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# SELECTION AND CORRECTION OF WEIGHTED RULES BASED ON ŁUKASIEWICZ'S FUZZY LOGIC WITH EVALUATED SYNTAX

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The core of the expert knowledge is typically represented by a set of rules (implications) assigned with weights specifying their (un)certainties. In the paper, a method for hierarchical selection and correction of expert's weighted rules is described particularly in the case when Łukasiewicz's fuzzy logic with evaluated syntax for dealing with weights is used.

*Keywords:* uncertain knowledge, fuzzy implication, rule base, Łukasiewicz's fuzzy logic with evaluated syntax, composition function

*Classification:* 28E10, 28E99

## 1. INTRODUCTION

The motivation of this research can be traced back to the seventies and eighties of the last century, to the time of the development of expert systems. The aim of an application of such a diagnostic expert system in a particular case was to weight each goal diagnosis using information about the values of the input attributes. The knowledge base of such a system contained a set of weighted rules usually obtained from experts. Rules were given in the form of implications

$$A_1 \wedge \ldots \wedge A_k \Longrightarrow C \quad (r)$$

where $A_i$ stood for propositions (values of different attributes) or their negations, and $r$ was a *weight* of the partial knowledge that the conjunction of $A_1, \ldots, A_k$ implied the conclusion $C$ or its negation. According to [5], the weights were typically real numbers from some interval. Basic examples were intervals $[0,1]$, $[-1,1]$ , and $[0,\infty]$. In this paper, we shall use the interval $[-1,1]$ where the weight 1 means certainly yes, $-1$ means certainly no, and 0 means unknown. We have several reasons for working with the interval $[-1,1]$, e.g.: this interval was used for dealing with uncertainty in the first compositional systems MYCIN, EMYCIN (weights as certainty factors); it is very suitable for many experts (own experience); its using makes a clear distinction between general weights and probabilities, there exists a neutral value (0) expressing

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"ignorance", and, as we will explain and employ later, it simply allows to separate positive and negative rules.

Sets of weighted rules along with (possibly weighted) input values of attributes are processed by some general inference mechanism to assign a resulting weight to each goal diagnosis (each conclusion which is not included in the condition part of any rule). During the process of inference that was usually realized by so called *forward chaining inference* from inputs through intermediate propositions to goals, the inference mechanism used several combination functions for weights calculation (for a general theory of combination functions see [3, 5]), namely:

- $NEG$ - when knowing the weight of a proposition, it calculated the weight of its negation;
- $CONJ$ - calculated the weight of propositions' conjunction from the weights of individual propositions;
- $CTR$ - which calculates the weight of a conclusion of a rule, when the weight of the rule's condition was given; if there were more rules with the same conclusion, this function yielded just the weight with which the rule contributed to its conclusion;
- $GLOB$ - if there were more rules with the same conclusion, this function determined the way how the results of individual rules, the contributions of rules, were composed together to assign the proper weight to the conclusion.

Let us consider a situation when a knowledge base is composed by an expert who designs rules and assigns some weights to them. To explain the basic idea in a simple way, let us assume the expert is constructing the set of rules systematically in a hierarchical way using Occam's razor principle: At first, he starts with the simplest rules of the form

$$A \Longrightarrow C \quad (b)$$

Then, when the expert considers a more complex rule of the form

$$A_1 \wedge A_2 \Longrightarrow C \quad (r)$$

he has to take into consideration whether its sub-rules were already inserted to the knowledge base, i. e. whether none, one, or both the two rules of the form

$$A_1 \Longrightarrow C \quad (r_1)$$

$$A_2 \Longrightarrow C \quad (r_2)$$

are contained in the knowledge base. Let for instance $q$ be the weight composed by the chosen inference mechanism from weights of these sub-rules using a particular composition function $GLOB$. If $q$ differs from the weight $r$ (expert's assumption) then the rule in question should be inserted into the knowledge base, however not with the weight $q$, but with some corrected weight $c$ such that the composition of weights $q, c$ by the function $GLOB$ gives the required weight $r$.

Similarly, the rule

$$A_1 \wedge \ldots \wedge A_k \Longrightarrow C \quad (r)$$

is considered by the expert as a new piece of knowledge only if the weight composed by the chosen inference mechanism from weights of all its sub-rules already existing in the knowledge base differs from the expert's assumption. Once more, the rules are included into the knowledge base with some correcting weights.

So the expert includes into the knowledge base only rules representing differences to the results of the general inference procedure used in the system. The size of the resulting rule base depends on the chosen inference mechanism, specially on its composition operation $GLOB$. The weights assigned to these rules are prescribed for correcting results obtained from the weights of their sub-rules.

This general approach of hierarchical selections and corrections of expert's weighted rules will be more specified in the next sections for the case when Łukasiewicz's fuzzy logic with evaluated syntax for dealing with weights is used.

