidly in each sampling interval, which also results in a large number of invalid calculations. In the framework of Model 3, a new time-dependent triggering condition will be generated

$$\|e(t_k)\| \le \mu \Upsilon^m. \tag{20}$$

It can be seen that (20) is just updated on each event interval. Actually, with the help of Lemma 1, the parameter $m$ can be modified to any positive integer $r(m)$ which satisfies $r(m) \geq m$ (including $k$). It should be pointed out that when $r(m)$ is larger, the exponential convergence rate of the closed-loop system will be faster. However, the triggered scheme will decay faster simultaneously. These two points need to be balanced in practical applications.

## 4. EVENT-TRIGGERED STABILIZATION OF A CLASS OF INHERENTLY NONLINEAR SYSTEMS

In this section, we discuss the event-triggered stabilization of a class of inherently nonlinear systems (2). By employing the following change of coordinates

$$x_1 = z_1, x_i = \delta_1^{\overline{\frac{1}{p_1 \cdots p_{i-1}}}} \dots \delta_{i-1}^{\overline{\frac{1}{p_{i-1}}}} z_i,$$

the system (2) becomes

$$\begin{aligned} &\dot{x}_i = x_{i+1}^{p_i} + \bar{\varrho}_i(\underline{z}_i), \ i = 1, \ \dots, \ n-1, \\ &\dot{x}_n = \delta u + \bar{\varrho}_n(\underline{z}_n), \ y = z_1, \end{aligned} \tag{21}$$

where $\bar{\varrho}_i(\underline{z}_i) = \frac{d(\delta_1^{\overline{\frac{1}{p_1 \cdots p_{i-1}}}} \dots \delta_{i-1}^{\overline{\frac{1}{p_{i-1}}}})}{dt} \frac{x_i}{\delta_1^{\overline{\frac{1}{p_1 \cdots p_{i-1}}}} \dots \delta_{i-1}^{\overline{\frac{1}{p_{i-1}}}}} + \delta_1^{\overline{\frac{1}{p_1 \cdots p_{i-1}}}} \dots \delta_{i-1}^{\overline{\frac{1}{p_{i-1}}}} \varrho_i(\underline{z}_i)$,
$\delta = \delta_1^{\overline{\frac{1}{p_1 \cdots p_{i-1}}}} \dots \delta_{n-1}^{\overline{\frac{1}{p_{n-1}}}} \delta_n$.

**Lemma 4.1.** Under Assumption 2.1 and 2.2, there exists a positive constant $\bar{l}$ satisfying the following inequality

$$\bar{\varrho}_i(\underline{z}_i) \leq \bar{l}(|y|^{\overline{\frac{1}{p_1 \cdots p_{i-1}}}} + |x_2|^{\overline{\frac{1}{p_2 \cdots p_{i-1}}}} + \dots + |x_i|).$$

Then, the following virtual state feedback controller can be designed for the system (21) recursively

$$u^* = -(\kappa_n x_n^{p_1 \cdots p_{n-1}} + \kappa_{n-1} x_{n-1}^{p_1 \cdots p_{n-2}} + \dots + \kappa_1 x_1)^{\overline{\frac{1}{p_1 \cdots p_{n-1}}}},$$

where $\kappa_1, \dots, \kappa_n$ are positive gains. Moreover, there exist a positive real number $\rho$ and a Lyapunov function $V_n$ such that

$$\dot{V}_n \leq -\rho \sum_{i=1}^{n} \xi_i^2 + \delta \xi_n^{2 - \overline{\frac{1}{p_1 \cdots p_{n-1}}}} (u - x_{n+1}^{*p_n}).$$

