Kybernetika

Tianwei Zhang; Yongzhi Liao

Existence and global attractivity of positive almost periodic solutions for a kind of fishing model with pure-delay

Kybernetika, Vol. 53 (2017), No. 4, 612–629

Persistent URL: http://dml.cz/dmlcz/146947

KYBERNETIKA — VOLUME *53* (2017), NUMBER 4, PAGES 612–629

EXISTENCE AND GLOBAL ATTRACTIVITY OF POSITIVE ALMOST PERIODIC SOLUTIONS FOR A KIND OF FISHING MODEL WITH PURE-DELAY

Tianwei Zhang and Yongzhi Liao

By using some analytical techniques, modified inequalities and Mawhin's continuation theorem of coincidence degree theory, some sufficient conditions for the existence of at least one positive almost periodic solution of a kind of fishing model with delay are obtained. Further, the global attractivity of the positive almost periodic solution of this model is also considered. Finally, three examples are given to illustrate the main results of this paper.

Keywords: almost periodic solution, coincidence degree, fishing model, global attractivity

Classification: 34K13, 92D25

1. INTRODUCTION

In 2008, Berezansky and Idels [2] proposed a kind of time-varying fishing model which describes how fish harvested in the form of

$$\dot{N}(t) = N(t)\left[\frac{a(t)}{1+\left[\frac{N(t-\theta(t))}{K(t)}\right]^r} - b(t)\right], \tag{1.1}$$

where x is is the population biomass, a is the per-capita fecundity rate, K is the carrying capacity of the environment, b is the per-capita mortality rate, $r > 0$, that controls how rapidly density dependence sets in, can be regarded as an abruptness parameter, $\theta(t)$ is the maturation time delay, a, K, b, $\theta \in C([0,\infty],[0,\infty])$. Berezansky and Idels [2] proposed Eq. (1.1) and studied its persistence, furthermore, the existence and stability of a positive periodic solution to Eq. (1.1) were also considered. Before continuing, Wang [23] investigated Eq. (1.1) and obtained some sufficient conditions for the existence of at least one positive periodic solution by using Mawhin's continuation theorem of coincidence degree theory. For more results in this direction, one could refer to [1, 11, 13, 14, 32] and the references cited therein.

In real world phenomenon, the environment varies due to the factors such as seasonal effects of weather, food supplies, mating habits and harvesting, etc. So it is usual to assume the periodicity of parameters in the systems. However, in applications, if

DOI: 10.14736/kyb-2017-4-0612

the various constituent components of the temporally nonuniform environment is with incommensurable (nonintegral multiples, see Example 1.1) periods, then one has to consider the environment to be almost periodic since there is no a priori reason to expect the existence of periodic solutions. Hence, if we consider the effects of the environmental factors, almost periodicity is sometimes more realistic and more general than periodicity. In recent years, the almost periodic solution of the continuous models in biological populations has been studied extensively (see [16, 21, 26, 27, 28, 29, 30, 31, 32] and the references cited therein).

Example 1.1. Let us consider the following simple fishing model:

$$\dot{N}(t) = N(t)\left[\frac{|\sin(\sqrt{2}t)|}{1+\left[\frac{N(t)}{2}\right]^{0.5}} - |\sin(\sqrt{3}t)|\right]. \tag{1.2}$$

In Eq. (1.2), $|\sin(\sqrt{2}t)|$ is $\frac{\sqrt{2}\pi}{2}$-periodic function and $|\sin(\sqrt{3}t)|$ is $\frac{\sqrt{3}\pi}{3}$-periodic function, which imply that Eq. (1.2) is with incommensurable periods. Then there is no a priori reason to expect the existence of periodic solutions of Eq. (1.2). Thus, it is significant to study the existence of almost periodic solutions of Eq. (1.2).

It is well known that Mawhin's continuation theorem of coincidence degree theory is an important method to investigate the existence of positive periodic solutions of some kinds of non-linear ecosystems (see [3, 4, 5, 6, 7, 8, 15, 16, 22, 25, 33, 34]). However, it is difficult to be used to investigate the existence of positive almost periodic solutions of non-linear ecosystems. Therefore, to the best of the author's knowledge, so far, there are scarcely any papers concerning with the existence of positive almost periodic solutions of Eq. (1.1) by using Mawhin's continuation theorem. Motivated by the above reason, the main purpose of this paper is to establish some new sufficient conditions on the existence of positive almost periodic solutions of Eq. (1.1) by using Mawhin's continuation theorem of coincidence degree theory.

Let $\mathbb{R}$, $\mathbb{Z}$ and $\mathbb{N}^+$ denote the sets of real numbers, integers and positive integers, respectively, $C(\mathbb{X},\mathbb{Y})$ and $C^1(\mathbb{X},\mathbb{Y})$ be the space of continuous functions and continuously differential functions which map $\mathbb{X}$ into $\mathbb{Y}$, respectively. Especially, $C(\mathbb{X}) := C(\mathbb{X},\mathbb{X})$, $C^1(\mathbb{X}) := C^1(\mathbb{X},\mathbb{X})$. Related to a continuous bounded function f, we use the following notations:

$$f^- = \inf_{s\in\mathbb{R}} f(s), \quad f^+ = \sup_{s\in\mathbb{R}} f(s), \quad |f|_\infty = \sup_{s\in\mathbb{R}} |f(s)|, \quad \bar{f} = \lim_{T\to\infty}\frac{1}{T}\int_0^T f(s)\,\mathrm{d}s.$$

The initial condition associated with Eq. (1.1) is of the form

$$N(s) = \varphi(s), \quad \forall s\in[-\theta^+,0], \quad N(0) = N_0 > 0, \quad \varphi\in C([-\theta^+,0],\mathbb{R}).$$

Throughout this paper, we always make the following assumption for Eq. (1.1):

(H_1) a, b, K and θ are nonnegative almost periodic functions with $K^- > 0$.

The paper is organized as follows. In Section 2, we give some basic definitions and necessary lemmas which will be used in later sections. In Section 3, we obtain some sufficient conditions for the existence of at least one positive almost periodic solution of Eq. (1.1) by means of Mawhin's continuation theorem of coincidence degree theory. In Section 4, we consider the global attractivity of a unique positive almost periodic solution to Eq. (1.1). Three examples are also given to illustrate our results in Section 5.

2. PRELIMINARIES

Definition 2.1. (He [12], Gaines and Mawhin [10]) $x \in C(\mathbb{R}, \mathbb{R}^n)$ is called almost periodic, if for any $\epsilon > 0$, it is possible to find a real number $l = l(\epsilon) > 0$, for any interval with length $l(\epsilon)$, there exists a number $\tau = \tau(\epsilon)$ in this interval such that $\|x(t+\tau) - x(t)\| < \epsilon$, $\forall t \in \mathbb{R}$, where $\|\cdot\|$ is arbitrary norm of $\mathbb{R}^n$. τ is called to the ϵ-almost period of x, $T(x, \epsilon)$ denotes the set of ϵ-almost periods for x and $l(\epsilon)$ is called to the length of the inclusion interval for $T(x, \epsilon)$. The collection of those functions is denoted by $AP(\mathbb{R}, \mathbb{R}^n)$. Let $AP(\mathbb{R}) := AP(\mathbb{R}, \mathbb{R})$.

