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ON THE INTERSECTION GRAPH OF A FINITE GROUP

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Abstract. For a finite group G, the intersection graph of G which is denoted by $\Gamma(G)$ is an undirected graph such that its vertices are all nontrivial proper subgroups of G and two distinct vertices H and K are adjacent when $H \cap K \neq 1$. In this paper we classify all finite groups whose intersection graphs are regular. Also, we find some results on the intersection graphs of simple groups and finally we study the structure of $\mathrm{Aut}(\Gamma(G))$.

Keywords: intersection graph; regular graph; simple group; automorphism group

MSC 2010: 05C25, 20E32

1. Introduction

For a graph Γ, the set of vertices and the set of edges of Γ are denoted by $V(\Gamma)$ and $E(\Gamma)$, respectively. A *null graph* is a graph with no edge and a graph in which all vertices are adjacent is called a *complete graph*. The degree of a vertex v in Γ is denoted by $\deg(v)$. A *regular graph* is a graph in which $\deg(u) = \deg(v)$ for all $u, v \in V(\Gamma)$. If u and v are two distinct vertices of a graph Γ, then a *path* between u and v is defined as a sequence of distinct vertices $u = v_0, v_1, \ldots, v_n = v$ such that $(v_i, v_{i+1}) \in E(\Gamma)$ for $0 \leqslant i \leqslant n-1$, and n is called the *length* of this path. The *distance* between u and v is the length of a shortest path between u and v and is denoted by $d(u, v)$. If there is no path connecting u and v we define $d(u, v)$ to be infinite. A graph is *connected* whenever there exists a path between each two distinct vertices. An automorphism of a graph Γ is a bijection φ on the vertices of Γ such that φ preserves adjacency and nonadjacency of every pair of vertices.

For a finite group G, the intersection graph of G which is denoted by $\Gamma(G)$ is an undirected graph such that its vertices are all nontrivial proper subgroups of G and two distinct vertices H and K are adjacent when $H \cap K \neq 1$. Csákány and Pollák in [4] introduced the intersection graph of a finite group. Zelinka in [12] considered

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the intersection graphs of finite abelian groups. Kayacan and Yaraneri in [7] studied finite groups with planar intersection graphs. Also, in [6] they considered finite abelian groups with isomorphic intersection graphs. Shen in [11] determined all finite groups with disconnected intersection graphs. Xuanlong Ma in [9] gave an upper bound for the diameter of intersection graphs of finite simple groups. Akbari et al. in [1] classified finite groups whose intersection graphs are triangle-free. Also, they determined all finite groups with null intersection graphs or complete intersection graphs. Obviously these groups have regular intersection graphs. It is a natural question to ask which finite groups have regular intersection graphs. In this paper we show that the only possibilities for the regular intersection graphs are the null graphs and the complete graphs. Also, we get a criterion for the simplicity of a finite group G according to its intersection graph. Finally, we show that if Γ is a graph with more than one vertex such that $\mathrm{Aut}(\Gamma) = 1$, then Γ is not the intersection graph of a finite group. Throughout this paper, following [3], for prime numbers p and q, where $q \mid p-1$, by $p : q$ we mean the semidirect product of $\mathbb{Z}_p$ and $\mathbb{Z}_q$ which is defined as $p : q = \langle a, b;\ a^p = b^q = 1,\ b^{-1}ab = a^r,\ r^q \equiv 1 \pmod{p},\ r \not\equiv 1 \pmod{p}\rangle$.

2. Finite groups with regular intersection graphs

For finite groups, Akbari and his coauthors in [1] proved the following lemmas:

Lemma 2.1. *Let G be a finite group. Then $\Gamma(G)$ is a complete graph if and only if G is isomorphic to $\mathbb{Z}_{p^n}$ for some prime number p and some integer $n \geqslant 2$ or Q_{2^n} for some integer $n \geqslant 3$.*

Lemma 2.2. *Let G be a finite group. Then $\Gamma(G)$ is a null graph if and only if G is isomorphic to one of the following groups: $\mathbb{Z}_{p^2}$, $\mathbb{Z}_p \times \mathbb{Z}_p$, $\mathbb{Z}_{pq}$ or $p : q$ for some primes $q < p$.*

We know that null graphs and complete graphs are special cases of regular graphs. In this section we show that the only possible regular intersection graphs are exactly the same.