*Proof.* First, the upper bound of nonlinear is straightforward. Then, as in [18], we construct the following virtual controllers $x_k^*, (k = 1, \dots, n)$,

$$\begin{aligned} &x_1^* = 0, && \xi_1 = y - x_1^*, \\ &x_k^{*p_1 \cdots p_{k-1}} = -\lambda_{k-1} \xi_{k-1}, && \xi_k = x_k^{p_1 \cdots p_{k-1}} - x_k^{*p_1 \cdots p_{k-1}}, k = 2, \ \dots, \ n, \end{aligned}$$

where $\lambda_k,\ (k=1,\dots,n-1)$ are positive constants, the results can be derived. □

Next, the event-triggered reduced-order observer is constructed.

$$\dot{\hat{\phi}}_i = -b_{i-1}\hat{\phi}_i - b_{i-1}^2\hat{x}_{i-1}(h_m),\ i=2,\ \dots,\ n, \tag{22}$$

where $\hat{x}_1 = y$, $\hat{x}_i = (\hat{\phi}_i + b_{i-1}\hat{x}_{i-1})^{\frac{1}{p_{i-1}}}$, and $b_1,\ \dots,\ b_{n-1}$ are the observer gains to be determined later. The event-triggered scheme is given by (20) in Section 2. Based on the observer (22) and the event-triggered scheme (20), the control input is designed as

$$\begin{aligned} u(h_m) = &-(\kappa_n \hat{x}_n^{p_1\cdots p_{n-1}}(h_m) + \kappa_{n-1}\hat{x}_{n-1}^{p_1\cdots p_{n-2}}(h_m) + \dots \\ &+ \kappa_1 x_1(h_m))^{\frac{1}{p_1\cdots p_{n-1}}},\ t\in[t_k, t_{k+1}). \end{aligned} \tag{23}$$

**Remark 4.2.** The design of the dynamic output feedback controller (22) and (23) still follows our idea in Section 3, the update instants of other discrete variables in controller depend on the event-triggered scheme which is set for the output $y$.

**Theorem 4.3.** There exist positive constants $T_{\min}$ and $T_{\max}$ such that for any $0 < T_{\min} \le T_k \le T_{\max}$, the closed-loop system composed of (21), (22) and (23) is globally exponentially stable under the event-triggered scheme (20).

Proof. Let $\phi_i = x_i^{p_{i-1}} - b_{i-1}x_{i-1}$, and the estimation error

$$\varepsilon_i = (x_i^{p_{i-1}} - \hat{x}_i^{p_{i-1}})^{p_1\cdots p_{i-2}}.$$

Define

$$\hat{V}_i(x_{i-1}, x_i, \hat{\phi}_i) = \int_{\eta_i^{2p_1\cdots p_{i-2}-1}}^{x_i^{2p_1\cdots p_{i-1}-p_{i-1}}} (s^{\frac{1}{2p_1\cdots p_{i-2}-1}} - \eta_i)\,\mathrm{d}s,$$

where $\eta_i = \hat{\phi}_i + b_{i-1}x_{i-1}$ $(i=2,\ \dots,\ n)$.

Calculating the derivative of $\hat{V}_i$ with respect to $t$ yields

$$\dot{\hat{V}}_i = \frac{\partial \hat{V}_i}{\partial x_{i-1}}(x_i^{p_{i-1}} + \bar{\varrho}_{i-1}(\underline{z}_{i-1})) + \frac{\partial \hat{V}_i}{\partial x_i}\dot{x}_i + \frac{\partial \hat{V}_i}{\partial \hat{\phi}_i}(-b_{i-1}\hat{x}_i^{p_{i-1}} + b_{i-1}^2(\hat{x}_{i-1} - \hat{x}_{i-1}(h_m))). \tag{24}$$

We estimate each term of (24) in the following derivation, for $i=1,\dots,n$, then

$$\begin{aligned} \frac{\partial \hat{V}_i}{\partial x_{i-1}} &= -b_{i-1}(x_i^{2p_1\dots p_{i-1}-p_{i-1}} - \eta_i^{2p_1\dots p_{i-2}-1}), \\ \frac{\partial \hat{V}_i}{\partial \hat{\phi}_i} &= -(x_i^{2p_1\dots p_{i-1}-p_{i-1}} - \eta_i^{2p_1\dots p_{i-2}-1}). \end{aligned}$$

