Jianglin Lan; Weijie Sun; Yunjian Peng

Constrained robust adaptive stabilization for a class of lower triangular systems with unknown control direction

Kybernetika, Vol. 50 (2014), No. 3, 450–469

Persistent URL: `http://dml.cz/dmlcz/143885`

KYBERNETIKA — VOLUME 50 (2014), NUMBER 3, PAGES 450–469

CONSTRAINED ROBUST ADAPTIVE STABILIZATION FOR A CLASS OF LOWER TRIANGULAR SYSTEMS WITH UNKNOWN CONTROL DIRECTION

Jianglin Lan, Weijie Sun and Yunjian Peng

This paper studies the constrained robust adaptive stabilization problem for a class of lower triangular systems with unknown control direction. A robust adaptive feedback control law for the systems is proposed by incorporating the technique of Barrier Lyapunov Function with Nussbaum gain. Such a controlled system arises from the study of the constrained robust output regulation problem for a class of output feedback systems with the unknown control direction and a nonlinear exosystem. An application of the constrained robust adaptive stabilization design leads to the solution of the constrained robust output regulation problem in the sense that the output tracking error is constrained within the prescribed barrier limit while asymptotically approaching to zero and the closed loop signals are all bounded for all the time. A numerical example is provided to illustrate the performance of the proposed control.

Keywords: Barrier Lyapunov Function, output regulation, nonlinear exosystem, Nussbaum gain

Classification: 93E12, 62A10

1. INTRODUCTION

The stabilization design problem for nonlinear systems has been paid great attention since 1980s, see [9, 11, 14] and their references therein. Various techniques and insights have been extensively issued, among which the small-gain control and backstepping design method as well as their new developments have been well applied and shown their great effectiveness [9, 10, 11]. Recently, a framework has been proposed transforming the original output regulation problem into a stabilization problem for an augmented system based on the internal model principle [8]. Such a framework brings forward new stabilization problem for nonlinear systems [3, 20] and much more challenging stabilization issues [4, 12, 21]. As such, the constrained robust adaptive stabilization problem to be considered in this paper for the lower triangular system is just closely related to the constrained robust output regulation problem for a class of output feedback systems in Section 4.

The stabilization problem to be considered in this paper is much more challenging than the previous ones. Two new technical issues have to be addressed here. The first

DOI: 10.14736/kyb-2014-3-0450

challenging issue is imposed by the fact that the aimed output is not only required to asymptotically approach to zero, but also expected to be guaranteed within some desired region during its transient period. The second issue concerns the unknown control coefficient $b(w)$, which needs to be estimated by employing some adaptive parameter update law. It is in this sense that the problem addressed in this paper is called the constrained robust adaptive stabilization problem.

The raise of the constrained stabilization problem is motivated by the fact that physical systems are greatly influenced by many factors such as physical geometry constraints [15, 21]. For example, for the electrostatic torsional micromirror in microscale, since the durability of the micromirror plays a major role in deciding reliability and longevity of a micromirror trauma surgery device [6], scanning control of the movable micromirror is required to avoid its collision with the fixed bottom electrodes in order to reduce surface damage at each contact and greatly increase the device lifetime [15, 21]. Recently, [28] dealt with the enhanced output regulation performance for the linear systems with input saturation and [21] studied the robust output regulation problem for a one degree of freedom electrostatic microelectromechanical systems model with output constraints.

As for the issue of the control coefficient, in some particular cases, its direction can be known, which clearly offers convenience for the control design. Actually, under the conditions that the plant is stable and the high-frequency probing signals can be introduced into the system, the high-frequency gain can be identified off-line before applying the control design to the system. However, it is not realistic to make the assumption that the control direction is known in the general case. For instance, due to the existence of the parameter variation, the real values of the system parameters needed for the control design might not be obtained precisely and such a fact could lead the control direction to be unknown which makes the controller design less effectiveness. Therefore, the adaptive control problem with unknown control direction has gained great attention for the past decades, and large numbers of works either on the output regulation problem or stabilization problem for nonlinear systems with unknown control direction have been published, see [12, 13, 26, 27] and their references therein.

Considering the two issues of the output constraint and the unknown control direction, techniques of Barrier Lyapunov Function and Nussbaum gain will be incorporated to handle them. The Barrier Lyapunov Function technique has been proved to be efficient in solving the variable constrained problems [21, 23] and the Nussbaum gain method is the standard way to handle the unknown sign of the high-frequency gain [12, 13, 16, 26, 27].

The reminder of this paper is organized as follows. In Section 2, the constrained robust adaptive stabilization problem is formulated and some preliminaries are made. In Section 3, a robust adaptive feedback control law is designed to solve the constrained robust adaptive stabilization problem described in Section 2. In Section 4, the constrained robust adaptive stabilization design proposed in Section 3 is applied to solve the constrained robust adaptive output regulation problem for a class of output feedback systems subject to the unknown control direction and a nonlinear exosystem. In Section 5, a numerical example is provided to illustrate the effectiveness of the proposed control design. Finally, the conclusion is given in Section 6.

2. PROBLEM FORMULATION AND PRELIMINARIES

Consider the following class of lower triangular nonlinear systems

$$\begin{aligned} \dot{Z} &= F(Z, x_1, v, w) \\ \dot{x}_1 &= f_1(Z, x_1, v, w) + b(w)x_2 \\ \dot{x}_i &= f_i(x_1, \ldots, x_i) + x_{i+1}, i = 2, \ldots, r-1 \\ \dot{x}_r &= f_r(x_1, \ldots, x_r) + \bar{u} \\ y &= x_1 \end{aligned} \tag{1}$$

where $Z \in R^n$ and $x = (x_1, \ldots, x_r)^\top$ with $x_i \in R$, $i = 1, \ldots, r$, are the states, $b(w)$ is the control coefficient, $\bar{u} \in R$ is the control input, $y \in R$ is the output, $w \in R^{n_w}$ is the constant uncertain parameter, and $v : [0, \ \infty) \to R^q$ is a smooth bounded time-varying function. Suppose that all the functions $F(\cdot)$ and $f_i(\cdot)$, $i = 1, \ldots, r$, in (1) are sufficiently smooth, and $F(0, 0, v, w) = 0$.