## 2. INFERENCE MECHANISM BASED ON ŁUKASIEWICZ'S FUZZY LOGIC WITH EVALUATED SYNTAX

Our inference mechanism for uncertainty processing in rule-based knowledge systems was inspired by the complete Łukasiewicz's fuzzy logic with evaluated syntax $Ev_{Ł}$ introduced by J. Pavelka (see [4, 9, 10, 11]). It has been designed and implemented previously in the System of Automatic Consultations (SAK), see [6, 8], nowadays in its follower New Expert System (NEST).

Let us briefly mention the reasons which inspired us for applying Łukasiewicz's fuzzy logic with evaluated syntax $Ev_{Ł}$ in (un)certain knowledge processing. The essence of our approach consists in representing a knowledge base as a fuzzy axiomatic theory, i. e. a set of formulas in which each formula is assigned a degree of membership in the fuzzy set of special axioms. In such a way, the uncertainty degrees are interpreted as measures of law-likeness of rules, i. e. degrees in which the rules are axioms of the field. The whole knowledge base is represented as a fuzzy set of axioms (a fuzzy axiomatic theory). According to this principle, the task of inference in knowledge systems with uncertainties can be viewed as a deduction in a many-valued (fuzzy) logic with evaluated syntax. In this logic, $[\alpha;\, a]$ stands for an evaluated formula where $\alpha$ is a propositional formula and $a \in [0, 1]$ is a degree assigned to $\alpha$.

As a result of this theoretical considerations, Łukasiewicz's fuzzy logic with evaluated syntax $Ev_{Ł}$ is preferred because of its completeness property. Due to Pavelka's result [11], only Łukasiewicz's fuzzy logic with evaluated syntax $Ev_{Ł}$ (and its isomorphic variants) out of uncountably many possible variants of fuzzy logic with evaluated syntax with truth values in the unit interval (defined by different residuated lattices) has the property of completeness which means (roughly speaking): for any formula $\varphi$ and any axiomatic theory $T$ the degree to which $\varphi$ follows from $T$ semantically is equal to the degree to which $\varphi$ is provable from $T$ syntactically.

More precisely, the degree $t$ to which formula $\varphi$ follows from the theory $T$ semantically (i. e. $T \models_t \varphi$) is the infimum of the truth values of $\varphi$ in all models $val$ of the theory $T$. Recall that a model of the theory $T$ is a truth valuation $val$ of propositional variables such that the truth values of the axioms of the theory $T$ are at least their degrees stated in the theory. The truth values of propositional formulas are calculated from the truth valuation of propositional variables using the truth functions of the connectives in the given propositional fuzzy logic.

Correspondingly, the degree $s$ to which formula $\varphi$ is provable from $T$ syntactically (i.e. $T \vdash_s \varphi$) is the supremum of the values $d$ of all proofs of $\varphi$ in the given fuzzy theory $T$

$$[\varphi_0;\ d_0], [\varphi_1;\ d_1], \ldots, [\varphi_n;\ d_n], [\varphi;\ d],$$

where the value $d$ of a proof is obtained by the sequential applications of the evaluated inference rules of a particular logic (using fuzzy logical and special axioms from the given theory $T$ endowed with degrees). An inference rule

$$\frac{[\beta_1;\ b_1], \ldots, [\beta_k;\ b_k]}{[\beta;\ b]}$$

is endowed with instructions for calculations of the degree $b$, which should be assigned to the conclusion of the inference rule from degrees $b_1, \ldots, b_k$ having been assigned to its premises.

Because both the degree $t$ of truthfulness ($T \models_t \varphi$), and the degree $s$ of provability ($T \vdash_s \varphi$) coincide in the complete Łukasiewicz's fuzzy logic with evaluated syntax $Ev_{\text{Ł}}$ for every theory $T$ and each formula $\varphi$, we prefer this logic as a base for our inference mechanism.

We shall use the following logical connectives and notations for evaluated propositional formulas:

- $\neg$ is the negation with the truth function $1 - x$ for $x \in [0, 1]$;
- $\wedge$ is the conjunction with the truth function $\min(x, y)$ for $x, y \in [0, 1]$;
- & is Łukasiewicz's conjunction with the truth function $\max(0, x + y - 1)$ for $x, y \in [0, 1]$;
- $\curlyvee$ is Łukasiewicz's disjunction with the truth function $\min(1, x+y)$ for $x, y \in [0, 1]$;
- $\Rightarrow$ is Łukasiewicz's implication with the truth function $\min(1, 1 - x + y)$ for $x, y \in [0, 1]$;
- Propositional variables and formulas will be denoted by Greek letters.