A finite group G is called a *Schmidt group* if G is not a nilpotent group but every subgroup of G is a nilpotent group. The following lemma which was proved by Bolinches, Romero and Robinson in [2] gives us the classification of Schmidt groups.

Lemma 2.3. *The Schmidt groups are exactly of the types described in* I *to* III *below. Let p and q be distinct primes.*

Type I: *$G \cong P \rtimes Q$, where $Q = \langle z \rangle$ is cyclic of order $q^r > 1$, with q a prime not dividing $p-1$, and P is an irreducible Q-module over the field of p elements with centralizer $\langle z^q \rangle$ in Q.*

Type II: *$G \cong P \rtimes Q$, where P is a nonabelian special p-group of rank $2m$, the order of p modulo q being $2m$, $Q = \langle z \rangle$ is cyclic of order $q^r > 1$, z induces an automorphism in P such that $P/\Phi(P)$ is a faithful irreducible Q-module, and z centralizes $\Phi(P)$. Furthermore, $|P/\Phi(P)| = p^{2m}$ and $|P'| \leqslant p^m$.*

Type III: *$G \cong P \rtimes Q$, where $P = \langle a \rangle$ is a normal subgroup of order p, $Q = \langle z \rangle$ is cyclic of order $q^r > 1$, and $a^z = a^i$, where i is the least primitive qth root of unity modulo p.*

Before presenting the main result of this section, we need the following lemma which determines the structure of a special class of p-groups.

Lemma 2.4 ([5], Theorem 12.5.2)**.** *A p-group which contains only one subgroup of order p is cyclic or a generalized quaternion group.*

Now we are ready to state the main result of this section.

Theorem 2.5. *Let G be a finite group. Then the intersection graph of G is regular if and only if one of the following cases occurs:*

1. *$G \cong \mathbb{Z}_{p^n}$ for some prime power p^n,*
2. *$G \cong Q_{2^n}$, a generalized quaternion group,*
3. *$G \cong \mathbb{Z}_p \times \mathbb{Z}_q$, where p and q are primes,*
4. *$G \cong p : q$, where p and q are primes and $q \mid p-1$.*

Proof. Let G be a group of one of the above types. Using Lemmas 2.1 and 2.2 we get that $\Gamma(G)$ is a complete graph or a null graph, which are regular.

Conversely, let G be a finite group whose intersection graph is regular. For every vertex $u \in V(\Gamma(G))$ let $N_1(u)$ be the set of all vertices which are adjacent to u, i.e., $N_1(u) = \{v \in V(\Gamma(G))\colon (u,v) \in E(\Gamma(G))\}$. Regularity of $\Gamma(G)$ is equivalent to $|N_1(u)| = |N_1(v)|$ for all vertices $u, v \in V(\Gamma(G))$. Let M be a maximal subgroup of G and L be a minimal subgroup of M. Obviously, if H is an arbitrary subgroup of G such that $H \cap L \neq 1$, then $H \cap M \neq 1$ and hence $N_1(L) - \{M\} \subseteq N_1(M) - \{L\}$. By the assumption $|N_1(L)| = |N_1(M)|$. Therefore $N_1(L) - \{M\} = N_1(M) - \{L\}$. Now if L' is a minimal subgroup of M and $L' \neq L$, then $L' \in N_1(M) - \{L\}$ and $L' \notin N_1(L) - \{M\}$, which is a contradiction. Thus every maximal subgroup of G contains a unique minimal subgroup, which implies that every maximal subgroup has only one prime divisor. Now we consider two cases:

Case i: Let G be a nilpotent group. Then G is a direct product of its Sylow subgroups. If there is more than one Sylow subgroup, by taking a maximal subgroup of one of them, we see that G is a direct product of two cyclic subgroups of prime order, i.e., $G \cong \mathbb{Z}_p \times \mathbb{Z}_q$. So let G be a p-group for a prime p. Since $Z(G) \neq 1$, let L be a minimal subgroup of $Z(G)$. If G has a minimal subgroup S, where $S \neq L$, then SL cannot be contained in a proper subgroup of G. Consequently, G is isomorphic to $\mathbb{Z}_p \times \mathbb{Z}_p$. Otherwise, G has a unique minimal subgroup and by Lemma 2.4, G is cyclic of prime power order or G is a generalized quaternion group.