The constrained robust adaptive stabilization problem addressed in this paper can be defined as follows.

Definition 2.1. (Constrained Robust Adaptive Stabilization Problem) Given any compact set $\Omega \subset R^q \times R^{n_w}$ and some smooth positive definite time-varying function $k_{b_1}(t)$ with the property that $0 < k_{b_1}(t) \le \bar{K}_{b_0}$ and whose time derivatives satisfy $|k_{b_1}^{(i)}(t)| \le \bar{K}_{b_i}$ for positive constants $\bar{K}_{b_i}, i = 0, \ldots, r-1$, for all $t \ge 0$, find a feedback control law for the lower triangular system (1) to ensure that the closed loop system satisfies the following properties:

P1. For all $(v, w) \in \Omega$, the solution of the closed loop system exists and is bounded for all $t \ge 0$.

P2. The output y asymptotically approaches to zero, i. e., $\lim_{t\to\infty} x_1 = 0$. In particular, if the initial output satisfies $|x_1(0)| < k_{b_1}(0)$, then $|x_1(t)| < k_{b_1}(t)$ for all $t \ge 0$.

In what follows, a constrained robust adaptive stabilization design will be proposed to solve this constrained robust adaptive stabilization problem. To this end, some assumptions about system (1) and a lemma based on the results in [23] and [26] are established firstly in the following.

Assumption 2.1. For all $w \in R^{n_w}$, $b(w) \neq 0$ and there exist positive constants b_m and b_M such that $b_m < |b(w)| < b_M$.

Assumption 2.2. For any compact subset $\Omega \subset R^q \times R^{n_w}$, there exists some $\mathcal{C}^1$ function $U_0(Z)$ satisfying $\underline{\alpha}_{0Z}(\|Z\|) \le U_0(Z) \le \bar{\alpha}_{0Z}(\|Z\|)$ for some class $\mathcal{K}_\infty$ functions $\underline{\alpha}_{0Z}(\cdot)$ and $\bar{\alpha}_{0Z}(\cdot)$ such that, for all $(v, w) \in \Omega$, the derivative of $U_0(Z)$ along the trajectory of the subsystem $\dot{Z} = F(Z, x_1, v, w)$ satisfies

$$\dot{U}_0(Z) \quad \le \quad -\alpha(\|Z\|) + \delta_0 \gamma_0(x_1)$$

with some unknown positive constant δ_0, some known class $\mathcal{K}_\infty$ functions $\alpha(\cdot)$ satisfying $\lim_{s\to 0^+} \sup(\alpha^{-1}(s^2)/s) < \infty$, and a known smooth positive definite function $\gamma_0(x_1)$.

Remark 2.1. From Assumption 2.2, it can be concluded that the subsystem $\dot{Z} = F(Z, x_1, v, w)$ is input-to-state stable, considering x_1 as the input and Z as the state. By employing the changing supply functions technique in [18], for any smooth function $\Delta(Z) > 0$, there exists some $\mathcal{C}^1$ function $U(Z)$ satisfying $\underline{\alpha}_Z(\|Z\|) \leq U(Z) \leq \bar{\alpha}_Z(\|Z\|)$ with some class $\mathcal{K}_\infty$ functions $\underline{\alpha}_Z(\cdot)$ and $\bar{\alpha}_Z(\cdot)$ such that, for all $(v, w) \in \Omega$, the following inequality holds

$$\dot{U}(Z) \quad \leq \quad -\Delta(Z)\|Z\|^2 + \delta\gamma(x_1)x_1^2$$

with some unknown positive constant δ and a known smooth positive definite function $\gamma(x_1)$.

Lemma 2.1. For any positive definite function $k_{b_1}(t)$, let $\mathcal{Z} = \{x_1(t) \in R : |x_1(t)| < k_{b_1}(t)\} \subset R$ and $\mathcal{N} = R^l \times \mathcal{Z} \subset R^{l+1}$ be open sets. Consider the system

$$\dot{\zeta} = h(t, \zeta) \tag{2}$$

where $\zeta = (\chi, x_1)^\top \in \mathcal{N}$, and $h : R_+ \times \mathcal{N} \to R^{l+1}$ is piecewise continuous in t, locally Lipschitz in ζ, and uniformly in t, on $R_+ \times \mathcal{N}$. Suppose that there exist functions $U : R^l \to R_+$ and $V_1 : \mathcal{Z} \to R_+$, which are continuously differentiable and positive definite in their respective domains, such that

$$\begin{aligned} V_1(x_1) \to \infty \quad \text{as} \quad |x_1| \to k_{b_1} \\ \varphi_1(\|\chi\|) \leq U(\chi) \leq \varphi_2(\|\chi\|) \end{aligned} \tag{3}$$

where $\varphi_1(\cdot)$ and $\varphi_2(\cdot)$ are class $\mathcal{K}_\infty$ functions. Let $V(\zeta) = V_1(x_1) + U(\chi)$ and $|x_1(0)| < k_{b_1}(0)$. If the following holds

$$V(\zeta) \quad \leq \quad \int_0^t (2b(w)N(k(\tau)) + c)\dot{k}(\tau)\,\mathrm{d}\tau + const, \quad \forall\, t \in [0, t_f) \tag{4}$$

where $k(\cdot)$ is a smooth function defined on $[0, t_f)$, $N(\cdot)$ is an even smooth Nussbaum-type function, $b(w)$ is a nonzero constant that takes values in the unknown closed interval $I = [b_m,\ b_M]$ with $0 \notin I$ for each fixed $w \in R^{n_w}$, c is any positive number and $const$ is some suitable constant, then $V(\zeta)$ is bounded and $x_1(t) \in \mathcal{Z}$ for all $t \geq 0$.