Lemma 2.1. (Zhang [27]) Assume that $x \in AP(\mathbb{R}) \cap C^1(\mathbb{R})$ with $\dot{x} \in C(\mathbb{R})$. For arbitrary interval $[a, b]$ with $b - a = \omega > 0$, let $\xi, \eta \in [a, b]$ and

$$I_1 = \big\{s \in [\xi, b] : \dot{x}(s) \ge 0\big\}, \quad I_2 = \big\{s \in [\eta, b] : \dot{x}(s) \le 0\big\},$$

then ones have

$$x(t) \le x(\xi) + \int_{I_1} \dot{x}(s)\,\mathrm{d}s, \quad \forall t \in [\xi, b], \quad x(t) \ge x(\eta) + \int_{I_2} \dot{x}(s)\,\mathrm{d}s, \quad \forall t \in [\eta, b].$$

Lemma 2.2. (Zhang [27]) If $x \in AP(\mathbb{R})$, then for arbitrary interval $I = [a, b]$ with $b - a = \omega > 0$, there exist $\xi \in [a, b]$, $\underline{\xi} \in (-\infty, a]$ and $\bar{\xi} \in [b, +\infty)$ such that

$$x(\underline{\xi}) = x(\bar{\xi}) \quad \text{and} \quad x(\xi) \le x(s), \ \ \forall s \in [\underline{\xi}, \bar{\xi}].$$

Lemma 2.3. (Zhang [27]) If $x \in AP(\mathbb{R})$, then for arbitrary interval $[a, b]$ with $I = b - a = \omega > 0$, there exist $\eta \in [a, b]$, $\underline{\eta} \in (-\infty, a]$ and $\bar{\eta} \in [b, +\infty)$ such that

$$x(\underline{\eta}) = x(\bar{\eta}) \quad \text{and} \quad x(\eta) \ge x(s), \ \ \forall s \in [\underline{\eta}, \bar{\eta}].$$

Lemma 2.4. (Zhang [27]) If $x \in AP(\mathbb{R})$, then for $\forall n \in \mathbb{N}^+$, there exists $\alpha_n \in \mathbb{R}$ such that $x(\alpha_n) \in [x^* - \frac{1}{n}, x^*]$, where $x^* = \sup_{s \in \mathbb{R}} x(s)$.

For $x \in AP(\mathbb{R})$, we denote by

$$a(x, \varpi) = \lim_{T \to \infty} \frac{1}{T} \int_0^T x(s) e^{-\mathrm{i}\varpi s}\,\mathrm{d}s,$$

$$\Lambda(x) = \left\{\varpi \in \mathbb{R} : \lim_{T \to \infty} \frac{1}{T} \int_0^T x(s) e^{-\mathrm{i}\varpi s} \mathrm{d}s \ne 0\right\}$$

the Bohr transform and the set of Fourier exponents of x, respectively.

Lemma 2.5. (Zhang [27]) Assume that $x \in AP(\mathbb{R})$ and $\bar{x} > 0$, then for $\forall t_0 \in \mathbb{R}$, there exists a positive constant T_0 independent of t_0 such that

$$\frac{1}{T}\int_{t_0}^{t_0+T} x(s)\,\mathrm{d}s \in \left[\frac{\bar{x}}{2}, \frac{3\bar{x}}{2}\right], \quad \forall T \geq T_0.$$

Following we recall the famous Mawhin's continuation theorem.

Let $\mathbb{X}$ and $\mathbb{Y}$ be real Banach spaces, $L : \mathrm{Dom}L \subseteq \mathbb{X} \to \mathbb{Y}$ be a linear mapping and $N : \mathbb{X} \to \mathbb{Y}$ be a continuous mapping. The mapping L is called a Fredholm mapping of index zero if the following conditions hold:

- $\mathrm{Im}L$ is closed in $\mathbb{Y}$;
- $\mathrm{dimKer}L = \mathrm{codimIm}L < +\infty$.

If L is a Fredholm mapping of index zero and there exist continuous projectors $P : \mathbb{X} \to \mathbb{X}$ and $Q : \mathbb{Y} \to \mathbb{Y}$ such that $\mathrm{Im}P = \mathrm{Ker}L$, $\mathrm{Ker}Q = \mathrm{Im}L = \mathrm{Im}(I - Q)$. It follows that $L|_{\mathrm{Dom}L\cap\mathrm{Ker}P} : (I - P)\mathbb{X} \to \mathrm{Im}L$ is invertible and its inverse is denoted by K_P. If Ω is an open bounded subset of $\mathbb{X}$, the mapping N will be called L-compact on $\bar{\Omega}$ if the following conditions are satisfied:

- $QN(\bar{\Omega})$ is bounded;
- $K_P(I - Q)N : \bar{\Omega} \to \mathbb{X}$ is compact.

Since $\mathrm{Im}Q$ is isomorphic to $\mathrm{Ker}L$, there exists an isomorphism $J : \mathrm{Im}Q \to \mathrm{Ker}L$.

Mawhin's Continuous Theorem 1. (Gaines and Mawhin [10]) Let $\Omega \subseteq \mathbb{X}$ be an open bounded set, L be a Fredholm mapping of index zero and N be L-compact on $\bar{\Omega}$. If all the following conditions hold:

(*a*) $Lx \neq \lambda Nx$, $\forall x \in \partial\Omega \cap \mathrm{Dom}L$, $\lambda \in (0, 1)$;

(*b*) $QNx \neq 0$, $\forall x \in \partial\Omega \cap \mathrm{Ker}L$;

(*c*) $\deg\{JQN, \Omega \cap \mathrm{Ker}L, 0\} \neq 0$, where $J : \mathrm{Im}Q \to \mathrm{Ker}L$ is an isomorphism.

Then $Lx = Nx$ has a solution on $\bar{\Omega} \cap \mathrm{Dom}L$.

Under the invariant transformation $N = e^x$, Eq. (1.1) reduces to

$$\dot{x}(t) = \frac{a(t)}{1 + \left[\frac{e^{x(t-\theta(t))}}{K(t)}\right]^{r}} - b(t). \tag{2.1}$$

Set $\mathbb{X} = \mathbb{Y} = \mathbb{V}_1 \bigoplus \mathbb{V}_2$, where

$$\mathbb{V}_1 = \Big\{x \in AP(\mathbb{R}) : \mathrm{mod}(x) \subseteq \mathrm{mod}(L_x), \forall \varpi \in \Lambda(x), |\varpi| \geq \gamma_0\Big\}, \quad \mathbb{V}_2 = \big\{x \equiv k, k \in \mathbb{R}\big\},$$

where

$$L_x = L_x(t,\varphi) = \frac{a(t)}{1+\left[\frac{e^{\varphi(-\theta(0))}}{K(t)}\right]^r} - b(t),$$

$$\mathrm{mod}(x) = \{\mu : \mu = \sum_{j=1}^{N} n_j \varpi_j, n_j, N \in \mathbb{Z}, N \geq 1, \varpi_j \in \Lambda(x)\},$$

$\varphi \in C([-\theta^+, 0], \mathbb{R})$, γ_0 is a given positive constant. Define the norm $\|x\|_{\mathbb{X}} = \sup_{s\in\mathbb{R}} |x(s)|$, $\forall x \in \mathbb{X} = \mathbb{Y}$.

Lemma 2.6. $\mathbb{X}$ and $\mathbb{Y}$ are Banach spaces endowed with $\|\cdot\|_{\mathbb{X}}$.