Case ii: Let G be a nonnilpotent group. As we stated above, every maximal subgroup of G is a p-group. So G is a minimal nonnilpotent group and by Lemma 2.3, G is of type I, II or III.

The groups of type I have a proper noncyclic, elementary abelian Sylow p-subgroup. The groups of type II have a Sylow p-subgroup P with nontrivial Frattini subgroup, and so if Q is a Sylow q-subgroup, then $\Phi(P)Q$ is not a group of prime power order. The only remaining possibility is a group of type III, a semidirect product of a subgroup P of order p with a cyclic subgroup Q of order q^r with $r \geqslant 1$. Finally $r = 1$, since the product of P and the maximal subgroup of Q should be of prime power order. In this case we have $G \cong p : q$. □

3. Intersection graphs of simple groups

As an application of intersection graphs, we get some criteria for the simplicity of a group. The following lemma, which is the main theorem of [11], shows that if Γ is a disconnected graph, then Γ is not the intersection graph of a simple group.

Lemma 3.1 ([11], Theorem)**.** *A finite group with a disconnected intersection graph is $\mathbb{Z}_p \times \mathbb{Z}_q$, where both p, q are primes, or a Frobenius group whose complement is a prime order group and the kernel is a minimal normal subgroup.*

Theorem 3.2. *If $\Gamma(G)$ contains a vertex u such that u is a fixed point of every automorphism of $\Gamma(G)$, then G is not a simple group or a direct product of isomorphic simple groups.*

P r o o f. Let H and K be two subgroups of G. Then for all $\varphi \in \mathrm{Aut}(G)$ we have $H \cap K \neq 1$ if and only if $\varphi(H) \cap \varphi(K) \neq 1$. This implies that every automorphism of G preserves adjacency (and nonadjacency) of vertices of $\Gamma(G)$ and hence every automorphism of G induces an automorphism of $\Gamma(G)$. Now if u is a vertex of $\Gamma(G)$ such that u is a fixed point of every automorphism of $\Gamma(G)$, then u is a fixed point of every automorphism of G, i.e., u is a characteristic subgroup of G. So we get the result. □

In the following example we investigate the intersection graph of the simple group $G = \mathrm{PSL}_2(23)$, and we see that for every vertex $u \in V(\Gamma(G))$, there exists a vertex $v \in V(\Gamma(G))$ such that $d(u, v) \geqslant 3$.

Example 3.3. Let $G = \mathrm{PSL}_2(23)$. We first show that for every maximal subgroup M, there exists a nontrivial proper subgroup L such that $d(M, L) \geqslant 3$ in $\Gamma(G)$. By [3], page 15, every maximal subgroup of G is isomorphic to one of the groups $23 : 11$, S_4, D_{24} or D_{22}. If $M \cong 23 : 11$, then let L be an arbitrary subgroup of order 3. Suppose that $d(M, L) = 2$ and $M - H - L$ is a shortest path of length 2 between M and L. Note that $M \cap H \neq 1$ and hence the order of H must be divisible by 23 or 11. Similarly we see that the order of H must be divisible by 3. Consequently, the order of every maximal subgroup containing H must be divisible by $23 \cdot 3$ or $11 \cdot 3$, a contradiction. Thus $d(M, L) \geqslant 3$. If $M \cong S_4$ or D_{24}, then let L be an arbitrary subgroup of order 23 and similarly we get the result. Now suppose that $M \cong D_{22}$. Note that M contains a unique subgroup of order 11, say K. If K is contained in all maximal subgroups of type $23 : 11$, then $1 \neq K \subseteq \mathrm{Core}_G(U)$ for some maximal subgroup U of G, where $U \cong 23 : 11$, which is a contradiction. Thus there exists a maximal subgroup of order $23 \cdot 11$, say M_1, such that $M \cap M_1 = 1$. Let $L \leqslant M_1$ be the unique subgroup of M_1 of order 23. Since every subgroup of G isomorphic to $23 : 11$ is uniquely determined by an element of order 11 and an element of order 23, then M_1 is the only subgroup of G containing properly L. If $d(M, L) = 2$ and $M - H - L$ is a path between M and L, then we have $H = M_1$, which implies that $M \cap M_1 \neq 1$, a contradiction. Thus $d(M, L) \geqslant 3$. So far, we have shown that for every maximal subgroup M, there exists a nontrivial proper subgroup L such that $d(M, L) \geqslant 3$. Now let K be any nontrivial proper subgroup of G and M be a maximal subgroup containing K. Suppose that $d(K, L) \leqslant 2$ for all subgroups $L \leqslant G$. Note that if a subgroup has nontrivial intersection with K, then it has nontrivial intersection with M and so $d(M, L) \leqslant 2$ for all subgroups $L \leqslant G$, a contradiction. Thus for every vertex $u \in V(\Gamma(\mathrm{PSL}_2(23)))$, there exists a vertex v such that $d(u, v) \geqslant 3$.