Proof. According to the Theorem 5.4 in [19], it can be seen that the conditions on function $h(\cdot)$ ensure that there exists a unique maximal solution $\zeta(t)$ on time interval $[0, t_f)$, which indicates the existence of $V(\zeta)$ for all $t \in [0, t_f)$.

By the proof of Lemma 1 in [26] and from the inequality (4), the boundedness of $k(t)$ on $[0, t_f)$ can be concluded. Thus, $V(\zeta)$ and $\int_0^t (2b(w)N(k(\tau)) + c)\dot{k}(\tau)\,\mathrm{d}\tau$ are also bounded on $[0, t_f)$. From the definition that $V(\zeta) = V_1(x_1) + U(\chi)$ with positive definite functions $V_1(x_1)$ and $U(\chi)$, it is known that $V_1(x_1)$ is also bounded on $t \in [0, t_f)$. Thus, it can be seen from (3) that $|x_1| \neq k_{b_1}$, which yields the fact that $x_1(t) \in \mathcal{Z}$, for all $t \in [0, t_f)$, if $|x_1(0)| < k_{b_1}(0)$.

In this respect, it is obvious that there exists a compact subsect $\Theta \subseteq \mathcal{N}$ ensures that the maximal solution $\zeta(t)$ of the system (2) fulfills that $\zeta(t) \in \Theta$ for all $t \in [0, t_f)$. Further, the Proposition C.3.6 in [19] indicates that $\zeta(t)$ is actually defined on the interval $[0, \infty)$, which directly leads to the conclusion that $V(\zeta)$ is bounded and $x_1(t) \in \mathcal{Z}$ for all $t \geq 0$. $\square$

3. CONSTRAINED ROBUST ADAPTIVE STABILIZATION DESIGN

In this section, a robust adaptive feedback control law will be proposed to solve the constrained robust adaptive stabilization problem described in Section 2. The techniques of Barrier Lyapunov Function and Nussbaum gain are to be combined to handle the nontrivial issues of the constrained output x_1 and the unknown control coefficient $b(w)$.

For simplicity, the following notations $f_1 = f_1(Z, x_1, v, w)$, $\alpha_1 = \alpha_1(x_1, k, k_{b_1})$, $f_i = f_i(x_1, \ldots, x_i)$, $\alpha_i = \alpha_i(x_1, \ldots, x_i, k, k_{b_1}, \hat{b})$, $i = 2, 3, \ldots, r$, and $\rho_{x_1} = \rho(\tilde{x}_1, k_{b_1})$, are used during the design procedure.

Step 1. Define $\tilde{x}_1 = x_1$ and

$$\begin{aligned}
\alpha_1 &= N(k)(k_{b_1}^2 - \tilde{x}_1^2)\rho_{x_1}\tilde{x}_1 \\
N(k) &= k^2 \cos(k) \\
\dot{k} &= \rho_{x_1}\tilde{x}_1^2 \\
\tilde{x}_2 &= x_2 - \alpha_1
\end{aligned}$$

where $N(k)$ is a Nussbaum-type function and ρ_{x_1} is some smooth function to be given later.

Consider the following Barrier Lyapunov Function candidate

$$V_{blf} = \log \frac{k_{b_1}^2}{k_{b_1}^2 - \tilde{x}_1^2}. \tag{5}$$

It can be calculated that

$$\begin{aligned}
\dot{V}_{blf} &= \frac{2\tilde{x}_1}{k_{b_1}^2 - \tilde{x}_1^2}\left(\dot{\tilde{x}}_1 - \tilde{x}_1\frac{\dot{k}_{b_1}}{k_{b_1}}\right) \\
&\leq \left(\frac{2}{(k_{b_1}^2 - \tilde{x}_1^2)^2} + \left(\frac{\dot{k}_{b_1}}{k_{b_1}}\right)^2\right)\tilde{x}_1^2 + f_1^2 + \frac{2b(w)\tilde{x}_1\tilde{x}_2}{k_{b_1}^2 - \tilde{x}_1^2} + 2b(w)N(k)\dot{k}.
\end{aligned} \tag{6}$$

Remark 3.1. It could be concluded that V_{blf} is a suitable Barrier Lyapunov Function candidate according to [23]. On one hand, for all $\tilde{x}_1 \in (-k_{b_1}, k_{b_1})$, it is obvious that $V_{blf} \geq 0$ and $V_{blf} = 0$ if and only if $\tilde{x}_1 = 0$. Thus, V_{blf} is continuous and positive definite in the set $(-k_{b_1}, k_{b_1})$. On the other hand, $V_{blf} \to \infty$ whenever $|\tilde{x}_1| \to k_{b_1}$.