The main deduction rule of modus ponens in Łukasiewicz's fuzzy logic with evaluated syntax $Ev_{\text{Ł}}$:

$$\frac{[\alpha;\ a], [\alpha \Rightarrow \beta;\ w]}{[\beta;\ \max(0, a + w - 1)]}$$

means: if the premises $(\alpha), (\alpha \Rightarrow \beta)$ have been derived in degrees $a, w$ respectively, then the conclusion $(\beta)$ may be derived in the degree $\max(0, a + w - 1)$.

## 3. RULE BASE REPRESENTATION IN ŁUKASIEWICZ'S FUZZY LOGIC WITH EVALUATED SYNTAX

When representing weights from the interval $[-1, 1]$ by degrees from $[0, 1]$, we have to cope with one basic problem. A proposition is assigned a weight 0 if we do not have any reasons supporting either the proposition or its negation; in the case of total ignorance. But there is not such a value in fuzzy logic. This is why we represent the weight of each proposition by two numbers: one expressing the truth value of the considered proposition, the other expressing the truth value of its negation.

To make this type of representation unambiguous and clear, in the sequel we will represent each proposition $V$ from the given knowledge base by a couple consisting of two propositional variables $\vartheta^-, \vartheta^+$. In addition, some other variables are introduced to represent formally needed steps of the composition process.

So, a knowledge base given as a set of weighted rules and input propositions (noted by Latin letters) with weights from the interval $[-1, +1]$ is in our approach represented as a fuzzy theory (noted by Greek letters) that can be described in the following recurrent way:

1. Let $I$ be an input proposition (i. e. each proposition which is not a conclusion of any rule of the given knowledge base) and $a$ be its given weight from the interval $[-1, 1]$. For our representation, we introduce two propositional variables $\iota^+, \iota^-$.

   If $a \geqslant 0$ then we represent it by two special axioms: $[\iota^+;\ a], [\iota^-;\ 0]$

   If $a < 0$ then we represent it by two special axioms: $[\iota^+;\ 0], [\iota^-;\ -a]$

   Each occurrence of the proposition $I$ in every assumption of consequent rules is now changed for our representation to the formula:

   $$(\iota^+ \& \neg\iota^-).$$

   Analogically, each occurrence of negation of $I$ is represented by the formula:

   $$(\iota^- \& \neg\iota^+).$$

   Let us note that only one of the formulas $(\iota^+ \& \neg\iota^-), (\iota^- \& \neg\iota^+)$ can have a positive (non-zero) degree (if $a > 0$ then it is the first one, if $a < 0$ then it is the second one). So in this step formulas $(\iota^+ \& \neg\iota^-), (\iota^- \& \neg\iota^+)$ are only expressing a positive, or a negative weight of the input proposition $I$, respectively. For propositions which are conclusions of rules, these formulas describe a superiority of positive contributions above negative ones, or a superiority of negative contributions above positive ones, respectively (as it will be given in the next step).

2. Now, let us have the non-empty list of all rules from the knowledge base with the same conclusion $C$ and assume recurrently that each proposition $V$ occurring in any assumption of these rules was already processed by the previous representation steps. Each rule will be represented by a special axiom of a fuzzy theory in Łukasiewicz's fuzzy logic with evaluated syntax. We separate positive, and negative rules.

   Let following $k$ rules be those with the positive weights:

   $$R_1^+ \Longrightarrow C \qquad (r_1^+)$$
   $$\dots$$
   $$R_k^+ \Longrightarrow C \qquad (r_k^+).$$

For our representation, we introduce propositional variables $\gamma_1^+, \ldots, \gamma_k^+$, and $\gamma^+$. We are representing the above rules by the following special axioms of the corresponding fuzzy theory:

$$[\rho_1^+ \Rightarrow \gamma_1^+;\ r_1^+]$$
$$\ldots$$
$$[\rho_k^+ \Rightarrow \gamma_k^+;\ r_k^+]$$

where formulas $\rho_i^+$ are obtained from $R_i^+$ by the previous representation steps, i. e. each occurrence of the proposition $V$ in $R_i^+$ is represented by the formula

$$(\vartheta^+ \& \neg\vartheta^-),$$

each occurrence of negation of $V$ is represented by the formula

$$(\vartheta^- \& \neg\vartheta^+),$$

respectively, and the conjunction among components of $R_i^+$ is treated as usual fuzzy conjunction $\wedge$.

To formally express the needed composition of the rules' positive contributions, we use Łukasiewicz's disjunction and add the special axiom

$$[(\gamma_1^+ \curlyvee \ldots \curlyvee \gamma_k^+) \Rightarrow \gamma^+;\ 1]$$

(or eventually $[\gamma^+;\ 0]$ if $k = 0$).