In the following theorem, using distances in an intersection graph, we get a criterion for the simplicity of a finite group.

Theorem 3.4. *Let G be a finite group whose intersection graph $\Gamma(G)$ is connected. If for every vertex $u \in V(\Gamma(G))$ there exists a vertex $v \in V(\Gamma(G))$ such that $d(u, v) \geqslant 3$, then G is a simple group.*

Proof. To the contrary, suppose that $N \neq 1$ is a normal subgroup of G. Then by the hypothesis there exists a nontrivial subgroup H such that $d(N, H) \geqslant 3$. We know that NH is a subgroup of G and if $NH \neq G$, then we have the path $H - NH - N$, which is a contradiction and hence $G = NH$. Now if H contains a nontrivial proper subgroup K, then $NK \neq NH = G$, since $H \cap N = 1$. In addition, both $H \cap NK$ and $N \cap NK$ are nontrivial subgroups and consequently we have the path $H - NK - N$, a contradiction. Thus H has no nontrivial proper subgroup, which means that H is a minimal subgroup of G. Now let M be a maximal subgroup of G which properly contains H. Then $G = NH \leqslant NM$, which implies that $M \cap N \neq 1$. But in this case we have the path $H - M - N$, which is clearly a contradiction. Then H is a minimal subgroup of G which is a maximal subgroup of G, i.e., H is an isolated vertex of $\Gamma(G)$ and this is a contradiction by the connectivity of $\Gamma(G)$. Therefore G has no nontrivial normal subgroup and hence G is a simple group. $\square$

Now we give an example of a simple group which shows that the condition of the above theorem is not necessary for the simplicity of a finite group, and hence the converse of Theorem 3.4 is not true in general.

Example 3.5. Let $G = A_5$ which naturally acts on the set $\{1, 2, 3, 4, 5\}$. Now consider the stabilizer of an arbitrary element, say G_5, which is obviously isomorphic to A_4. We show that for every minimal subgroup L, there exists a path with length at most 2 between G_5 and L. Let $x \notin G_5$ be an element of prime order. Then x has one of the following permutation forms:

$$(a\,b)(c\,5), \quad (a\,b\,5) \quad \text{or} \quad (a\,b\,c\,d\,5), \qquad \text{where } \{a, b, c, d\} = \{1, 2, 3, 4\}.$$

If $x = (a\,b)(c\,5)$, choose $y = (a\,b\,d) \in G_5$. Obviously we have $\langle x, y\rangle \cong S_3$ and hence $\langle x, y\rangle < G$. Furthermore, $\langle x\rangle \cap \langle x, y\rangle \neq 1$ and also $\langle x, y\rangle \cap G_5 \neq 1$. Thus $\langle x\rangle - \langle x, y\rangle - G_5$ is a path between $L = \langle x\rangle$ and G_5. If $x = (a\,b\,5)$, choose $y = (a\,b\,c) \in G_5$. In this case $\langle x, y\rangle \cong A_4$ (the stabilizer subgroup of d) and similarly we get the result. If $x = (a\,b\,c\,d\,5)$, choose $y = (a\,d)(b\,c) \in G_5$ and let $K = \langle x, y\rangle$. Note that $K \cong D_{10}$ and hence $\langle x, y\rangle < G$. Therefore $K \neq G$ and we have the path $\langle x\rangle - \langle x, y\rangle - G_5$. Now it is obvious that if H is an arbitrary subgroup of G, then H contains a minimal subgroup L and so there exists a path with length at most 2 between H and G_5.