Let $V_1 = U(Z) + V_{blf}$. According to Remark 2.1 and the inequality (6), we have

$$\begin{aligned}
\dot{V}_1 &= \dot{U}(Z) + \dot{V}_{blf} \\
&\leq -\Delta(Z)\|Z\|^2 + \left(\delta\gamma(\tilde{x}_1) + \frac{2}{(k_{b_1}^2 - \tilde{x}_1^2)^2} + \left(\frac{\dot{k}_{b_1}}{k_{b_1}}\right)^2\right)\tilde{x}_1^2 + f_1^2 + \frac{2b(w)\tilde{x}_1\tilde{x}_2}{k_{b_1}^2 - \tilde{x}_1^2} \\
&\quad + 2b(w)N(k)\dot{k}.
\end{aligned}$$

Step 2. Define

$$\begin{aligned}\alpha_2 &= -f_2 - \tilde{x}_2 + \frac{\partial \alpha_1}{\partial k}\dot{k} + \frac{\partial \alpha_1}{\partial k_{b_1}}\dot{k}_{b_1} + \hat{b}\frac{\partial \alpha_1}{\partial \tilde{x}_1}x_2 - \frac{1}{4}(\frac{\partial \alpha_1}{\partial \tilde{x}_1})^2\tilde{x}_2 \\ \tilde{x}_3 &= x_3 - \alpha_2 \\ \gamma_1 &= -\frac{\partial \alpha_1}{\partial \tilde{x}_1}\tilde{x}_2 x_2\end{aligned}$$

where $\hat{b}$ is designed to estimate the unknown $b(w)$.

Consider the Lyapunov function

$$V_2 = V_1 + \tilde{x}_2^2 + (\hat{b} - b(w))^2.$$

Note that

$$\begin{aligned}\tilde{x}_2\dot{\tilde{x}}_2 &= \tilde{x}_2(\dot{x}_2 - \dot{\alpha}_1) \\ &= \tilde{x}_2\Big(f_2 + \tilde{x}_3 + \alpha_2 - (\frac{\partial \alpha_1}{\partial \tilde{x}_1}\dot{\tilde{x}}_1 + \frac{\partial \alpha_1}{\partial k}\dot{k} + \frac{\partial \alpha_1}{\partial k_{b_1}}\dot{k}_{b_1})\Big) \\ &\leq \tilde{x}_2\tilde{x}_3 - \tilde{x}_2^2 + f_1^2 + (\hat{b} - b(w))\frac{\partial \alpha_1}{\partial \tilde{x}_1}\tilde{x}_2 x_2,\end{aligned}$$

then it can be calculated that

$$\begin{aligned}\dot{V}_2 &= \dot{V}_1 + 2\tilde{x}_2\dot{\tilde{x}}_2 + 2(\hat{b} - b(w))\dot{\hat{b}} \\ &\leq -\Delta(Z)\|Z\|^2 + \Big(\delta\gamma(\tilde{x}_1) + \frac{2 + b^2(w)}{(k_{b_1}^2 - \tilde{x}_1^2)^2} + \Big(\frac{\dot{k}_{b_1}}{k_{b_1}}\Big)^2\Big)\tilde{x}_1^2 + 2b(w)N(k)\dot{k} \\ &\quad + 2(\hat{b} - b(w))(\dot{\hat{b}} - \gamma_1) + 3f_1^2 - \tilde{x}_2^2 + 2\tilde{x}_2\tilde{x}_3.\end{aligned}$$

Step i. Further define

$$\begin{aligned}\alpha_i &= -f_i - \tilde{x}_{i-1} - \tilde{x}_i + \frac{\partial \alpha_{i-1}}{\partial k}\dot{k} + \frac{\partial \alpha_{i-1}}{\partial k_{b_1}}\dot{k}_{b_1} + \frac{\partial \alpha_{i-1}}{\partial \hat{b}}\gamma_{i-1} + \sum_{j=2}^{i-1}\frac{\partial \alpha_{i-1}}{\partial \tilde{x}_j}\dot{\tilde{x}}_j \\ &\quad -\Big(\hat{b} - \sum_{j=2}^{i-2}\tilde{x}_{j+1}\frac{\partial \alpha_j}{\partial \hat{b}}\Big)\frac{\partial \alpha_{i-1}}{\partial \tilde{x}_1}x_2 - \frac{1}{4}(\frac{\partial \alpha_{i-1}}{\partial \tilde{x}_1})^2\tilde{x}_i \\ \tilde{x}_{i+1} &= x_{i+1} - \alpha_i \\ \gamma_{i-1} &= \gamma_{i-2} - \frac{\partial \alpha_{i-1}}{\partial \tilde{x}_1}x_2\tilde{x}_i\end{aligned}$$

with $i = 3, \ldots, r$, and $x_{r+1} = \bar{u} \in R$.

Letting $V_i = V_{i-1} + \tilde{x}_i^2$ yields

$$\begin{aligned}\dot{V}_i &\leq -\Delta(Z)\|Z\|^2 + \Big(\delta\gamma(\tilde{x}_1) + \frac{2 + b^2(w)}{(k_{b_1}^2 - \tilde{x}_1^2)^2} + \Big(\frac{\dot{k}_{b_1}}{k_{b_1}}\Big)^2\Big)\tilde{x}_1^2 + 2b(w)N(k)\dot{k} \\ &\quad + 2(\hat{b} - b(w))(\dot{\hat{b}} - \gamma_{i-1}) + (2i-1)f_1^2 - \sum_{j=2}^{i}\tilde{x}_j^2 + 2\tilde{x}_i\tilde{x}_{i+1}.\end{aligned}$$

At the end of the design, by taking the control law $\bar{u} = \alpha_r$, $\dot{\hat{b}} = \gamma_{r-1}$ and noting $\tilde{x}_{r+1} = 0$, it can be obtained that

$$\begin{aligned} \dot{V}_r \;\; &\leq \;\; -\Delta(Z)\|Z\|^2 + \Big(\delta\gamma(\tilde{x}_1) + \frac{2 + b^2(w)}{(k_{b_1}^2 - \tilde{x}_1^2)^2} + \Big(\frac{\dot{k}_{b_1}}{k_{b_1}}\Big)^2\Big)\tilde{x}_1^2 + 2b(w)N(k)\dot{k} \\ &\quad + (2r-1)f_1^2 - \sum_{j=2}^{r} \tilde{x}_j^2. \end{aligned}$$