Based on Lemma 2.3, the following inequalities hold

$$|x_i - \hat{x}_i| \le 2^{1-\frac{1}{p_{i-1}}} |\varepsilon_i|^{\frac{1}{p_1\cdots p_{i-1}}}, \tag{25}$$

$$\left|\phi_i - \hat{\phi}_i\right| \le |\varepsilon_i|^{\frac{1}{p_1\cdots p_{i-2}}} + b_{i-1} 2^{1-\frac{1}{p_{i-2}}} |\varepsilon_{i-1}|^{\frac{1}{p_1\cdots p_{i-2}}}. \tag{26}$$

According to (25), (26), Lemmas 2.3 and 2.4, we get

$$\frac{\partial \hat{V}_i}{\partial x_{i-1}} x_i^{p_{i-1}} - b_{i-1}\frac{\partial \hat{V}_i}{\partial \hat{\phi}_i}\hat{x}_i^{p_{i-1}} \le -(c_{i,2}b_{i-1}-1)\varepsilon_i^2 + \xi_i^2 + \xi_{i-1}^2 + c_{i,1}(b_{i-1})\varepsilon_{i-1}^2, \tag{27}$$

where $c_{i,1}(b_{i-1})$ is a positive constant with respect to $b_{i-1}$, $c_{i,2} = 2^{2-2p_1\ldots p_{i-2}}$. Next, the following inequality will be established

$$\frac{\partial \hat{V}_i}{\partial x_{i-1}} \bar{\varrho}_{i-1}(\underline{z}_{i-1}) \le \sum_{j=1}^{i} \xi_j^2 + \varepsilon_i^2 + c_{i,3}(b_{i-1})\varepsilon_{i-1}^2, \tag{28}$$

where $c_{i,3}(b_{i-1})$ is a positive constant with respect to $b_{i-1}$.

Then, for $i = 2, \ \ldots, \ n-1$, we can easily obtain

$$\frac{\partial \hat{V}_i}{\partial x_i}\dot{x}_i \le \sum_{j=1}^{i+1} \xi_j^2 + c_{i,4}\varepsilon_i^2 + c_{i,5}(b_{i-1})\varepsilon_{i-1}^2, \tag{29}$$

where $c_{i,4}$ is a positive constant, $c_{i,5}(b_{i-1})$ is a positive constant with respect to $b_{i-1}$.

Note that

$$\delta u(h_m) \le c_{i,6}\left(\sum_{i=1}^{n} |\xi_i(h_m)|^{\frac{1}{p_1\cdots p_{n-1}}} + \sum_{i=2}^{n} |\varepsilon_i(h_m)|^{\frac{1}{p_1\cdots p_{n-1}}}\right),$$

where $c_{i,6}$ is a positive constant. Thus, for $i = n$, we also have

$$\frac{\partial \hat{V}_n}{\partial x_n}\dot{x}_n \le \sum_{i=1}^{n} \xi_i^2 + c_{n,4}\varepsilon_n^2 + c_{n,5}(b_{n-1})\varepsilon_{n-1}^2 + \sum_{i=1}^{n} \xi_i^2(h_m) + \sum_{i=2}^{n} \varepsilon_i^2(h_m), \tag{30}$$

where $c_{n,4}$ is a positive constant, $c_{n,5}(b_{i-1})$ is a positive constant with respect to $b_{i-1}$.