Proof. Obviously, $\mathbb{X}$ and $\mathbb{Y}$ are linear spaces. Assume that $x_n \in \mathbb{V}_1$ and $\lim_{n\to\infty} x_n = x_0$. Since $x_n \in \mathbb{V}_1$, for all $|\varpi| < \gamma_0$ we have

$$\lim_{T\to\infty} \frac{1}{T}\int_0^T x_n(s) e^{-\mathrm{i}\varpi s}\,\mathrm{d}s = 0.$$

Thus

$$\lim_{T\to\infty} \frac{1}{T}\int_0^T x_0(s) e^{-\mathrm{i}\varpi s}\,\mathrm{d}s = 0$$

which implies that $\forall \varpi \in \Lambda(x_0), |\varpi| \geq \gamma_0$. It is easy to see that $\mathbb{V}_1$ is a Banach space endowed with $\|\cdot\|_{\mathbb{X}}$. The same can be concluded for $\mathbb{X}$ and $\mathbb{Y}$. This completes the proof. □

Lemma 2.7. Let $L : \mathbb{X} \to \mathbb{Y}$, $Lx = \dot{x}$, then L is a Fredholm mapping of index zero.

Proof. It is obvious that L is a linear operator and $\mathrm{Ker}L = \mathbb{V}_2$. It remains to prove that $\mathrm{Im}L = \mathbb{V}_1$. Suppose that $\phi \in \mathrm{Im}L \subseteq \mathbb{Y}$, there exist $\phi_1 \in \mathbb{V}_1$ and $\phi_2 \in \mathbb{V}_2$ such that

$$\phi = \phi_1 + \phi_2.$$

By the definition of ϕ_1 and Lemma 4.12 in [9], we have $\int_0^t \phi_1(s)\,\mathrm{d}s$ is almost periodic. Since $\phi \in \mathrm{Im}L$, there exists $\upsilon \in \mathbb{X}$ such that

$$L\upsilon = \dot{\upsilon} = \phi,$$

which implies that

$$\left|\int_0^t \phi(s)\,\mathrm{d}s\right| = \left|\int_0^t \dot{\upsilon}(s)\,\mathrm{d}s\right| \leq |\upsilon(t) - \upsilon(0)|.$$

Since $\upsilon \in AP(\mathbb{R})$, there exists $K > 0$ such that $|\upsilon|_\infty \leq K$. Then

$$\left|\int_0^t \phi(s)\,\mathrm{d}s\right| \leq 2K,$$

which implies $\int_0^t \phi(s)\,\mathrm{d}s$ is almost periodic. By the almost periodic property of $\int_0^t \phi(s)\,\mathrm{d}s$ and $\int_0^t \phi_1(s)\,\mathrm{d}s$, it follows that

$$|\phi_2||t| = \left|\int_0^t \phi_2\,\mathrm{d}s\right| = \left|\int_0^t \big[\phi(s) - \phi_1(s)\big]\,\mathrm{d}s\right| < +\infty, \quad \forall t \in \mathbb{R},$$

which implies that $\phi_2 \equiv 0$. Therefore, $\phi = \phi_1 \in \mathbb{V}_1$. This tells us that $\mathrm{Im}L \subseteq \mathbb{V}_1$.

In the following, we will prove that $\mathbb{V}_1 \subseteq \mathrm{Im}L$. Suppose that $\varphi \in \mathbb{V}_1$, $\int_0^t \varphi(s)\,\mathrm{d}s \in AP(\mathbb{R})$. Indeed, if $\varpi \neq 0$, then we obtain

$$\lim_{T\to\infty} \frac{1}{T}\int_0^T \left[\int_0^t \varphi(s)\,\mathrm{d}s\right] e^{-\mathrm{i}\varpi t}\,\mathrm{d}t = \frac{1}{\mathrm{i}\varpi}\lim_{T\to\infty}\frac{1}{T}\int_0^T \varphi(s)e^{-\mathrm{i}\varpi s}\,\mathrm{d}s.$$

Let $\psi = \int_0^t \varphi(s)\,\mathrm{d}s - m\big(\int_0^t \varphi(s)\,\mathrm{d}s\big)$. So

$$\Lambda(\psi) = \Lambda(\varphi).$$

Therefore, $\psi \in \mathbb{V}_1 \subseteq \mathbb{X}$. Further, we have

$$\dot{\psi}(t) = \frac{\mathrm{d}}{\mathrm{d}t}\left(\int_0^t \varphi(s)\,\mathrm{d}s - m\left[\int_0^t \varphi(s)\,\mathrm{d}s\right]\right) = \varphi(t), \quad \forall t \in \mathbb{R},$$

which implies that $\varphi \in \mathrm{Im}L$. Hence, we deduce that $\mathbb{V}_1 \subseteq \mathrm{Im}L$.

Therefore, $\mathbb{V}_1 = \mathrm{Im}L$. Furthermore, one can easily show that $\mathrm{Im}L$ is closed in $\mathbb{Y}$ and

$$\dim \mathrm{Ker}L = 1 = \mathrm{codim}\,\mathrm{Im}L.$$

Therefore, L is a Fredholm operator of index zero. This completes the proof. □

Lemma 2.8. Define $N : \mathbb{X} \to \mathbb{Y}$, $P : \mathbb{X} \to \mathbb{X}$ and $Q : \mathbb{Y} \to \mathbb{Y}$ by

$$Nx = \frac{a(t)}{1 + \left[\frac{e^{x(t-\theta(t))}}{K(t)}\right]^r} - b(t), \quad Px = \bar{x} = Qw, \quad \forall x \in \mathbb{X} = \mathbb{Y}.$$

Then N is L-compact on $\bar{\Omega}$ (Ω is an open and bounded subset of $\mathbb{X}$).

Proof. Obviously, P and Q are continuous such that $\mathrm{Im}P = \mathrm{Ker}L$ and $\mathrm{Im}L = \mathrm{Ker}Q$. Further, we have $(I-Q)\mathbb{V}_2 = \{0\}$ and $(I-Q)\mathbb{V}_1 = \mathbb{V}_1$. Hence, $\mathrm{Im}(I-Q) = \mathbb{V}_1 = \mathrm{Im}L$. In view of

$$\mathrm{Im}P = \mathrm{Ker}L \quad \text{and} \quad \mathrm{Im}L = \mathrm{Ker}Q = \mathrm{Im}(I-Q),$$

we can conclude that the generalized inverse $K_P : \mathrm{Im}L \to \mathrm{Ker}P \cap \mathrm{Dom}L$ of L exists and is given by

$$K_P x = \int_0^t x(s)\,\mathrm{d}s - m\left[\int_0^t x(s)\,\mathrm{d}s\right], \quad \forall x \in \mathrm{Im}L.$$

Thus

$$QNx = m\big[Nx\big], \quad K_P(I-Q)Nx = f[x(t)] - Qf[x(t)], \quad \forall x \in \mathrm{Im}L,$$

where $f[x(t)]$ is defined by

$$f[x(t)] = \int_0^t \big[Nx(s) - QNx(s)\big]\, \mathrm{d}s.$$

Clearly, QN and $(I-Q)N$ are continuous. We claim that K_P is also continuous. Assume that $x_n \in \mathrm{Im}L = \mathbb{V}_1$ ($n \in \mathbb{N}^+$) such that

$$\lim_{n\to\infty} x_n = x_0.$$

By the completeness of $\mathbb{V}_1$, $x_0 \in \mathbb{V}_1$ and $x_n - x_0 \in \mathbb{V}_1$ ($n \in \mathbb{N}^+$). Then there exists a constant D such that

$$|K_P x_n - K_P x_0|_\infty \le D|x_n - x_0|_\infty, \quad n \in \mathbb{N}^+.$$

Therefore, $\lim_{n\to\infty} |K_P x_n - K_P x_0|_\infty = 0$. So K_P and $K_P(I-Q)$ are also continuous. In addition, $K_P(I-Q)x$ are uniformly bounded on $\bar{\Omega}$. It is not difficult to verify that $QN(\bar{\Omega})$ is bounded and $K_P(I-Q)Nx$ is equicontinuous on $\bar{\Omega}$. Hence, by the Arzela–Ascoli theorem, we can conclude that $K_P(I-Q)N(\bar{\Omega})$ is compact. Thus N is L-compact on $\bar{\Omega}$. This completes the proof. □

3. EXISTENCE OF POSITIVE ALMOST PERIODIC SOLUTIONS

In this section, we study the existence of at least one positive almost periodic solution of Eq. (1.1) by using Mawhin's continuous theorem of coincidence degree theory.