Denote $\mathcal{F}_r = (2r-1)f_1^2$. Since $f_1(Z, x_1, v, w)$ satisfies $f_1(0, 0, v, w) = 0$, by Lemma 7.8 of [7] and Taylor Theorem, there exist a positive constant s_1, some smooth function $\Delta_1(Z)$ and a known smooth function $\phi(\tilde{x}_1)$ such that

$$\mathcal{F}_r \;\; \leq \;\; \Delta_1(Z)\|Z\|^2 + s_1\phi(\tilde{x}_1)\tilde{x}_1^2.$$

Choose $\Delta(Z) \geq \Delta_1(Z) + 1$. Taking $c \geq \max\Big(\delta, s_1, 2 + b^2(w), \Big(\frac{\dot{k}_{b_1}}{k_{b_1}}\Big)^2\Big)$ and $\rho_{x_1} \geq \gamma(\tilde{x}_1) + \phi(\tilde{x}_1) + \frac{1}{(k_{b_1}^2 - \tilde{x}_1^2)^2} + 1$ gives $c\rho_{x_1} \geq \delta\gamma(\tilde{x}_1) + s_1\phi(\tilde{x}_1) + \frac{2+b^2(w)}{(k_{b_1}^2 - \tilde{x}_1^2)^2} + \Big(\frac{\dot{k}_{b_1}}{k_{b_1}}\Big)^2$. Thus, we get

$$\dot{V}_r \;\; \leq \;\; (2b(w)N(k) + c)\dot{k} - \|Z\|^2 - \sum_{j=2}^{r} \tilde{x}_j^2. \tag{7}$$

Denote $\tilde{x}_c = (Z, \tilde{x}, \hat{b})^\top$ with $\tilde{x} = (\tilde{x}_1, \ldots, \tilde{x}_r)^\top$. Applying Lemma 2.1 to the inequality (7) leads to the conclusion that $V_r(\tilde{x}_c)$, k and $\int_0^t (2b(w)N(k(\tau)) + c)\dot{k}(\tau)\,\mathrm{d}\tau$ are all bounded on $[0, \infty)$. Since $V_r(\tilde{x}_c)$ is a positive definite function of $\tilde{x}_c$, $\tilde{x}_c$ is bounded for all $t \geq 0$. According to this, all the signals of the closed loop system (1) are bounded due to the boundedness of v and w. Furthermore, it can be known that $(\dot{Z}, \dot{\tilde{x}})$ are bounded and square integrable on $[0, \infty)$. By Barbalat's Lemma, it is known that $(Z, \tilde{x})$ approaches to zero as $t \to \infty$. As a result, the output $x_1(t)$ converges to zero asymptotically. From Lemma 2.1, what can also be seen is that if $|\tilde{x}_1(0)| < k_{b_1}(0)$, then $|\tilde{x}_1(t)| < k_{b_1}(t)$ for all $t \in [0, \infty)$. To sum up, the following theorem for the constrained robust adaptive stabilization problem can be given.

Theorem 3.1. Under Assumption 2.1 and Assumption 2.2, the following robust adaptive feedback control law solves the constrained robust adaptive stabilization problem of the system (1)

$$\begin{aligned} \bar{u} &= \alpha_r \\ \dot{k} &= \rho_{x_1}\tilde{x}_1^2 \\ \dot{\hat{b}} &= \gamma_{r-1} \end{aligned} \tag{8}$$

with $i = 3, \ldots, r$,

$$\begin{aligned} N(k) &= k^2\cos(k) \\ \gamma_1 &= -\frac{\partial\alpha_1}{\partial\tilde{x}_1}x_2\tilde{x}_2 \\ \gamma_{i-1} &= \gamma_{i-2} - \frac{\partial\alpha_{i-1}}{\partial\tilde{x}_1}x_2\tilde{x}_i \end{aligned}$$

$$
\begin{aligned}
\alpha_1 &= N(k)(k_{b_1}^2-\tilde{x}_1^2)\rho_{x_1}\tilde{x}_1\\
\alpha_2 &= -f_2-\tilde{x}_2+\frac{\partial\alpha_1}{\partial k}\dot{k}+\frac{\partial\alpha_1}{\partial k_{b_1}}\dot{k}_{b_1}+\hat{b}\frac{\partial\alpha_1}{\partial\tilde{x}_1}x_2-\frac{1}{4}(\frac{\partial\alpha_1}{\partial\tilde{x}_1})^2\tilde{x}_2\\
\alpha_i &= -f_i-\tilde{x}_{i-1}-\tilde{x}_i+\frac{\partial\alpha_{i-1}}{\partial k}\dot{k}+\frac{\partial\alpha_{i-1}}{\partial k_{b_1}}\dot{k}_{b_1}+\frac{\partial\alpha_{i-1}}{\partial\hat{b}}\gamma_{i-1}+\sum_{j=2}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\tilde{x}_j}\dot{\tilde{x}}_j\\
&\quad -\Big(\hat{b}-\sum_{j=2}^{i-2}\tilde{x}_{j+1}\frac{\partial\alpha_j}{\partial\hat{b}}\Big)\frac{\partial\alpha_{i-1}}{\partial\tilde{x}_1}x_2-\frac{1}{4}(\frac{\partial\alpha_{i-1}}{\partial\tilde{x}_1})^2\tilde{x}_i.
\end{aligned}
$$