Moreover,

$$\begin{aligned} b_{i-1}^2 \frac{\partial \hat{V}_i}{\partial \hat{\phi}_i}(\hat{x}_{i-1} - \hat{x}_{i-1}(h_m)) &\le \xi_i^2 + \xi_{i-1}^2 + \varepsilon_i^2 + c_{i,7}(b_{i-1})\varepsilon_{i-1}^2 \\ &+ (y - y(h_m))^2 + \left(\hat{\phi}_{i-1} - \hat{\phi}_{i-1}(h_m)\right)^{2p_1\ldots p_{i-3}} + \ldots + \left(\hat{\phi}_2 - \hat{\phi}_2(h_m)\right)^2, \end{aligned} \tag{31}$$

where $c_{i,7}(b_{i-1})$ is a positive constant respects to $b_{i-1}$. Then, it follows from Hölder's inequality and Lemma 2.5 that

$$\begin{aligned} &\left(\hat{\phi}_i - \hat{\phi}_i(h_m)\right)^{2p_1\ldots p_{i-2}} \le \left(\int_{t-s^*T_{\max}}^{t} \dot{\hat{\phi}}_i(s)\,\mathrm{d}s\right)^{2p_1\ldots p_{i-2}} \\ &\le 2^{2p_1\ldots p_{i-2}-1}(s^*T_{\max})^{2p_1\ldots p_{i-2}} b_{i-2}^{4p_1\ldots p_{i-2}} \hat{x}_{i-1}^{2p_1\ldots p_{i-2}}(h_m) \\ &+ 2^{2p_1\ldots p_{i-2}-1}(s^*T_{\max})^{2p_1\ldots p_{i-2}-1} b_{i-1}^{2p_1\ldots p_{i-2}} \int_{t-s^*T_{\max}}^{t} F_1(s)\,\mathrm{d}s, \end{aligned} \tag{32}$$

$$(y - y(h_m))^2 \leq \vartheta_1 s^* T_{\max} \int_{t-s^*T_{\max}}^{t} F_2(s)\,\mathrm{d}s, \tag{33}$$

where $\vartheta_1 = 2\max(1, \bar{l})$, $F_1 = \hat{\phi}_2^2 + \ldots + \hat{\phi}_n^{2p_1\ldots p_{n-2}}$, $F_2 = x_n^{2p_1\ldots p_{n-1}} + \ldots + y^2$.

Then, substituting (32) and (33) into (31), we have

$$\begin{aligned} b_{i-1}^2 \frac{\partial \hat{V}_i}{\partial \hat{\phi}_i}(\hat{x}_{i-1} - \hat{x}_{i-1}(h_m)) &\leq \varepsilon_i^2 + \xi_i^2 + \xi_{i-1}^2 + c_{i,7}(b_{i-1})\varepsilon_{i-1}^2 + \psi_1(T_{\max})F_3(h_m) \\ &+ \psi_2(T_{\max}) \int_{t-s^*T_{\max}}^{t} F_1(s)\,\mathrm{d}s + \vartheta_1 s^* T_{\max} \int_{t-s^*T_{\max}}^{t} F_2(s)\,\mathrm{d}s, \end{aligned} \tag{34}$$

where $\psi_1(T_{\max}) = \max\limits_i \{2^{2p_1\ldots p_{i-2}-1}(s^*T_{\max})^{2p_1\ldots p_{i-2}} b_{i-2}^{4p_1\ldots p_{i-2}}\}$, $\psi_2(T_{\max}) = \max\limits_i\{2^{2p_1\ldots p_{i-2}-1}(s^*T_{\max})^{2p_1\ldots p_{i-2}-1} b_{i-1}^{2p_1\ldots p_{i-2}}\}$, $F_3 = \hat{x}_{n-1}^{2p_1\ldots p_{n-2}} + \ldots + \hat{x}_2^{2p_1} + x_1^2$