Similarly for $l$ rules with the negative weights:

$$R_1^- \Longrightarrow C \qquad (r_1^-)$$
$$\ldots$$
$$R_l^- \Longrightarrow C \qquad (r_l^-)$$

we introduce propositional variables $\gamma_1^-, \ldots, \gamma_k^-$, $\gamma^-$ and state the following special axioms of the corresponding fuzzy theory:

$$[\rho_1^- \Rightarrow \gamma_1^-;\ -\ r_1^-]$$
$$\ldots$$
$$[\rho_l^- \Rightarrow \gamma_l^-;\ -\ r_l^-]$$

where formulas $\rho_j^-$ are obtained from $R_j^-$ analogously as above for the case of positive rules, and $(-\ r_j^-)$ are now numbers in $[0,\ 1]$.

To formally express the needed composition of the rules' negative contributions, we use Łukasiewicz's disjunction and add the special axiom

$$([\gamma_1^- \curlyvee \ldots \curlyvee \gamma_l^-) \Rightarrow \gamma^-;\ 1]$$

(or eventually $[\gamma^-;\ 0]$ if $l = 0$).

3. Each occurrence of the proposition $C$ in some consequent rule assumption will be transformed for our representation to the formula:

$$(\gamma^{+}\&\neg\gamma^{-})$$

which describes the superiority of the positive contributions to $C$ above the negative ones.

Analogically, each occurrence of negation of $C$ is represented by the formula:

$$(\gamma^{-}\&\neg\gamma^{+})$$

which describes the superiority of the negative contributions to $C$ above the positive ones.

Clearly, only one of these formulas can have a positive (non-zero) degree.

4. Finally, each goal diagnosis $D$ (which is the conclusion of some rules but is not included in any assumption) is in this way represented by the introduced propositional variables $\delta^{+}$ and $\delta^{-}$, and formulas

$$(\delta^{+}\&\neg\delta^{-}), (\delta^{-}\&\neg\delta^{+}).$$

A positive (non-zero) degree of the first formula will mean a recommendation of the goal diagnosis $D$, a positive (non-zero) degree of the second formula will mean the goal diagnosis $D$ is not recommended.

Now, the whole knowledge (inputs and rules) is represented as a fuzzy set of axioms, so the knowledge processing can be understood as formal proving in the fuzzy theory (using Łukasiewicz's fuzzy logic with evaluated syntax $Ev_{\text{Ł}}$). It can be used to calculate the provability degrees of formulas corresponding to the goals of the knowledge base.

Evidently, the fuzzy theory composed by the above procedure from a rule base without cycles is not contradictory due to separate representation of positive and negative contributions of rules with the same conclusion by different propositional variables. Moreover, each propositional variable $\pi$ is either introduced by the special axiom $[\pi;\ p]$ or it is a conclusion of the unique special axiom $[\alpha\Rightarrow\pi;\ w]$.

The *forward chaining* inference procedure will start with the input propositional variables introduced in the above representation step 1 and will arrange proofs of all intermediate formulas (and finally also of formulas corresponding to the goals of the knowledge base) according to the above described representation steps in such a way that each propositional variable $\pi$ will be derived by the deduction rule of modus ponens applied to the special axiom $[\alpha\Rightarrow\pi;\ w]$ immediately after the premise $\alpha$ has been derived.

The degrees of propositional variables obtained by these proofs create the truth valuation *val* of propositional variables. First, there are assigned the truth values $val(\pi) = p$ to all propositional variables $\pi$ which are introduced by the special axioms of the form $[\pi;\ p]$. Further, iteratively according to the representation steps described above in the point 2, the truth value $y$ is assigned to the propositional variable $\pi$ when the deduction rule of modus ponens is applied in the corresponding forward chaining proof to the special axiom $[\alpha\Rightarrow\pi;\ w]$ and its premise $[\alpha;\ a]$ with the conclusion $[\pi;\ y]$.

This valuation guarantees all axioms of our theory hold true in the requested degrees ($val$ is a model of the theory). To verify it, let us consider a special axiom of the form $[\alpha \Rightarrow \pi;\ w]$. Assume recurrently that its premise $\alpha$ was proved with value $a$ which is also the truth value of $\alpha$ in the created truth valuation $val$. Then $\pi$ is immediately proved by the application of the deduction rule of modus ponens in Łukasiewicz's fuzzy logic with evaluated syntax $Ev_{Ł}$ with the value $y = \max(0, a + w - 1)$. Let us assign $val(\pi) = y$. The truth function of Łukasiewicz's implication gives in the truth valuation $val$ for the formula $\alpha \Rightarrow \pi$ the truth value

$$\min(1, 1 - a + y) = \min(1, 1 - a + \max(0, a + w - 1)) = \begin{cases} w & \text{for } w > 1 - a \\ 1 - a & \text{for } w \leq 1 - a \end{cases}$$

which means that the special axiom $[\alpha \Rightarrow \pi;\ w]$ has in the created truth valuation $val$ the truth value which is at least its degree $w$ stated in the theory.