Remark 3.2. The proposed constrained robust adaptive stabilization design offers us a greater flexibility to incorporate various desired constraint areas for the aimed constrained output. In fact, any function $k_{b_1}(t)$ that is smooth positive definite, monotone decreasing, asymptotically converges to a positive steady value when time approaches to infinity, and whose time derivatives satisfy $|k_{b_1}^{(i)}(t)|\leq\bar{K}_{b_i}$ for positive constants $\bar{K}_{b_i}, i=0,\ldots,r-1$, for all $t\geq 0$, can be a candidate. Many functions satisfy such properties, e.g. $k_{b_1}(t)=\frac{1}{t+c}+\varepsilon$ with some suitable positive constants c and ε. However, due to easy physical realization, the constraint barrier $k_{b_1}(t)$ we select in practice may be much more to be an exponential function $k_{b_1}(t)=Le^{-ct}+\varepsilon$ with L, ε and c some positive constants. Its corresponding constraint area for the output is shown in Figure 1. Though $k_{b_1}(t)$ can also be chosen to be static defined as a positive constant depending on the allowed worst case constant bound of the output $x_1(t)$, i.e., $\max_{t\geq 0}\{|x_1(t)|\}$, as can be seen from Figure 1, the exponential convergence constraint theoretically imposes a much more stringent requirement on the transient behavior of the output than the static constraint $L+\varepsilon$. And the shorter time for the output to approach zero could be expected by an appropriate selection of the exponential convergence rate c and parameter ε.

Remark 3.3. We can compare the initial output requirement of Barrier Lyapunov Function based design with that of Quadratic Lyapunov Function. According to Lemma 2.1, from the inequality (7) and the definition of V_{blf} in the equation (5), it can be seen that, for all $t\geq 0$, $|\tilde{x}_1(t)|<k_{b_1}(t)$ if the initial output requirement satisfies the inequality

$$|\tilde{x}_1(0)| \quad < \quad k_{b_1}(0). \tag{9}$$

However, the initial output requirement is more stringent with Quadratic Lyapunov Function design. If we replace the Barrier Lyapunov Function V_{blf} with $\tilde{x}_1^2$ and let $V_r=\sum_{i=1}^r\tilde{x}_i^2+U(Z)+(\hat{b}-b(w))^2$, then the controller obtained is in the same form as the equation (8) except that $\rho_{x_1}=\beta$, $\alpha_1=N(k)\rho_{x_1}\tilde{x}_1^2$ with some positive constant β. The inequality (7) can also be obtained and further reduced into

$$\dot{V}_r \quad\leq\quad (2b(w)N(k)+c)\dot{k}.$$

Integrating both sides of the above inequality over $[0,t)$, for all $t\geq 0$, gives

$$V_r(t) \quad\leq\quad \int_0^t(2b(w)N(k(\tau))+c)\dot{k}(\tau)\,\mathrm{d}\tau+V_r(0).$$

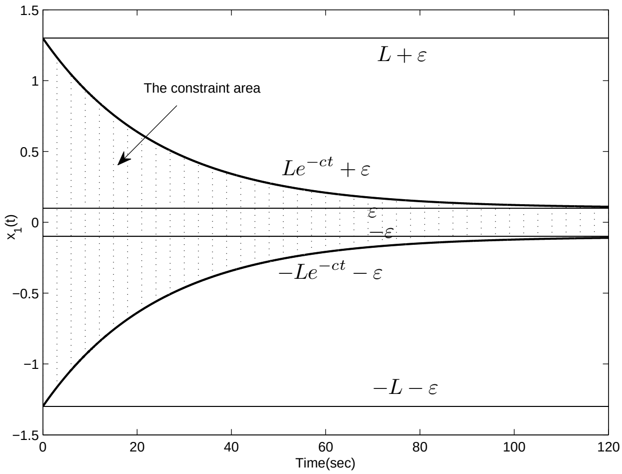

Fig. 1. The constraint area.

Since there exists some positive constant N_0 such that $\int_0^t (2b(w)N(k(\tau))+c)\dot{k}(\tau)\,\mathrm{d}\tau \le N_0$ for all $t \ge 0$, it can be obtained that $0 \le V_r(t) \le \bar{V}_r$ with the upper bound

$$\begin{aligned} \bar{V}_r &= N_0 + \bar{\alpha}_Z(\|Z(0)\|) + \sum_{j=1}^{r} \tilde{x}_j^2(0) + 2(\hat{b}^2(0) + b_M^2) \\ &= M_0 + \sum_{j=1}^{r} \tilde{x}_j^2(0) \end{aligned} \tag{10}$$

where $M_0 = N_0 + \bar{\alpha}_Z(\|Z(0)\|) + 2(\hat{b}^2(0) + b_M^2)$. Additionally, it holds that $|\tilde{x}_1(t)| \le \sqrt{\bar{V}_r}$. Thus, a sufficient condition for $|\tilde{x}_1(t)| < k_{b_1}(t)$ is $\sqrt{\bar{V}_r} < k_{b_1}(t)$. Since $k_{b_1}(t)$ is monotone decreasing, it holds that $k_{b_1}(t) \le k_{b_1}(0)$, and then also $\bar{V}_r < k_{b_1}^2(0)$. Finally, together with the equation (10), the initial output requirement with Quadratic Lyapunov Function can be given as

$$|\tilde{x}_1(0)| \quad < \quad \sqrt{k_{b_1}^2(0) - M_0 - \sum_{j=2}^{r} \tilde{x}_j^2(0)}. \tag{11}$$

Comparing the inequality (11) with the inequality (9), it is apparent that the initial output requirement is much more stringent when employing the Quadratic Lyapunov Function, due to its relationship with M_0 and $\sum_{j=2}^{r} \tilde{x}_j^2(0)$. Noting the fact that the virtual control functions $\alpha_1, \alpha_2, \ldots, \alpha_r$ are associated with $\sum_{j=2}^{r} \tilde{x}_j^2$, the control parameters in $\alpha_1, \alpha_2, \ldots, \alpha_r$ should be selected carefully so as to satisfy the condition $k_{b_1}^2(0) - M_0 - \sum_{j=2}^{r} \tilde{x}_j^2(0) > 0$. And even if such a condition were satisfied, the allowed feasible initial condition that guarantees the satisfaction of the constrained output is much smaller than employing the Barrier Lyapunov Function technique.