According to (25), Lemmas 2.3 and 2.4, we can deduce that

$$\begin{aligned} \delta \xi_n^{2-\frac{1}{p_1\cdots p_{n-1}}}(u(h_m) - u^*) &\leq g_1 \sum_{i=1}^{n} \xi_i^2 + \sum_{i=2}^{n} \varepsilon_i^2(h_m) \\ &+ (x_n - x_n(h_m))^{2p_1\ldots p_{n-1}} + \ldots + (x_1(h_m) - x_1)^2, \end{aligned} \tag{35}$$

where $g_1$ is a positive constant. Similarly to (32) and (33), the following inequalities can be established

$$\begin{aligned} &(x_i - x_i(h_m))^{2p_1\ldots p_{i-1}} \leq \vartheta_i (s^*T_{\max})^{2p_1\ldots p_{i-1}-1} \int_{t-s^*T_{\max}}^{t} F_2(s)\,\mathrm{d}s, \\ &i = 2,\ \ldots,\ n-1, \end{aligned} \tag{36}$$

$$\begin{aligned} &(x_n - x_n(h_m))^{2p_1\ldots p_{n-1}} \leq g_2 T_{\max}^{2p_1\ldots p_{n-1}} F_2(h_m) \\ &+ \vartheta_n (s^*T_{\max})^{2p_1\ldots p_{n-1}-1} \int_{t-s^*T_{\max}}^{t} F_2(s)\,\mathrm{d}s + \sum_{i=2}^{n} \varepsilon_i^2(h_m), \end{aligned} \tag{37}$$

where $g_2,\ \vartheta_i,\ (i = 2,\ \ldots,\ n)$ are positive constants.

Consequently, from (35)–(37), we have

$$\begin{aligned} \delta \xi_n^{2-\frac{1}{p_1\cdots p_{n-1}}}(u(h_m) - u^*) &\leq g_1 \sum_{i=1}^{n} \xi_i^2 + \sum_{i=2}^{n} \varepsilon_i^2(h_m) + \\ g_2 T_{\max}^{2p_1\ldots p_{n-1}} F_2(h_m) &+ \psi_3(T_{\max}) \int_{t-s^*T_{\max}}^{t} F_2(s)\,\mathrm{d}s, \end{aligned} \tag{38}$$

where $\psi_3(T_{\max}) = \max\limits_i \{\vartheta_i (s^*T_{\max})^{2p_1\ldots p_{i-1}-1}\}$.

Let $U = V_n + \sum_{i=2}^{n} \hat{V}_i$. According to (27)−(30), (34) and (38), and selecting the observer gains $b_i$ in following order

$$\begin{aligned}
b_{n-1} &= \frac{1}{c_{n,2}}(\sigma + c_{n,4} + 3),\\
b_i &= \frac{1}{c_{i+1,2}}(\sigma + c_{i+1,4} + 3 + c_{i+2,8}(b_{i+1})), i = n-2\ ,\dots\ ,1,
\end{aligned}$$

where $c_{i,8}(b_{i-1}) = c_{i,1}(b_{i-1}) + c_{i,3}(b_{i-1}) + c_{i,5}(b_{i-1}) + c_{i,7}(b_{i-1})$, $\sigma$ is a positive constant, we arrive at

$$\begin{aligned}
\dot{U} \le{}& -(\rho - 4(n-1) - g_1)\sum_{i=1}^{n}\xi_i - \sigma\sum_{i=2}^{n}\varepsilon_i^2 + \sum_{i=2}^{n}\xi_i^2(h_m) + 2\sum_{i=2}^{n}\varepsilon_i^2(h_m)\\
&+ \psi_4(T_{\max})F_4(h_m) + (n-1)\psi_2(T_{\max})\int_{t-s^*T_{\max}}^{t} F_1(s)\,\mathrm{d}s\\
&+ (n-1)\psi_3(T_{\max})\int_{t-s^*T_{\max}}^{t} F_2(s)\,\mathrm{d}s,
\end{aligned}$$

where $\psi_4(T_{\max}) = \max\{(n-1)\psi_1(T_{\max}), g_2 T_{\max}^{2p_1\dots p_{n-1}}\}$, $F_4(h_m) = F_2(h_m) + F_3(h_m)$.