So, the created truth valuation $val$ is the model of our theory. It also means the proofs with maximum values are identified for all necessary formulas by the forward chaining inference procedure. Due to the completeness of Łukasiewicz's fuzzy logic with evaluated syntax $Ev_{Ł}$, we have for our special theory the provability degrees which are the degrees to which formulas follow from the theory semantically as well.

## 4. ILLUSTRATING EXAMPLE

We shall illustrate the described method of formal representation of rule bases in Łukasiewicz's fuzzy logic with evaluated syntax $Ev_{Ł}$ on a simple example.

Let us assume a rule base:

$$\begin{array}{rl} A \Longrightarrow F & (0.4) \\ B \Longrightarrow F & (0.8) \\ F \Longrightarrow G & (0.7) \\ \neg C \wedge D \Longrightarrow G & (-0.5) \\ \neg E \Longrightarrow G & (-0.7) \end{array}$$

and the following assignment of weights to input propositions:

$$\begin{array}{ll} A & (1) \\ B & (0.4) \\ C & (-0.6) \\ D & (0.8) \\ E & (-1). \end{array}$$

Let us represent these inputs in the form of a fuzzy axiomatic theory:
Input propositions are represented by special fuzzy axioms (evaluated formulas) applying

the operations of Section 3, point 1:

$$[\alpha^+;\ 1],\ [\alpha^-;\ 0]$$
$$[\beta^+;\ 0.4],\ [\beta^-;\ 0]$$
$$[\gamma^+;\ 0],\ [\gamma^-;\ 0.6]$$
$$[\delta^+;\ 0.8],\ [\delta^-;\ 0]$$
$$[\varepsilon^+;\ 0],\ [\varepsilon^-;\ 1].$$

The rules

$$A \Longrightarrow F \qquad (0.4)$$
$$B \Longrightarrow F \qquad (0.8)$$

and their composition are represented by special fuzzy axioms (evaluated formulas) applying the operations of Section 3, points 1, 2:

$$[(\alpha^+ \& \neg\alpha^-) \Rightarrow \varphi_1^+;\ 0.4]$$
$$[(\beta^+ \& \neg\beta^-) \Rightarrow \varphi_2^+;\ 0.8]$$
$$[(\varphi_1^+ \curlyvee \varphi_2^+) \Rightarrow \varphi^+;\ 1]$$
$$[\varphi^-;\ 0].$$

The rule

$$F \Longrightarrow G \qquad (0.7)$$

is represented by a special fuzzy axiom (evaluated formula) applying the operations of Section 3, points 2, 3:

$$[(\varphi^+ \& \neg\varphi^-) \Rightarrow \psi_1^+;\ 0.7].$$

The rules

$$\neg C \wedge D \Longrightarrow G \qquad (-0.5)$$
$$\neg E \Longrightarrow G \qquad (-0.7)$$

and their composition are represented by special fuzzy axioms (evaluated formulas) applying the operations of Section 3, points 1, 2:

$$[(\gamma^- \& \neg\gamma^+) \wedge (\delta^+ \& \neg\delta^-) \Rightarrow \psi_1^-;\ 0.5]$$
$$[(\varepsilon^- \& \neg\varepsilon^+) \Rightarrow \psi_2^-;\ 0.7]$$
$$[\psi_1^+ \Rightarrow \psi^+;\ 1]$$
$$[(\psi_1^- \curlyvee \psi_2^-) \Rightarrow \psi^-;\ 1].$$

Finally, the goal diagnosis $G$ is represented applying the operations of Section 3, point 4 by the formulas

$$(\psi^+ \& \neg\psi^-), (\psi^- \& \neg\psi^+).$$

The provability degrees of the proposition variables obtained by sequential applications of the deduction rule of modus ponens in Łukasiewicz's fuzzy logic with evaluated syntax $Ev_Ł$ to special fuzzy axioms of our theory (and logical axioms for used connectives) look as follows:

$$[\varphi_1^+;\ 0.4], [\varphi_2^+;\ 0.2], [\varphi^+;\ 0.6],$$
$$[\psi_1^+;\ 0.3], [\psi^+;\ 0.3],$$
$$[\psi_1^-;\ 0.1], [\psi_2^-;\ 0.7], [\psi^-;\ 0.8].$$

Finally, the formulas representing the goal $G$ are proved with the degrees:

$$[(\psi^+ \& \neg\psi^-);\ 0]$$
$$[(\psi^- \& \neg\psi^+);\ 0.5].$$

So, the resulting weight of the goal $G$ is $-0.5$.