4. APPLICATION TO CONSTRAINED ROBUST OUTPUT REGULATION PROBLEM

In this section, the proposed control design in Section 3 will be applied to solve the constrained robust output regulation for a class of nonlinear systems in the following output feedback form

$$\begin{aligned}\dot{x} &= F(w)x + G(y,v,w) + g(w)u + D_1(v,w)\\ \dot{y} &= H(w)x + K(y,v,w) + D_2(v,w)\\ e &= y - q(v,w)\end{aligned} \tag{12}$$

where $(x,y)^\top \in R^n$ is the state, $u \in R$ and $y \in R$ are the input and output, respectively, and $e \in R$ is the tracking error, $w \in R^{n_w}$ is the uncertain parameter. The exogenous signal $v \in R^q$ is generated by the nonlinear exosystem

$$\dot{v} = a(v). \tag{13}$$

It is assumed that the system (12) holds a uniform relative degree $r \geq 2$. The control coefficient $g(w) \neq 0$ and there exist positive constants g_m and g_M such that $g_m < |g(w)| < g_M$ for all $w \in R^{n_w}$. All the functions in the system (12) are sufficiently smooth with $G(0,v,w) = 0$, $K(0,v,w) = 0$, $D_1(0,w) = 0$, $D_2(0,w) = 0$ and $q(0,w) = 0$, while $F(w)$ is Hurwitz for all $w \in R^{n_w}$. To have the problem well posed, it is further assumed that the solution of the exosystem (13) starting from any initial state $v(0)$ exists for all $t \geq 0$ and is globally bounded. The typical cases are those nonlinear systems which exhibit (chaotic) attractors or asymptotically stable limit cycles.

As in [5], since $a(0) = 0$, the nonlinear exosystem (13) can be decomposed as

$$a(v) = A_1 v + \sum_{k=2}^{K} A_k v a_k(v)$$

with some integer $K \geq 2$, some matrices $A_k \in R^{q\times q}$, and some sufficiently smooth functions $a_k(v) : R^q \to R$ satisfying $a_k(0) = 0$.

The constrained robust output regulation problem studied in this section is to design a robust adaptive output feedback control law u for the system composed of (12) and (13) such that, all the closed loop signals are bounded and the output y could asymptotically track the reference signal $q(v,w)$, i.e., $\lim_{t\to\infty} e(t) = 0$, and furthermore, the constraint barrier of the tracking error will not be violated, i.e., $|e(t)| < k_{b_1}(t)$ for all $t \geq 0$.

It is well known that the output regulation problem for output feedback systems can be converted into a stabilization problem for augmented lower triangular systems. The existence of a suitable internal model is the key to achieve such a conversion [2, 5]. For the linear exosystem case, the work [7] has given several conditions for the existence of the internal model. When the exosystem is nonlinear, it is more complicated and difficult to find an appropriate internal model. But for the reason that the output regulation technique would have a much wider practical application with a nonlinear exosystem, the output regulation problem for nonlinear systems under a nonlinear exosystem has attracted great attention recently, see [5, 20, 22, 25] and their references therein. Though [5, 22] have proposed the kind of internal model that involves the exogenous signal, it is not zero input asymptotically stable [20]. Recently, another kind of internal model also involving the exogenous signal is given in [25], which is zero input globally asymptotically stable and consequently can achieve the global stabilization for the augmented system. In this section, the kind of internal model given in [25] will be introduced to handle the constrained robust output regulation problem for the system (12) subject to a nonlinear exosystem and a unknown control coefficient.

By introducing a dynamic extension and performing the coordinate transformation like that in [14], the system (12) can be converted into a familiar lower triangular form

$$\begin{aligned} \dot{z} &= \bar{F}(w)z + \bar{G}(y,v,w) + \bar{D}_1(v,w) \\ \dot{y} &= \bar{H}(w)z + \bar{K}(y,v,w) + b(w)\xi_1 + \bar{D}_2(v,w) \\ \dot{\xi}_i &= -\lambda_i\xi_i + \xi_{i+1}, i = 1,\dots,r-2 \\ \dot{\xi}_{r-1} &= -\lambda_{r-1}\xi_{r-1} + u \\ e &= y - q(v,w) \end{aligned} \tag{14}$$

with the control coefficient $b(w)$ and the functions $\bar{F}(w), \bar{G}(y,v,w), \bar{D}_1(v,w), \bar{H}(w)$, $\bar{K}(y,v,w)$ and $\bar{D}_2(v,w)$ defined in [7].

In this respect, by employing the internal model principle in [25], the robust output regulation problem for the system (12) with output error constraint can be further converted into a robust stabilization problem for a transformed augmented system with output constraint. To achieve this, a few standard assumptions are listed.

Assumption 4.1. For all $v \in R^q$ and $w \in R^{n_w}$, there exists a sufficiently smooth function $\mathbf{z}(v,w)$ with $\mathbf{z}(0,0) = 0$ such that

$$\frac{\partial \mathbf{z}(v,w)}{\partial v}a(v) = \bar{F}(w)\mathbf{z}(v,w) + \bar{G}(q(v,w),v,w)q(v,w) + \bar{D}_1(v,w). \tag{15}$$

Remark 4.1. It is noticed that, unlike the special case where $a(v)$ is linear and the power series approach [1, 7] can be well applied to obtain the solution of the regulator equations, the solution of the equation (15) with nonlinear exosystem remains difficult [17] or impossible even in the case where all functions in (15) are of polynomial nonlinearity.