Construct the following functions

$$M_i = \int_{t-s^*T_{\max}}^{t}\int_{s}^{t} F_i(\rho)\,\mathrm{d}\rho\,\mathrm{d}s,\ i = 1,\ 2,$$

and differentiate $M_i$ with respect to $t$,

$$\dot{M}_i = s^*T_{\max}F_1 - \int_{t-s^*T_{\max}}^{t} F_i(s)\,\mathrm{d}s,\ i = 1,\ 2.$$

At the same time, we also have

$$M_i \le s^*T_{\max}\int_{t-s^*T_{\max}}^{t} F_i(s)\,\mathrm{d}s,\ i = 1,\ 2.$$

Let $N = M_1 + M_2 + U$, then,

$$\begin{aligned}
\dot{N} \le{}& -(\rho - 4n + 4 - g_1)\sum_{i=1}^{n}\xi_i - \sigma\sum_{i=2}^{n}\varepsilon_i^2 + \sum_{i=2}^{n}\xi_i^2(h_m) + 2\sum_{i=2}^{n}\varepsilon_i^2(h_m)\\
&+ \psi_4(T_{\max})F_4(h_m) + s^*T_{\max}F_5 - (1-(n-1)\psi_2(T_{\max}))\int_{t-s^*T_{\max}}^{t} F_1(s)\,\mathrm{d}s\\
&- (1-(n-1)\psi_3(T_{\max}))\int_{t-s^*T_{\max}}^{t} F_2(s)\,\mathrm{d}s.
\end{aligned}$$

where $F_5 = F_1 + F_2$.

Next, based on the weighted homogeneity theory [1, 11, 23], we can build some inequalities. Define the vector

$$\chi = (y, x_2, \ldots, x_n, \hat{x}_2, \ldots, \hat{x}_n),$$

and choose the dilation weight

$$\Omega = (1, \frac{1}{p_1}, \ldots, \frac{1}{p_1 \ldots p_{n-1}}, \frac{1}{p_1}, \ldots, \frac{1}{p_1 \ldots p_{n-1}}).$$

Define $F_6 = \xi_1^2 + \ldots + \xi_n^2 + \varepsilon_2^2 + \ldots + \varepsilon_n^2$. It is easily to verify that $F_4,\ F_5,\ F_6$ are homogeneous of degree 2. Then, there exist positive constants $g_3,\ g_4,\ g_5,\ g_6$ such that

$$F_4(h_m) \leq g_3 F_6(h_m),\ F_5 \leq g_4 F_6,\ g_5 F_6 \leq U \leq g_6 F_6.$$

We also have

$$M_i \leq s^* T_{\max} \int_{t - s^* T_{\max}}^{t} F_i(s)\, \mathrm{d}s,\ i = 1,\ 2.$$

Consider the error function $y(h_m) = e(t_k) + y(t_k)$ in the interval $[t_k,\ t_{k+1})$, the following inequalities finally hold

$$\begin{aligned}
&\dot{N} \leq -\bar{\rho} F_6 + (2 + \psi_4(T_{\max}) g_3) F_6(h_m) + g_4 s^* T_{\max} F_6 \\
&+ 2(2 + g_3 \psi_4(T_{\max}))(e^2(t_k) + y^2(t_k)) \\
&- \frac{1}{s^* T_{\max}}(1 - (n-1)\psi_2(T_{\max})) M_1 - \frac{1}{s^* T_{\max}}(1 - (n-1)\psi_3(T_{\max})) M_2 \\
&\leq -\frac{1}{g_6}(\bar{\rho} - g_4 s^* T_{\max}) U + \frac{2}{g_5}(2 + \psi_4(T_{\max}) g_3) U(t_k) \\
&- \frac{1}{s^* T_{\max}}(1 - (n-1)\psi_2(T_{\max})) M_1 - \frac{1}{s^* T_{\max}}(1 - (n-1)\psi_3(T_{\max})) M_2 \\
&+ \frac{1}{g_5}(2 + g_3 \psi_4(T_{\max})) U(h_m) + 2(2 + g_3 \psi_4(T_{\max})) \mu \Upsilon^m \\
&\leq -\alpha N + \beta N(t_k) + \gamma N(h_m) + \omega \Upsilon^m,
\end{aligned}$$