## 5. COMBINATION FUNCTIONS BASED ON ŁUKASIEWICZ'S FUZZY LOGIC WITH EVALUATED SYNTAX

It is clear that the calculations of provability degrees in the fuzzy theory representing a rule base can be realized also with original rules and weights in the interval $[-1, +1]$ which is frequently used in rule-based systems. The corresponding combination functions on $[-1, +1]$ deduced from Łukasiewicz's fuzzy logic with evaluated syntax $Ev_Ł$ can be written as follows:

$$NEG(a) = -a$$

calculates the weight of a proposition's negation given its weight $a$;

$$CONJ(a_1, a_2) = \min(a_1, a_2)$$

calculates the weight of propositions' conjunction given their weights $a_1, a_2$;

$$CTR(a, b) = \begin{cases} sgn(b) \cdot (\max(0, a + |b| - 1) & \text{for } a > 0 \\ 0 & \text{for } a \leq 0 \end{cases}$$

calculates the weight, with which the rule $A_1 \wedge \ldots \wedge A_k \implies B$ with the weight $b$ contributes to its conclusion when its assumption has the weight $a$. It corresponds to the deduction rule of modus ponens of Łukasiewicz's fuzzy logic with evaluated syntax $Ev_Ł$ adopted for the case of weights taken from $[-1, +1]$.

Finally, the function $GLOB$ realizes the composition of contributions $c_1, \ldots, c_n$ of rules with the same conclusion $C$:

$$GLOB(c_1, \ldots, c_n) = \min\left(1, \sum_{c_i > 0} c_i\right) + \max\left(-1, \sum_{c_i < 0} c_i\right).$$

This function is based on Łukasiewicz's disjunction modified for the case of weights taken from $[-1, +1]$. The positive and negative weights are treated here separately. Let us note that this composition function is commutative and associative inside the subgroup of positive arguments, and inside the subgroup of negative arguments, respectively. As a conclusion, the value $GLOB(c_1, \ldots, c_n)$ does not depend on the ordering of arguments $c_1, \ldots, c_n$.

The inference mechanism based on Łukasiewicz's fuzzy logic with evaluated syntax $Ev_{Ł}$ has some advantages, namely connected to the simplicity of combining uncertainties using only operations of limited addition and subtraction of weights. This leads to the effect that the resulting weights are in the same scale as the weights in the knowledge base and the inputs during its processing in applications. All computations with weights are clearly understandable for experts. Experiments with the knowledge bases processed by standard (Prospector and MYCIN) inference mechanisms and our inference mechanism confirmed the comparability of results [6, 8]. Nevertheless, one more advantage arises from the possibility of introducing the special types of rules we are going to describe in the next section.

## 6. CORRECTING AND TURNING RULES

Let us apply our general approach of hierarchical selections and corrections of expert's weighted rules outlined in the introductory section for the case when dealing with weights based on Łukasiewicz's fuzzy logic with evaluated syntax $Ev_{Ł}$ is used.

A problem is arising when the difference between the required weight $r$ and the composed weight $q$ is greater than 1. Although this situation is rather exceptional in the case of knowledge base formulation by an expert, the correction process should be arranged to also cope with it. We propose for that purpose the idea of turning rules and the corresponding modification of the inference mechanism.

First of all, let us describe the idea of turning rules on the simplest case mentioned in Section 1: Let us consider a rule of the form

$$A \wedge B \Longrightarrow C \quad (r)$$

with the required weight $r$. Let $q$ be the weight composed by the function $GLOB$ from weights $r_1, r_2$ of sub-rules

$$A \Longrightarrow C \quad (r_1), \quad B \Longrightarrow C \quad (r_2).$$

If $q$ differs from the expert's assumption $r$ and $|r - q| \leq 1$ then the rule in question is inserted to the knowledge base with the corrected weight $c = r - q$. As the result, the sequential composition of the weights $r_1, r_2, c$ gives the assumed weight $r$.

On the other hand, if $|r - q| > 1$ then it is not possible to reach the assumed weight $r$ by any correcting weight $c$ from $[-1, +1]$. To solve this problem, the rule in question is inserted to the knowledge base two times: first as a *turning* rule with the weight $t = sgn(r - q)$, and second as a *correcting* rule with the weight $c = r - q - sgn(r - q)$. As the result, the sequential composition of the weights $r_1, r_2, t, c$ gives the required weight $r$.

Let us note that our method of rule base representation in Łukasiewicz's fuzzy logic with evaluated syntax (described in Section 3) could be extended for this new case of knowledge bases with two types of rules.