Remark 3.6. It is a natural question to ask which simple groups admit the conditions of Theorem 3.4.

The above counterexample shows that Theorem 3.4 gives only a sufficient condition for the simplicity of a group. In the next theorem we give a necessary condition for the simplicity of G.

Theorem 3.7. *If G is a nonabelian simple group, then there exist at least two vertices $u, v \in V(\Gamma(G))$ such that $d(u, v) \geqslant 3$.*

P r o o f. King in [8] has shown that every nonabelian finite simple group can be generated by an element of order 2 and an element of some odd prime order p. So let x and y be the generators of G which are of orders 2 and p, respectively. It is easy to check that if H is a subgroup of G which has nontrivial intersections with $\langle x\rangle$ and $\langle y\rangle$, then H contains both x and y, and hence $H = G$. Thus every path between $\langle x\rangle$ and $\langle y\rangle$ has length at least 3. $\square$

We see that in the intersection graphs of groups D_8 and S_5, there exist some vertices with distance at least 3, while obviously they are not simple. So the converse of the above theorem is not true in general. In the following lemma we give a necessary condition for the existence of vertices with distance at least 3 in the intersection graph of a finite group.

Lemma 3.8. *Let G be a finite group. If there exist two vertices $u, v \in V(\Gamma(G))$ such that $d(u, v) \geqslant 3$, then G can be generated by two elements of some prime orders.*

P r o o f. Let H and K be two subgroups of G such that $d(H, K) \geqslant 3$ in $\Gamma(G)$. Suppose that x and y are some prime order elements of H and K, respectively, and let $L = \langle x, y\rangle$. If L is a proper subgroup of G, then L has nontrivial intersections with H and K, and hence we have the path $H - L - K$, a contradiction. So $L = G$ and we get the result. $\square$

4. On the group of automorphisms of $\Gamma(G)$

In this section we get some results on Aut(Γ(G)). We first state the following lemma, which determines the structure of finite groups in which all subgroups are normal (these groups are called *Dedekind groups*).

Lemma 4.1 ([10], Theorem 5.3.7, Dedekind, Baer)**.** *All the subgroups of a group G are normal if and only if G is abelian or the direct product of a quaternion group of order* 8*, an elementary abelian* 2*-group and an abelian group with all its elements of odd order.*

This has the following useful corollary.

Corollary 4.2. *Let G be a finite group. Then for each subgroup H of G and each $\varphi \in \mathrm{Aut}(G)$, we have $\varphi(H) = H$ if and only if G is a cyclic group.*

P r o o f. If G is a cyclic group, then G has a unique subgroup of order d for each divisor d of $|G|$ and we get the result. Conversely, if $\varphi(H) = H$ for all $H \leqslant G$ and all $\varphi \in \mathrm{Aut}(G)$, then every subgroup of G is a normal subgroup, i.e., G is a Dedekind group. If G contains Q_8 as a direct factor, then let φ be an automorphism of G such that $\varphi(i) = j$ and $\varphi(j) = i$ and $\varphi(H) = H$ for all subgroups not containing i or j, where $Q_8 = \langle i, j \colon i^4 = 1,\ i^2 = j^2,\ j^{-1}ij = i^{-1}\rangle$. Obviously φ is a nontrivial automorphism of G such that $\varphi(\langle i\rangle) \neq \langle i\rangle$, which is a contradiction. Thus by Lemma 4.1, G is an abelian group. To show that G is a cyclic group, it suffices to prove it in the case when G is a p-group, this can be easily verified by induction on $|G|$ and we get the result. □