where

$$\begin{aligned}
&\bar{\rho} = \max\{\rho - 4n + 4 - g_1, \sigma\}, \\
&\alpha = \min\{\frac{1}{g_6}(\bar{\rho} - g_4 s^* T_{\max}), \frac{1}{s^* T_{\max}}(1 - (n-1)\psi_2(T_{\max})), \frac{1}{s^* T_{\max}}(1 - (n-1)\psi_3(T_{\max}))\}, \\
&\beta = \frac{2}{g_5}(2 + \psi_4(T_{\max}) g_3), \qquad \gamma = \frac{1}{g_5}(2 + g_3 \psi_4(T_{\max})), \qquad \omega = 2(2 + g_3 \psi_4(T_{\max})) \mu.
\end{aligned}$$

Note that $\psi_i(T_{\max}),\ (i = 2,\ 3,\ 4)$ are increasing functions with respect to $T_{\max}$, and satisfy $\psi_i(0) = 0$. There exists $0 < T_{\max} < T^*$ such that $\alpha > \beta + s^* \gamma$ holds. Then, based on Theorem 3.2, we can determine $T_{\min}$ such that the closed-loop system is globally exponentially stable. □

## 5. NUMERICAL SIMULATIONS

Consider the famous benchmark system shown in [5] with an unknown time-varying control coefficient $\delta$.

$$\dot{z}_1 = z_2^3 + 0.5z_1,\ \ \dot{z}_2 = \delta u,$$

where the unknown time-varying control coefficients $\delta = 1 + 0.1 \sin t$. We employ the reduced-order observer (22) and the discrete-time control input (23), and set $b_1 = 6.2,\ \kappa_1 = 42.9,\ \kappa_2 = 77.1$. The parameters of the event-triggered scheme are set as $\mu = 50,\ \Upsilon = 0.9$. The initial values are set as $(x_1,\ x_2,\ \hat{\phi}_2) = (12,\ -10,\ 0)$. The time-varying sampling periods are randomly generated on the interval $[0.03s,\ 0.05s]$. The simulation results under different triggering conditions (17) and (20) are shown in Figures 2−3 and Table 1 below. Clearly, under the same initial conditions, the date-releasing rate of the proposed triggered strategy is much lower than that of the strategy (17) investigated in [2, 25, 28], which implies that our event-triggered scheme (20) can effectively reduce the computing resource. At the same time, comparing Figure 1 and 2, we can notice that the slower the threshold decays, the slower the convergence rate of the closed-loop system. This is consistent with our conclusion in Section 3.