Theorem 3.1. Assume that (H_1) holds, suppose further that

(H_2) $\bar{b} > 0$ and $\bar{c} := m[a(s) - b(s)] > 0$,

then Eq. (1.1) admits at least one positive almost periodic solution.

Proof. It is easy to see that if Eq. (2.1) has one almost periodic solution $\bar{x}$, then $\bar{N}=e^{\bar{x}}$ is a positive almost periodic solution of Eq. (1.1). Therefore, to complete the proof it suffices to show that Eq. (2.1) has one almost periodic solution.

In order to use the Mawhin's continuous theorem, we set the Banach spaces $\mathbb{X}$ and $\mathbb{Y}$ as those in Lemma 2.6 and L, N, P, Q the same as those defined in Lemmas 2.7 and 2.8, respectively. It remains to search for an appropriate open and bounded subset $\Omega \subseteq \mathbb{X}$.

Corresponding to the operator equation $Lx = \lambda x$, $\lambda \in (0, 1)$, we have

$$\dot{x}(t) = \lambda \left[\frac{a(t)}{1 + \left[\frac{e^{x(t-\theta(t))}}{K(t)}\right]^r} - b(t) \right]. \tag{3.1}$$

Suppose that $x \in \mathrm{Dom}L \subseteq \mathbb{X}$ is a solution of Eq. (3.1) for some $\lambda \in (0, 1)$, where $\mathrm{Dom}L = \{x \in \mathbb{X} : x \in C^1(\mathbb{R}), \dot{x} \in C(\mathbb{R})\}$. By Lemma 2.4, there exists a sequence $\{\alpha_n : n \in \mathbb{N}^+\}$ such that

$$x(\alpha_n) \in \left[x^* - \frac{1}{n}, x^*\right], \quad x^* = \sup_{s\in\mathbb{R}} x(s), \quad n \in \mathbb{N}^+. \tag{3.2}$$

By (H_2) and Lemma 2.5, for $\forall t_0 \in \mathbb{R}$, there exists a constant $\omega \in (2\theta^+, +\infty)$ independent of t_0 such that

$$\frac{1}{T}\int_{t_0}^{t_0+T} b(s)\,\mathrm{d}s \in \left[\frac{\bar{b}}{2}, \frac{3\bar{b}}{2}\right], \quad \frac{1}{T}\int_{t_0}^{t_0+T} [a(s)-b(s)]\,\mathrm{d}s \in \left[\frac{\bar{c}}{2}, \frac{3\bar{c}}{2}\right], \quad \forall T \geq \frac{\omega}{2}. \tag{3.3}$$

For $\forall n_0 \in \mathbb{N}^+$, we consider $[\alpha_{n_0} - \omega, \alpha_{n_0}]$, where ω is defined as that in (3.3). By Lemma 2.2, there exists $\xi_{n_0} \in [\alpha_{n_0} - \omega, \alpha_{n_0}]$, $\underline{\xi}_{n_0} \in (-\infty, \alpha_{n_0} - \omega]$ and $\bar{\xi}_{n_0} \in [\alpha_{n_0}, +\infty)$ such that

$$x(\underline{\xi}_{n_0}) = x(\bar{\xi}_{n_0}) \quad \text{and} \quad x(\xi_{n_0}) \leq x(s), \;\; \forall s \in [\underline{\xi}_{n_0}, \bar{\xi}_{n_0}]. \tag{3.4}$$

Integrating Eq. (3.1) from $\underline{\xi}_{n_0}$ to $\bar{\xi}_{n_0}$ leads to

$$\begin{aligned} & \int_{\underline{\xi}_{n_0}}^{\bar{\xi}_{n_0}} \left[\frac{a(s)}{1+\left(\frac{e^{x(s-\theta(s))}}{K(s)}\right)^r} - b(s)\right]\mathrm{d}s \\ = & \int_{\underline{\xi}_{n_0}}^{\bar{\xi}_{n_0}} \left(\frac{[a(s)-b(s)][K(s)]^r - b(s)e^{rx(s-\theta(s))}}{[K(s)]^r + e^{rx(s-\theta(s))}}\right)\mathrm{d}s \\ = & \; 0, \end{aligned}$$

which yields that

$$\int_{\underline{\xi}_{n_0}}^{\bar{\xi}_{n_0}} b(s)e^{rx(s-\theta(s))}\,\mathrm{d}s = \int_{\underline{\xi}_{n_0}}^{\bar{\xi}_{n_0}} [a(s)-b(s)][K(s)]^r\,\mathrm{d}s. \tag{3.5}$$

By the definitions of ω, $\underline{\xi}_{n_0}$, $\bar{\xi}_{n_0}$ and (3.3), there exists $s_0 \in [\underline{\xi}_{n_0} + \theta^+, \bar{\xi}_{n_0}]$ ($s_0 - \theta(s_0) \in [\underline{\xi}_{n_0}, \bar{\xi}_{n_0}]$) such that

$$\begin{aligned} \frac{1}{\bar{\xi}_{n_0} - \underline{\xi}_{n_0}} \int_{\underline{\xi}_{n_0}}^{\bar{\xi}_{n_0}} b(s)e^{rx(s-\theta(s))}\,\mathrm{d}s & \geq \frac{1}{\bar{\xi}_{n_0} - \underline{\xi}_{n_0}} \int_{\underline{\xi}_{n_0}+\theta^+}^{\bar{\xi}_{n_0}} b(s)e^{rx(s-\theta(s))}\,\mathrm{d}s \\ & \geq \frac{(\bar{\xi}_{n_0} - \underline{\xi}_{n_0} - \theta^+)e^{rx(s_0-\theta(s_0))}}{\bar{\xi}_{n_0} - \underline{\xi}_{n_0}} \\ & \quad \times \frac{1}{\bar{\xi}_{n_0} - \underline{\xi}_{n_0} - \theta^+} \int_{\underline{\xi}_{n_0}+\theta^+}^{\bar{\xi}_{n_0}} b(s)\,\mathrm{d}s \\ & \geq \frac{e^{rx(s_0-\theta(s_0))}}{2} \times \frac{\bar{b}}{2} \\ & = \frac{\bar{b}e^{rx(s_0-\theta(s_0))}}{4}. \end{aligned} \tag{3.6}$$

Substituting (3.6) into (3.5), we obtain

$$\frac{\bar{b}e^{rx(s_0-\theta(s_0))}}{4} \leq \frac{1}{\bar{\xi}_{n_0} - \underline{\xi}_{n_0}} \int_{\underline{\xi}_{n_0}}^{\bar{\xi}_{n_0}} [a(s)-b(s)][K(s)]^r\,\mathrm{d}s \leq a^+(K^+)^r,$$

which implies from (3.4) that

$$x(\xi_{n_0}) \le x(s_0-\theta(s_0)) \le \frac{1}{r}\ln\left[\frac{4a^+(K^+)^r}{\bar b}\right]. \tag{3.7}$$