**Example.** To illustrate this, let the weights in the above simplest case be $r_1 = 0.6, r_2 = 0.3$, so $q = 0.9$, but the required weight $r = -0.5$. Then the turning rule with the weight $t = sgn(r-q) = -1$ and the correcting rule with the weight $c = r-q-sgn(r-q) = -0.4$ will be included to the knowledge base. Our representation of this knowledge base will be:

$$[(\alpha^+\&\neg\alpha^-)\Rightarrow\gamma_1^+;\ 0.6]$$
$$[(\beta^+\&\neg\beta^-)\Rightarrow\gamma_2^+;\ 0.3]$$
$$[(\gamma_1^+ \curlyvee \gamma_2^+)\Rightarrow\gamma_q^+;\ 1]$$
$$[\gamma_q^-;\ 0]$$
$$[(\alpha^+\&\neg\alpha^-) \wedge (\beta^+\&\neg\beta^-)\Rightarrow\gamma_t^-;\ 1]$$
$$[\gamma_t^+;\ 0]$$
$$[((\gamma_q^+\&\neg\gamma_q^-) \curlyvee (\gamma_t^+\&\neg\gamma_t^-))\Rightarrow\gamma_s^+;\ 1]$$
$$[((\gamma_q^-\&\neg\gamma_q^+) \curlyvee (\gamma_t^-\&\neg\gamma_t^+))\Rightarrow\gamma_s^-;\ 1]$$
$$[(\alpha^+\&\neg\alpha^-) \wedge (\beta^+\&\neg\beta^-)\Rightarrow\gamma_c^-;\ 0.4]$$
$$[\gamma_c^+;\ 0]$$
$$[((\gamma_s^+\&\neg\gamma_s^-) \curlyvee (\gamma_c^+\&\neg\gamma_c^-))\Rightarrow\gamma^+;\ 1]$$
$$[((\gamma_s^-\&\neg\gamma_s^+) \curlyvee (\gamma_c^-\&\neg\gamma_c^+))\Rightarrow\gamma^-;\ 1]$$

where the propositional variables $\gamma_q^+, \gamma_q^-, \gamma_t^+, \gamma_t^-, \gamma_s^+, \gamma_s^-, \gamma_c^+, \gamma_c^-$ and the corresponding axioms are newly introduced in such a way that the sequential composition of the weights $r_1, r_2, t, c$ is ensured. Suppose input propositions $A, B$ are true so they are represented by formulas

$$[\alpha^+;\ 1], [\alpha^-;\ 0], [\beta^+;\ 1], [\beta^-;\ 0].$$

Provability degrees obtained by sequential applications of the deduction rule of modus ponens in Łukasiewicz's fuzzy logic with evaluated syntax $Ev_{\text{Ł}}$ to the above fuzzy axioms (and logical axioms for the connectives used) look as follows:

$$[\gamma_q^+;\ 0.9], [\gamma_q^-;\ 0]$$
$$[\gamma_t^+;\ 0], [\gamma_t^-;\ 1]$$
$$[\gamma_s^+;\ 0.9], [\gamma_s^-;\ 1]$$
$$[\gamma_c^+;\ 0], [\gamma_c^-;\ 0.4]$$
$$[\gamma^+;\ 0], [\gamma^-;\ 0.5].$$

Then the formulas representing the conclusion $C$ are proved with the degrees:

$$[(\gamma^+\&\neg\gamma^-);\ 0]$$
$$[(\gamma^-\&\neg\gamma^+);\ 0.5].$$

So, the resulting weight of the proposition $C$ is $-0.5$ as it was requested.

In general, the extension of our method of rule base representation in Łukasiewicz's fuzzy logic with evaluated syntax (described in Section 3) for the case of knowledge bases with two types of rules (turning and correcting ones) is rather formally complicated. The paper is primarily determined to the reader interested in construction of knowledge bases and their applications to real-world problems. Therefore the rest of the paper neglects the formal apparatus and concentrates on the practical instructions for hierarchical knowledge base construction.

The function $GLOB$ which realizes the composition of contributions $c_1, \dots, c_n$ of rules with the same conclusion $C$ should be adopted for working with both correcting ($CR$) and turning ($TR$) rules. The modified function $GLOB^*$ is defined in such a way that $GLOB$ is applied sequentially according to lengths of rules.

Let $TR_i, CR_i$ be the sets of all turning rules, and correcting rules, respectively, with the conclusion $C$ and the length $i$. Let us define partial compositions:

- $t_i$ as the result of $GLOB$ on the weights of rules from $TR_i$ (in fact, $t_i \in \{-1, 0, 1\}$ reflects the result of voting of positive turning rules against negative ones), and
- $w_i$ as the result of $GLOB$ on the weights of rules from $CR_i$.

Let us mention that $t_i, w_i$ do not depend on the ordering of rules in $TR_i$, and $CR_i$, respectively, because of $GLOB(c_1, \dots, c_n)$ does not depend on the ordering of arguments.