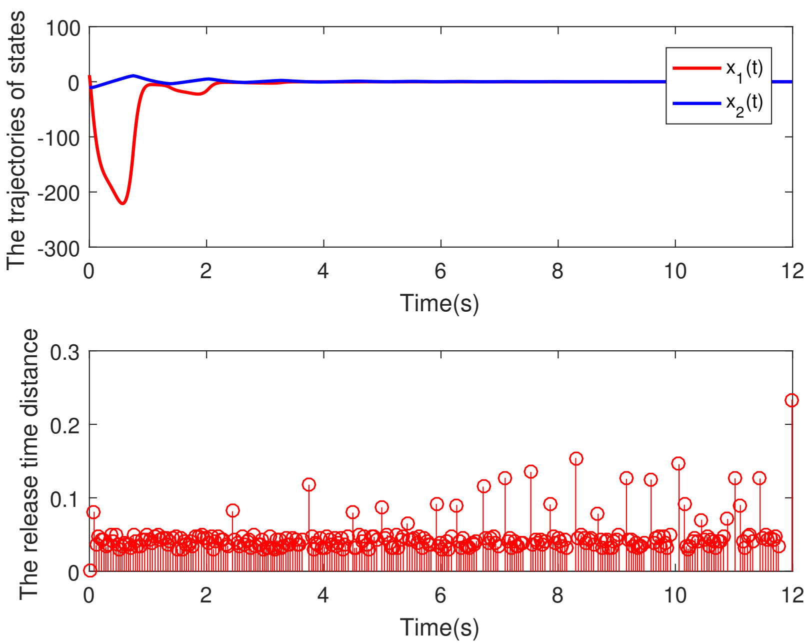


**Fig. 2.** The simulation results with the triggering condition (17).

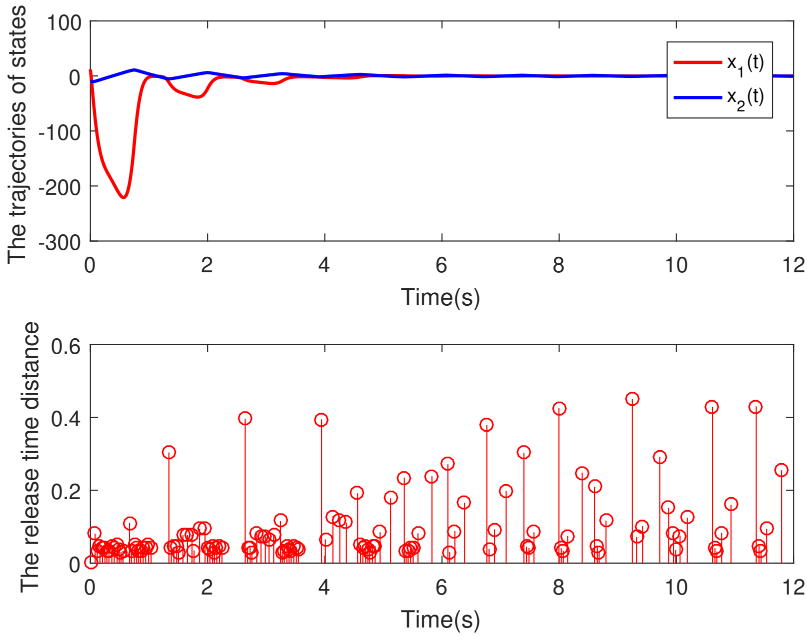


**Fig. 3.** The simulation results with the triggering condition (20).

| Triggering condition | Number of sampling times | Date-releasing rate | Maximum release intervals |
|---|---|---|---|
| (17) | 300 | 86.67% | 0.2335s |
| (20) | 300 | 40.67% | 0.4507s |

**Tab. 1.** Comparison with triggering condition (17).

## 6. CONCLUSION

In this paper, exponential stability for sampled-data-based event-triggered system was discussed. A general framework for designing these strategies was presented. Then, we applied the developed theory to the problem of stabilization of a class of inherently nonlinear systems. A novel event-triggered strategy with nonuniform sampled-data and a fully discretized dynamic output controller were designed. Finally, the rationality of the theory was verified by numerical simulations.

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*Zhang Li, College of Electrical Engineering and New Energy, China Three Gorges University, Yichang, Hubei, 443002. P. R. China.*
*e-mail: shenyj@ctgu.cn*

*Gang Yu, College of Electrical Engineering and New Energy, China Three Gorges University, Yichang, Hubei, 443002. P. R. China.*
*e-mail: 1415364634@qq.com*

*Yanjun Shen, Corresponding author. College of Electrical Engineering and New Energy, China Three Gorges University, Yichang, Hubei, 443002. P. R. China.*
*e-mail: shenyj@ctgu.cn*