Let $I=[\xi_{n_0},\alpha_{n_0}]$ and $I_1=\{s\in I:\dot x(s)\ge 0\}$. It follows from Eq. (3.1) that

$$\begin{aligned}\int_{I_1}\dot x(s)\,\mathrm{d}s &= \int_{I_1}\lambda\left[\frac{a(s)}{1+\left(\frac{e^{x(s-\theta(s))}}{K(s)}\right)^r}-b(s)\right]\mathrm{d}s\\ &\le \int_I \frac{a(s)}{1+\left(\frac{e^{x(s-\theta(s))}}{K(s)}\right)^r}\,\mathrm{d}s\\ &\le \int_{\alpha_{n_0}-\omega}^{\alpha_{n_0}} a(s)\,\mathrm{d}s\\ &\le a^+\omega.\end{aligned}$$

By Lemma 2.1, it follows from (3.7) – (3.8) that

$$x(t)\le x(\xi_{n_0})+\int_{I_1}\dot x(s)\,\mathrm{d}s \le \frac{1}{r}\ln\left[\frac{4a^+(K^+)^r}{\bar b}\right]+a^+\omega := \rho_1,\quad \forall t\in[\xi_{n_0},\alpha_{n_0}],$$

which implies that

$$x(\alpha_{n_0})\le \rho_1.$$

In view of (3.2), letting $n_0\to+\infty$ in the above inequality leads to

$$x^*=\lim_{n_0\to+\infty} x(\alpha_{n_0})\le\rho_1. \tag{3.8}$$

Taking

$$l_0 := \max\left\{\omega, \frac{4\theta^+b^+e^{r\rho_1}}{(K^-)^r\bar c}\right\}.$$

For $\forall n_0\in\mathbb{Z}$, by Lemma 2.3, there exist $\eta_{n_0}\in[n_0l_0,(n_0+1)l_0]$, $\underline{\eta}_{n_0}\in(-\infty,n_0l_0]$ and $\bar\eta_{n_0}\in[(n_0+1)l_0,+\infty)$ such that

$$x(\underline{\eta}_{n_0})=x(\bar\eta_{n_0})\quad\text{and}\quad x(\eta_{n_0})\ge x(s),\ \ \forall s\in[\underline{\eta}_{n_0},\bar\eta_{n_0}]. \tag{3.9}$$

Similar to (3.5), integrating Eq. (3.1) from $\underline{\eta}_{n_0}$ to $\bar\eta_{n_0}$ leads to

$$\int_{\underline{\eta}_{n_0}}^{\bar\eta_{n_0}} b(s)e^{rx(s-\theta(s))}\,\mathrm{d}s=\int_{\underline{\eta}_{n_0}}^{\bar\eta_{n_0}}[a(s)-b(s)][K(s)]^r\,\mathrm{d}s. \tag{3.10}$$

By the definitions of ω, $\underline{\eta}_{n_0}$, $\bar{\eta}_{n_0}$ and (3.3), there exists $s_1 \in [\underline{\eta}_{n_0} + \theta^+, \bar{\eta}_{n_0}]$ $(s_1 - \theta(s_1) \in [\underline{\eta}_{n_0}, \bar{\eta}_{n_0}])$ such that

$$\begin{aligned} \frac{1}{\bar{\eta}_{n_0} - \underline{\eta}_{n_0}} \int_{\underline{\eta}_{n_0}}^{\bar{\eta}_{n_0}} b(s) e^{rx(s-\theta(s))}\, \mathrm{d}s &\le \frac{1}{\bar{\eta}_{n_0} - \underline{\eta}_{n_0}} \int_{\underline{\eta}_{n_0}+\theta^+}^{\bar{\eta}_{n_0}} b(s) e^{rx(s-\theta(s))}\, \mathrm{d}s \\ &\quad + \frac{1}{\bar{\eta}_{n_0} - \underline{\eta}_{n_0}} \int_{\underline{\eta}_{n_0}}^{\underline{\eta}_{n_0}+\theta^+} b(s) e^{rx(s-\theta(s))}\, \mathrm{d}s \\ &\le \frac{(\bar{\eta}_{n_0} - \underline{\eta}_{n_0} - \theta^+) b^+ e^{rx(s_1-\theta(s_1))}}{\bar{\eta}_{n_0} - \underline{\eta}_{n_0}} + \frac{\theta^+ b^+ e^{r\rho_1}}{\bar{\eta}_{n_0} - \underline{\eta}_{n_0}} \\ &\le b^+ e^{rx(s_1-\theta(s_1))} + \frac{\theta^+ b^+ e^{r\rho_1}}{l_0} \\ &= b^+ e^{rx(s_1-\theta(s_1))} + \frac{(K^-)^r \bar{c}}{4}. \end{aligned} \tag{3.11}$$

Substituting (3.12) into (3.11), we obtain

$$b^+ e^{rx(s_1-\theta(s_1))} + \frac{(K^-)^r \bar{c}}{4} \ge \frac{1}{\bar{\eta}_{n_0} - \underline{\eta}_{n_0}} \int_{\underline{\eta}_{n_0}}^{\bar{\eta}_{n_0}} [a(s) - b(s)][K(s)]^r\, \mathrm{d}s \ge \frac{(K^-)^r \bar{c}}{2},$$

which implies from (3.10) that

$$x(\eta_{n_0}) \ge x(s_1 - \theta(s_1)) \ge \frac{1}{r} \ln \left[\frac{(K^-)^r \bar{c}}{4b^+} \right].$$

Further, we obtain from Eq. (3.1) that

$$\begin{aligned} \int_{n_0 l_0}^{(n_0+1)l_0} |\dot{x}(s)|\, \mathrm{d}s &= \int_{n_0 l_0}^{(n_0+1)l_0} \lambda \left| \frac{a(s)}{1 + \left(\frac{e^{x(s-\theta(s))}}{K(s)}\right)^r} - b(s) \right| \mathrm{d}s \\ &\le (a^+ + b^+) l_0. \end{aligned}$$

It follows from (3.13)–(3.14) that

$$\begin{aligned} x(t) &\ge x(\eta_{n_0}) - \int_{n_0 l_0}^{(n_0+1)l_0} |\dot{x}(s)|\, \mathrm{d}s \\ &\ge \frac{1}{r} \ln \left[\frac{(K^-)^r \bar{c}}{4b^+} \right] - (a^+ + b^+) l_0 := \rho_2, \quad \forall t \in [n_0 l_0, (n_0+1) l_0]. \end{aligned}$$

Obviously, ρ_2 is a constant independent of n_0. So it follows from (3.15) that

$$x_* = \inf_{s \in \mathbb{R}} x(s) = \inf_{n_0 \in \mathbb{Z}} \Big\{ \min_{s \in [n_0 l_0, (n_0+1) l_0]} x(s) \Big\} \ge \inf_{n_0 \in \mathbb{Z}} \{\rho_2\} = \rho_2. \tag{3.12}$$

Set $C = |\rho_1| + |\rho_2| + 1$. Clearly, C is independent of $\lambda \in (0,1)$. Consider the algebraic equations $QNx_0 = 0$ for $x_0 \in \mathbb{R}$ as follows:

$$0 = m\left[\frac{a(t)}{1 + \left(\frac{e^{x_0}}{K(t)}\right)^r} - b(t) \right] \Longrightarrow e^{x_0} m(b(s)) = m\big[(a(s) - b(s)) K^r(s)\big].$$

So we can easily obtain that

$$\rho_2 \leq \frac{1}{r} \ln \left[\frac{(K^-)^r \bar{c}}{\bar{b}} \right] \leq x_0 \leq \frac{1}{r} \ln \left[\frac{(K^+)^r \bar{c}}{\bar{b}} \right] \leq \rho_1.$$

Then $\|x_0\|_{\mathbb{X}} < C$. Let $\Omega = \{x \in \mathbb{X} : \|x\|_{\mathbb{X}} < C\}$, then Ω satisfies conditions (a) and (b) of Mawhin's continuous theorem.