Then we apply the operation

$$g(a,b) = \quad \begin{cases} \min(1, a+b) & \text{for } a,b > 0 \\ \max(-1, a+b) & \text{for } a,b < 0 \\ a+b & \text{for } a \cdot b < 0 \end{cases}$$

for the sequence of values $t_1, w_1, t_2, w_2, \dots$ (from the left) to obtain the resulting value $w^*$ of the modified composition function $GLOB^*$ applied on weights of correcting and turning rules with the conclusion $C$ .

## 7. HIERARCHICAL SELECTION AND CORRECTION OF WEIGHTED RULES

The composition function $GLOB^*$ (which is now defined for working with both correcting and turning rules) allows us to realize our hierarchical selection and correction of weighted rules.

Keep in mind that the hierarchical process of a knowledge base construction proceed from shorter rules to longer ones. Therefore, when considering a rule

$$A_1 \wedge \dots \wedge A_k \Longrightarrow C \quad (r),$$

all shorter rules have already been considered (and in the case of necessity included into the knowledge base as the correcting rules or even turning ones).

Let $q^*$ be the weight composed by the function $GLOB^*$ from the weights of all sub-rules of the considered rule which have already been included in the rule base:

$$q^* = GLOB^*(t_1, w_1, \dots, t_{k-1}, w_{k-1})$$

where $t_i$ is the partial composition of the weights of the turning sub-rules of the length $i$, and $w_i$ is the partial composition of the weights of the correcting sub-rules of the length $i$, respectively.

1. If $q^*$ is equal to the expert's assumption $r$, then the rule in question is redundant and will not be inserted to the rule base under construction.

2. If $q^*$ differs from the expert's assumption $r$ and $|r - q^*| \leq 1$ then the rule in question is inserted to the rule base with the corrected weight $c = r - q^*$. As the result, the composition of all the weights of sub-rules and the corrected weight $c$ by function $GLOB^*$ gives the required weight $r$ because

$$GLOB^*(t_1, w_1, \ldots, t_{k-1}, w_{k-1}, t_k, w_k) = GLOB^*(t_1, w_1, \ldots, t_{k-1}, w_{k-1}, 0, c)$$

$$= g(q^*, r - q^*) = r.$$

3. On the other hand, if $|r - q^*| > 1$ then the rule in question is inserted to the knowledge base two times: first as a turning rule with the weight $t = sgn(r - q^*)$, and second as a correcting rule with the weight $c = r - q^* - sgn(r - q^*)$. As the result, the composition of all the weights of sub-rules and the weights $t, c$ gives the required weight $r$.

   To show this, let us discuss two possible cases:

   - Let $r > 0 > q^*$. Then $t = sgn(r - q^*) = 1$ and

     $$GLOB^*(t_1, w_1, \ldots, t_{k-1}, w_{k-1}, t_k, w_k) = GLOB^*(t_1, w_1, \ldots, t_{k-1}, w_{k-1}, t, c)$$

     $$= g(g(q^*, 1), r - q^* - 1) = g(q^* + 1, r - q^* - 1) = r$$

     because $q^* + 1 > 0, r - q^* - 1 > 0$.

   - Let $r < 0 < q^*$. Then $t = sgn(r - q^*) = -1$ and

     $$GLOB^*(t_1, w_1, \ldots, t_{k-1}, w_{k-1}, t_k, w_k) = GLOB^*(t_1, w_1, \ldots, t_{k-1}, w_{k-1}, t, c)$$

     $$= g(g(q^*, -1), r - q^* + 1) = g(q^* - 1, r - q^* + 1) = r$$

     because $q^* - 1 < 0, r - q^* + 1 < 0$.

As the result, the rule base will be in our case composed by two sets of rules:

- $TR$ is the set of turning rules with weights from $\{-1, 1\}$,

- $CR$ is the set of correcting rules with weights from $[-1, +1]$.

Let us mention once more that a particular turning rule reflects an exceptional situation when the direction of the rule proposed by the expert is totally opposite to the direction deduced from its already stated sub-rules.

## 8. CONCLUSION

We proposed an application of the correction principle during hierarchical constructions of the rule base in the frame of the inference mechanism based on Łukasiewicz's fuzzy logic with evaluated syntax $Ev_Ł$. More complex rules are added into the rule set only when the requested weight of the rule in question differs from the composed weight (value obtained when composing weights of all sub-rules which have been inserted into the rule base already). At this moment, a weight correcting the composed weight to the requested one is calculated. We proposed addition of turning rules to the rule base in situations when the composed and the required weight are strongly opposite. This leads to some modifications of the composition function.

The described approach to a hierarchical selection and correction of weighted rules can be used in the frame of some other approaches for dealing with uncertainties of rules. As an example we can remind our method for automatic knowledge base construction from categorical data (see [1, 2, 7]). Possible applications of Łukasiewicz's fuzzy logic with evaluated syntax $Ev_Ł$ in this case are a topic of an ongoing research.

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