Theorem 4.3. *Let G be a finite group and $\Gamma(G)$ be its intersection graph. Then there is a homomorphism from* $\mathrm{Aut}(G)$ *to* $\mathrm{Aut}(\Gamma(G))$. *Furthermore, this homomorphism is nontrivial if and only if G is not a cyclic group.*

Proof. Let $\varphi \in \mathrm{Aut}(G)$. We define $\overline{\varphi}$ on the set of all proper nontrivial subgroups of G by $\overline{\varphi}(H) := \varphi(H)$, where $\varphi(H) = \{\varphi(h) \colon h \in H\}$. Obviously $\overline{\varphi}$ is a bijection on the set of all proper nontrivial subgroups of G. Also $\overline{\varphi}(H \cap K) = \overline{\varphi}(H) \cap \overline{\varphi}(K)$ for all subgroups H, K and hence $\overline{\varphi}$ is an automorphism of $\Gamma(G)$. Define $f \colon \mathrm{Aut}(G) \to \mathrm{Aut}(\Gamma(G))$ by $f(\varphi) := \overline{\varphi}$. Obviously f is a homomorphism and the kernel of f is $\mathrm{A}(G)$, the group of automorphisms of G which map every subgroup of G onto itself (these automorphisms are called *power automorphisms*). Consequently $\mathrm{Aut}(G)/\mathrm{A}(G) \hookrightarrow \mathrm{Aut}(\Gamma(G))$. By Corollary 4.2, groups in which $\mathrm{Aut(G)}/\mathrm{A}(G) = 1$ are only cyclic groups. Thus, if G is a noncyclic group, then the intersection graph of G has at least one nontrivial automorphism which is induced by an automorphism of G. □

Theorem 4.4. *Let G be a finite group with at least two nontrivial proper subgroups. Then* $\mathrm{Aut}(\Gamma(G))$ *is a nontrivial group.*

Proof. By Theorem 4.3, it suffices to prove the result in the case when G is a cyclic group. We consider two cases:

Case a. If G contains a subgroup $H \cong \mathbb{Z}_{p^2}$ for some prime divisor p of $|G|$, then H contains a unique subgroup L of order p and obviously we have $L \cap K \neq 1$ if and only if $H \cap K \neq 1$ for each $K \leqslant G$. Now define $\sigma \colon V(\Gamma(G)) \to V(\Gamma(G))$ by $\sigma(H) = L$, $\sigma(L) = H$ and if $K \notin \{H, L\}$, define $\sigma(K) = K$. Clearly σ is a bijection on $V(\Gamma(G))$ and preserves adjacency and nonadjacency in $\Gamma(G)$. Therefore σ is a nontrivial automorphism of $\Gamma(G)$.

Case b. Suppose that G does not contain any subgroup $H \cong \mathbb{Z}_{p^2}$ for each prime p. Then $G \cong \mathbb{Z}_{p_1} \times \mathbb{Z}_{p_2} \times \ldots \times \mathbb{Z}_{p_n}$ for some distinct primes $p_1, p_2, \ldots, p_n$. Let L_1 and L_2 be two distinct minimal subgroups of G. If a subgroup K_1 contains L_1 and $L_2 \nleqslant K_1$, then K_1 is a split extension of L_1 and hence K_1/L_1 is isomorphic to a unique subgroup of G, say T_1, which does not contain L_1. Now let $K_2 = T_1 L_2$, which obviously contains L_2. Using this correspondence define $\sigma \colon V(\Gamma(G)) \to V(\Gamma(G))$ by

$\sigma(K_1) = K_2$ if K_1 contains L_1 and $L_2 \nleq K_1$, similarly $\sigma(K_2) = K_1$ if K_2 contains L_2 and $L_1 \nleq K_2$, and otherwise $\sigma(K) = K$. Obviously σ is a bijection and preserves adjacency and nonadjacency in $\Gamma(G)$. So we get the result. $\square$

In the sequel we pose the following question:

Question. There are many groups in which $\mathrm{Aut}(\Gamma(G))$ is of even order. Is there any group such that $\mathrm{Aut}(\Gamma(G))$ is of odd order?

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