Finally, we will show that condition (c) of Mawhin's continuous theorem is satisfied. Let us consider the homotopy

$$H(\iota, x) = m\left[\frac{a(t)}{1 - \iota + \left(\frac{e^x}{K(t)}\right)^r} - b(t) \right], \quad (\iota, x) \in [0,1] \times \Omega \cap \mathrm{Ker}L,$$

From the above discussion it is easy to verify that $H(\iota, x) \neq 0$ on $\partial\Omega \cap \mathrm{Ker}L$, $\forall \iota \in [0,1]$. Further, by $H(1,x) = 0 (x \in \mathbb{R})$, we obtain

$$\frac{m[a(t)K^r(t)]}{e^{rx}} - \bar{b} = 0 \Longrightarrow x = \frac{1}{r} \ln \frac{m[a(t)K^r(t)]}{\bar{b}} \in \Omega.$$

Then

$$\deg\big(H(1,x), \Omega \cap \mathrm{Ker}L, 0\big) = \mathrm{sign}\big(- rm[a(t)K^r(t)]e^{-rx}\big) = -1.$$

By the invariance property of homotopy, direct calculation produces

$$\begin{aligned} \deg\big(JQN, \Omega \cap \mathrm{Ker}L, 0\big) &= \deg\big(H(0,x), \Omega \cap \mathrm{Ker}L, 0\big) \\ &= \deg\big(H(1,x), \Omega \cap \mathrm{Ker}L, 0\big) \\ &= -1, \end{aligned}$$

where $\deg(\cdot,\cdot,\cdot)$ is the Brouwer degree and J is the identity mapping since $\mathrm{Im}Q = \mathrm{Ker}L$. Obviously, all the conditions of Mawhin's continuous theorem are satisfied. Therefore, Eq. (2.1) has one almost periodic solution, that is, Eq. (1.1) has at least one positive almost periodic solution. This completes the proof. □

Corollary 3.1. Assume that $(H_1)-(H_2)$ hold, suppose further that a, b, K and θ in Eq. (1.1) are continuous nonnegative periodic functions with periods α, β, σ and δ, respectively, then Eq. (1.1) has at least one positive almost periodic solution.

Remark 3.1. By Corollary 3.1, it is easy to obtain the existence of at least one positive almost periodic solution of Eq. (1.2) in Example 1.1, although there is no a priori reason to expect the existence of positive periodic solutions of Eq. (1.2).

Corollary 3.2. Assume that $(H_1)-(H_2)$ hold, suppose further that a, b, K and θ in Eq. (1.1) are continuous nonnegative ω-periodic functions, then Eq. (1.1) has at least one positive ω-periodic solution.

4. STABILITY

Theorem 4.1. Assume that (H_1)-(H_2) hold. Suppose further that

(H_3) $a(t) > b(t) > 0$, $\forall t \in \mathbb{R}$.

(H_4) $r \int_{t-\theta(t)}^{t} a(s)\,\mathrm{d}s < 6$, $\forall t \in \mathbb{R}$.

Then Eq. (1.1) has a unique almost periodic solution, which is globally attractive.

Proof. The proof of this theorem is similar to Theorem 3.1 in [23] and we should omit it. This completes the proof. □

Together with Corollaries 3.1 – 3.2, we obtain

Corollary 4.1. Assume that $(H_1)-(H_4)$ hold, suppose further that a, b, K and θ in Eq. (1.1) are continuous nonnegative periodic functions with periods α, β, σ and δ, respectively, then Eq. (1.1) has a unique positive almost periodic solution, which is globally attractive.

Corollary 4.2. (Wang [23]) Assume that $(H_1)-(H_4)$ hold, suppose further that a, b, K and θ in Eq. (1.1) are continuous nonnegative ω-periodic functions, then Eq. (1.1) has a unique positive ω-periodic solution, which is globally attractive.

5. THREE EXAMPLES

Example 5.1. Consider the following fishing model:

$$\dot{x}(t) = x(t)\left[\frac{2|\cos(\sqrt{3}t)|}{1+\left[\frac{x(t-1)}{2+\sin(\sqrt{3}t)}\right]^{0.5}} - |\cos(\sqrt{7}t)|\right]. \tag{5.1}$$

Then Eq. (5.1) has at least one positive almost periodic solution.

Proof. Corresponding to Eq. (1.1), we have $\bar{a} = m[2|\cos(\sqrt{3}t)|] = \frac{4}{\pi}$ and $\bar{b} = m[|\cos(\sqrt{7}t)|] = \frac{2}{\pi}$. So, (H_1)-(H_2) in Theorem 3.1 hold. By Theorem 3.1, Eq. (5.1) has at least one positive almost periodic solution (see Figure 1). This completes the proof. □

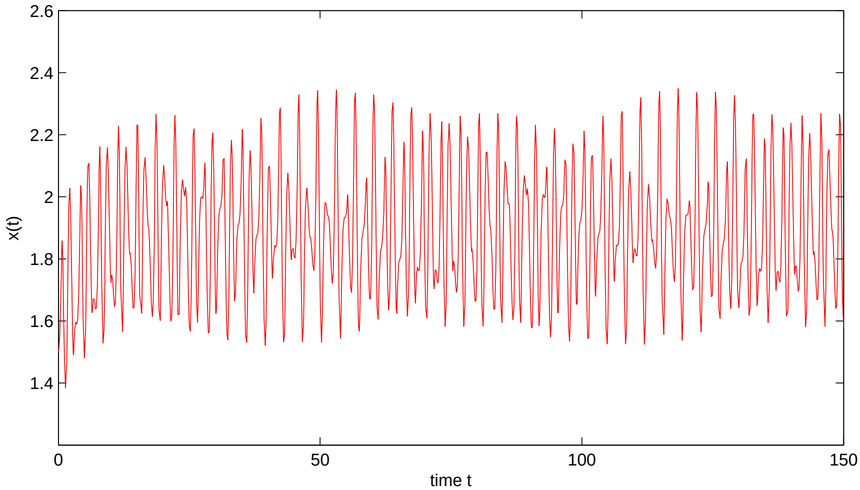

Fig. 1. State variable x of Eq. (5.1).

Remark 5.1. In Eq. (5.1), $|\cos(\sqrt{3}t)|$ is $\frac{\sqrt{3}\pi}{3}$-periodic function and $|\cos(\sqrt{7}t)|$ is $\frac{\sqrt{7}\pi}{7}$-periodic function. So Eq. (5.1) is with incommensurable periods. Through all the coefficients of Eq. (5.1) are periodic functions, the positive periodic solutions of Eq. (5.1) could not possibly exist. However, by Theorem 3.1, the positive almost periodic solutions of Eq. (5.1) exactly exist.

Example 5.2. Consider the following fishing model:

$$\dot{x}(t) = x(t)\left[\frac{4+\cos(\sqrt{3}t)}{1+\left[\frac{x(t-1)}{2+\sin(\sqrt{3}t)}\right]^{0.5}} - 2 - \cos(\sqrt{7}t)\right]. \tag{5.2}$$

Then Eq. (5.2) has a unique positive almost periodic solution, which is globally attractive.

Proof. Obviously, $(H_1)-(H_4)$ in Theorem 4.1 hold. By Theorem 4.1, Eq. (5.2) has a unique positive almost periodic solution, which is globally attractive (see Figures 2–3). This completes the proof. $\square$

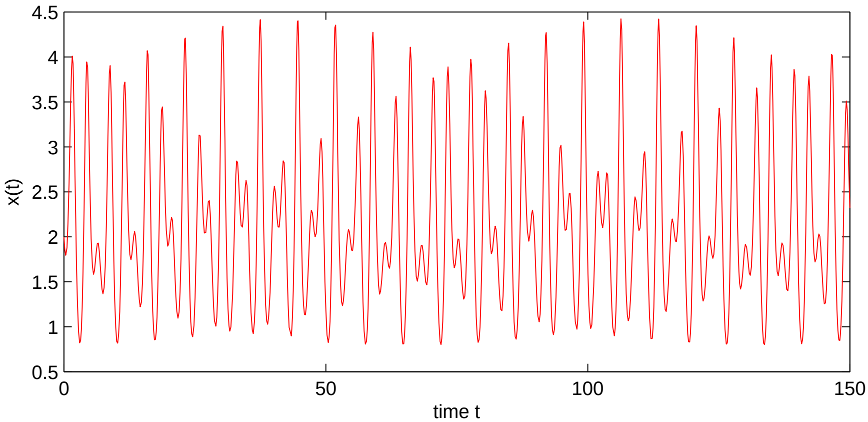

Fig. 2. State variable x of Eq. (5.2)

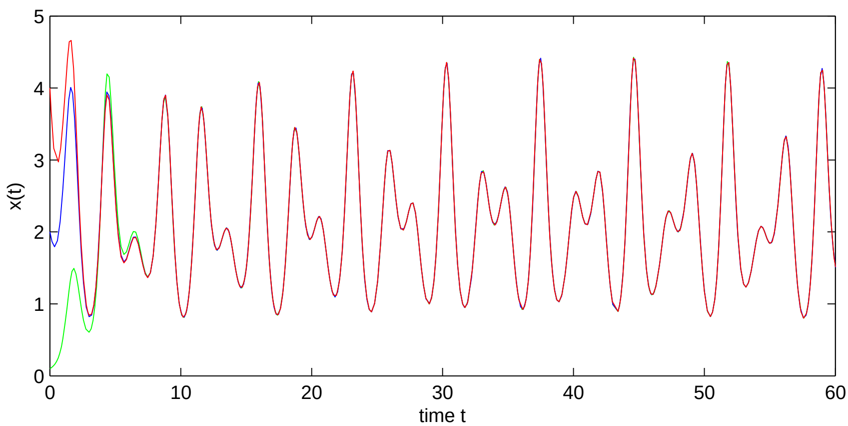

Fig. 3. Stability of state variable x of Eq. (5.2).

Example 5.3. Consider the following almost periodic fishing model:

$$\dot{x}(t) = x(t)\left[\frac{8+\cos(\sqrt{2}t)+\cos(\sqrt{3}t)}{2+2\left[\frac{x(t-0.5)}{2+\sin(\sqrt{3}t)}\right]^{0.5}} - 2 - \cos(\sqrt{7}t)\right]. \tag{5.3}$$

In system (5.3), $\cos(\sqrt{2}t)+\cos(\sqrt{3}t)$ is almost periodic, which is not periodic. Similar to the argument as that in Example 5.2, it is easy to obtain that system (5.3) has a unique positive almost periodic solution, which is globally attractive (see Figures 4−5).

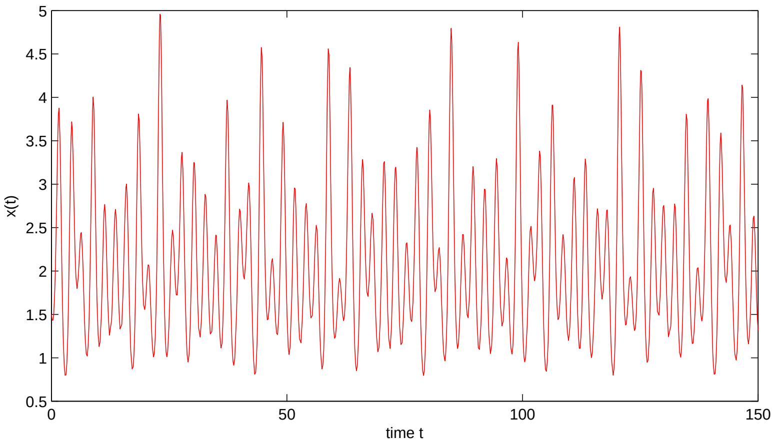

Fig. 4. State variable x of Eq. (5.3)

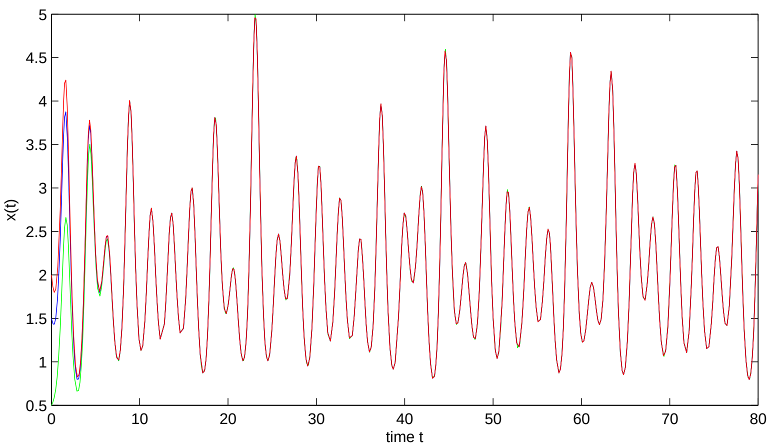

Fig. 5. Stability of state variable x of Eq. (5.3).

6. DISCUSSION

In [16, 28, 29, 32], the authors studied the existence of positive almost periodic solutions of some discrete population models (such as fishing model, predator-prey model and mutualism model) by using the Lyapunov functional method. By a similar method in [16, 28, 29, 32], the authors [31] studied the existence of positive almost periodic solutions of continuous Schoener's competition model. In [21, 26], the multiplicity of positive almost periodic solutions are obtained for some continuous population models with harvesting terms by using Mawhin's continuation theorem. But for the continuous fishing model (1.1), there are scarcely any papers concerning with the existence of positive almost periodic solutions. Therefore, in this paper, some criterions for the existence and stability of positive almost periodic solution of a kind of fishing model with delay are obtained by using some analytical techniques, modified inequalities and Mawhin's continuation theorem of coincidence degree theory.

Theorem 3.1 (i. e., $(H_1)-(H_2)$) indicates that model (1.1) must contain a positive almost periodic oscillation if the per-capita fecundity rate (i. e., $a(t)$) is greater than the per-capita mortality rate (i. e., $b(t)$). Theorem 4.1 indicates that the maturation time delay (i. e., $\theta(t)$) is harm for the stability of the model. The method used in this paper provides a possible method to study the existence and global attractivity of positive almost periodic solution of the models in biological populations.

ACKNOWLEDGEMENT

This work are supported by the Funding for Applied Technology Research and Development of Panzhihua City under Grant 2015CY-S-14 and Natural Science Foundation of Education Department of Sichuan Province under Grant 15ZB0419.

(Received May 10, 2016)

REFERENCES

[1] L. Berezansky, E. Braverman, and L. Idels: On delay differential equations with Hills type growth rate and linear harvesting. Comput. Math. Appl. *49* (2005), 549–563. DOI:10.1016/j.camwa.2004.07.015

[2] L. Berezansky and L. Idels: Stability of a time-varying fishing model with delay. Appl. Math. Lett. *21* (2008), 447–452. DOI:10.1016/j.aml.2007.03.027

[3] B. X. Dai, H. Su, and D. W. Hu: Periodic solution of a delayed ratio-dependent predator-prey model with monotonic functional response and impulse. Nonlinear Anal. TMA *70* (2009), 126–134. DOI:10.1016/j.na.2007.11.036

[4] B. Du, M. Hu, and X. Lian: Dynamical behavior for a stochastic predator-prey model with HV type functional response. Bull. Malays. Math. Sci. Soc. *40* (2016), 1, 487–503. DOI:10.1007/s40840-016-0325-3

[5] B. Du, Y. Liu, and H. A. Batarfi: Almost periodic solution for a neutral-type neural networks with distributed leakage delays on time scales. Neurocomputing *173* (2016), 921–929. DOI:10.1016/j.neucom.2015.08.047

[6] B. Du, Y. Liu, and A. I. Atiatallah: Existence and asymptotic behavior results of periodic solution for discrete-ime neutral-type neural networks. J. Franklin Inst., Engrg. Appl. Math. *353* (2016), 448–461. DOI:10.1016/j.jfranklin.2015.11.013

[7] C. Egami: Positive periodic solutions of nonautonomous delay competitive systems with weak Allee effect. Nonlinear Anal. RWA *10* (2009), 494–505. DOI:10.1016/j.nonrwa.2007.10.010

[8] Y. H. Fan and L. L. Wang: Periodic solutions in a delayed predator-prey model with nonmonotonic functional response. Nonlinear Anal. RWA *10* (2009), 3275–3284. DOI:10.1016/j.nonrwa.2008.10.032

[9] A. M. Fink: Almost Periodic Differential Equation. Spring-Verlag, Berlin, Heidleberg, New York, 1974. DOI:10.1007/bfb0070324

[10] R. Gaines and J. Mawhin: Coincidence Degree and Nonlinear Differential Equations. Springer Verlag, Berlin 1977. DOI:10.1007/bfb0089537

[11] K. Gopalsamy: Stability and Oscillations in Delay Differential Equations of Population Dynamics. Kluwer Acad. Publ., 1992. DOI:10.1007/978-94-015-7920-9

[12] C. Y. He: Almost Periodic Differential Equations. Higher Education Publishing House, Beijing, 1992 (in Chinese).

[13] M. Kot: Elements of Mathematical Ecology. Cambr. Univ. Press, 2001. DOI:10.1017/cbo9780511608520

[14] Y. Kuang: Delay Differential Equations With Applications in Population Dynamics. Academic Press, Inc., 1993. DOI:10.1016/s0076-5392(08)x6164-8

[15] R. X. Liang and J. H. Shen: Positive periodic solutions for impulsive predator-prey model with dispersion and time delays. Appl. Math. Comput. *217* (2010), 661–676. DOI:10.1016/j.amc.2010.06.003

[16] Y. Z. Liao and T. W. Zhang: Almost periodic solutions of a discrete mutualism model with variable delays. Discrete Dynamics in Nature and Society Volume 2012, Article ID 742102, 27 pages. DOI:10.1155/2012/742102

[17] X. L. Lin, Y. L. Jiang, and X. Q. Wang: Existence of periodic solutions in predator-prey with Watt-type functional response and impulsive effects. Nonlinear Anal. TMA *73* (2010), 1684–1697. DOI:10.1016/j.na.2010.05.003

[18] S. Lu: Applications of topological degree associated condensing field to the existence of periodic solutions for neutral functional differential equations with nonlinear difference operator. Acta Mathematica Sinica, English Series, to appear.

[19] S. Lu, T. Zhong, and L. Chen: Periodic solutions for p-Laplacian Rayleigh equations with singularities. Boundary Value Problems 2016, *96* (2016).

[20] S. Lu and L. Chen: The problem of existence of periodic solutions for neutral functional differential system with nonlinear difference operator. J. Math. Anal. Appl. *387* (2012), 1127–1136. DOI:10.1016/j.jmaa.2011.10.022

[21] J. Y. Shu and T. W. Zhang: Multiplicity of almost periodic oscillations in a harvesting mutualism model with time delays. Dynam. Cont. Disc. Impul. Sys. B: Appl. Algor. *20* (2013), 463–483.

[22] K. Wang: Existence and global asymptotic stability of positive periodic solution for a predator-prey system with mutual interference. Nonlinear Anal. RWA *10* (2009), 2774–2783. DOI:10.1016/j.nonrwa.2008.08.015

[23] X. P. Wang: Stability and existence of periodic solutions for a time-varying fishing model with delay. Nonlinear Anal.: RWA *11* (2010), 3309–3315. DOI:10.1016/j.nonrwa.2009.11.023

[24] K. Wang: Periodic solutions to a delayed predator-prey model with Hassell–Varley type functional response. Nonlinear Anal. RWA *12* (2011), 137–145. DOI:10.1016/j.nonrwa.2010.06.003

[25] Q. Wang, M. M. Ding, Z. J. Wang, and H. Y. Zhang: Existence and attractivity of a periodic solution for an N-species Gilpin–Ayala impulsive competition system. Nonlinear Anal. RWA *11* (2010), 2675–2685. DOI:10.1016/j.nonrwa.2008.08.015

[26] T. W. Zhang: Multiplicity of positive almost periodic solutions in a delayed Hassell–Varleytype predator-prey model with harvesting on prey. Math. Meth. Appl. Sci. *37* (2013), 686–697. DOI:10.1002/mma.2826

[27] T. W. Zhang: Almost periodic oscillations in a generalized Mackey–Glass model of respiratory dynamics with several delays. Int. J. Biomath. *7* (2014), 1450029 (22 pages). DOI:10.1142/s1793524514500296

[28] T. W. Zhang and X. R. Gan: Existence and permanence of almost periodic solutions for Leslie–Gower predator-prey model with variable delays. Elect. J. Differ. Equa. *2013* (2013), 1–21.

[29] T. W. Zhang and X. R. Gan: Almost periodic solutions for a discrete fishing model with feedback control and time delays. Commun. Nonlinear Sci. Numer. Simul. *19* (2014), 150–163. DOI:10.1016/j.cnsns.2013.06.019

[30] T. W. Zhang and Y. K. Li: Positive periodic solutions for a generalized impulsive n-species Gilpin–Ayala competition system with continuously distributed delays on time scales. Int. J. Biomath. *4* (2011), 23–34. DOI:10.1142/s1793524511001131

[31] T. W. Zhang, Y. K. Li, and Y. Ye: On the existence and stability of a unique almost periodic solution of Schoener's competition model with pure-delays and impulsive effects. Commun. Nonlinear Sci. Numer. Simul. *17* (2012), 1408–1422. DOI:10.1016/j.cnsns.2011.08.008

[32] T. W. Zhang, Y. K. Li, and Y. Ye: Persistence and almost periodic solutions for a discrete fishing model with feedback control. Commun. Nonlinear Sci. Numer. Simul. *16* (2011), 1564–1573. DOI:10.1016/j.cnsns.2010.06.033

[33] G. D. Zhang, Y. Shen, and B. S. Chen: Positive periodic solutions in a non-selective harvesting predator-prey model with multiple delays. J. Math. Anal. Appl. *395* (2012), 298–306. DOI:10.1016/j.jmaa.2012.05.045

[34] Y. L. Zhu and K. Wang: Existence and global attractivity of positive periodic solutions for a predator-prey model with modified Leslie–Gower Holling–type II schemes. J. Math. Anal. Appl. *384* (2011), 400–408. DOI:10.1016/j.jmaa.2011.05.081

Tianwei Zhang, City College, Kunming University of Science and Technology, Kunming 650051. P. R. China.
e-mail: zhang@kmust.edu.cn

Yongzhi Liao, School of Mathematics and Computer Science, Panzhihua University, Panzhihua, Sichuan 617000. P. R. China.
e-mail: mathyzliao@163.com