Assumption 4.1 guarantees that the regulator equations associated with the extended system (14) and the nonlinear exosystem (13) have a global solution given as

$$\begin{aligned} \mathbf{y}(v,w) &= q(v,w) \\ \mathbf{\Xi}_1(v,w) &= \frac{1}{b(w)}(\mathcal{L}_a\mathbf{y}(v,w) - \bar{H}(w)\mathbf{z}(v,w) - \bar{K}(\mathbf{y}(v,w),v,w) - \bar{D}_2(v,w)) \\ \mathbf{\Xi}_i(v,w) &= \mathcal{L}_a\mathbf{\Xi}_{i-1}(v,w) + \lambda_{i-1}\mathbf{\Xi}_{i-1}(v,w), \quad i = 2,\dots,r-1 \\ \mathbf{u}(v,w) &= \mathcal{L}_a\mathbf{\Xi}_{r-1}(v,w) + \lambda_{r-1}\mathbf{\Xi}_{r-1}(v,w) \end{aligned}$$

where $\mathcal{L}_a\mathbf{y}(v,w)$ and $\mathcal{L}_a\Xi_i(v,w)$ are the Lie derivatives of $\mathbf{y}(v,w)$ and Ξ_i along $a(v)$, respectively.

Assumption 4.2. There exist some integer s and sufficiently smooth scalar functions $b_i(v), i=1,\dots,s$, such that

$$\frac{d^s\Xi_1(v,w)}{dt^s} = b_0(v)\Xi_1(v,w)+b_1(v)\frac{d\Xi_1(v,w)}{dt}+\ldots+b_{s-1}(v)\frac{d^{s-1}\Xi_1(v,w)}{dt^{s-1}}.$$

Under Assumption 4.2, a steady-state input generator with output Ξ_1 can be given as

$$\begin{aligned}\frac{d\tau(v,w)}{dt} &= \Phi(v)\tau(v,w)\\ \Xi_1(v,w) &= \Gamma\tau(v,w)\end{aligned}$$

with $\tau(v,w)=(\tau_1,\tau_2,\dots,\tau_s)^\top=(\Xi_1,\dot{\Xi}_1,\dots,\Xi_1^{s-1})^\top$, $\Phi(v)=\begin{bmatrix} b_{s-1}(v) & \\ \vdots & I_{s-1}\\ b_0(v) & 0\end{bmatrix}$ and $\Gamma=[1\ \ 0\ \ \dots\ \ 0]$. Moreover, the matrix $\Phi(v)$ can be rewritten as $\Phi(v)=\Phi_b+b(v)\Gamma$, with $\Phi_b=\begin{bmatrix} 0 & \\ \vdots & I_{s-1}\\ 0 & 0\end{bmatrix}$ and $b(v)=\begin{bmatrix} b_{s-1}(v)\\ \vdots\\ b_0(v)\end{bmatrix}$.

Since the pair (Φ_b,Γ) is observable, for any Hurwitz matrix $M\in R^{s\times s}$ and $N\in R^{s\times 1}$ such that (M,N) is controllable, there is a unique and nonsingular matrix T satisfying the Sylvester equation $T\Phi_b-MT=N\Gamma$. Let $\theta(v,w)=T\tau(v,w)$ and $\beta_1(\theta)=\Gamma(T^{-1}\theta)$, then $\dot{\theta}(v,w)=T\Phi(v)T^{-1}\theta(v,w)$. Further, let $\beta_i(\theta,v)=\frac{\partial\beta_{i-1}(\theta,v)}{\partial\theta}\dot{\theta}+\frac{\partial\beta_{i-1}(\theta,v)}{\partial v}a(v)+\lambda_{i-1}\beta_{i-1}(\theta,v), i=2,\dots,r$. It is noticed that the functions $\beta_i, i=1,\dots,r$, are independent on $b(w)$. Denote $N(v)=N+Tb(v)$, an internal model of the system (14) with the output ξ_1 [25] is

$$\dot{\eta} = M\eta+N(v)\xi_1. \tag{16}$$

Performing on the augmented system composed of (14) and (16) the following coordinate and input transformation

$$\begin{aligned}\bar{z} &= z-\mathbf{z}(v,w)\\ e &= y-q(v,w)\\ \bar{\xi}_1 &= \xi_1-\beta_1(\eta)\\ \bar{\xi}_i &= \xi_i-\beta_i(\eta,v),\quad i=2,\dots,r-1\\ \bar{\eta} &= \eta-\theta(v,w)-N(v)b^{-1}(w)e\\ \bar{u} &= u-\beta_r(\eta,v)\end{aligned}$$

yields the augmented system in a lower triangular form

$$
\begin{aligned}
\dot{\bar{z}} &= \bar{F}(w)\bar{z} + \widetilde{G}(x_1, v, w) \\
\dot{\bar{\eta}} &= M\bar{\eta} + f_0(\bar{z}, \bar{\eta}, x_1, v, w) \\
\dot{x}_1 &= f_1(\bar{z}, \bar{\eta}, x_1, v, w) + b(w)x_2 \\
\dot{x}_i &= f_i(x_1, \ldots, x_i, \eta, v) + x_{i+1}, \ i = 2, \ldots, r-1 \\
\dot{x}_r &= f_r(x_1, \ldots, x_r, \eta, v) + \bar{u}
\end{aligned} \tag{17}
$$

where $x = (x_1, \ldots, x_r)^\top = (e, \bar{\xi}_1, \ldots, \bar{\xi}_{r-1})^\top$, $\Psi = \Gamma T^{-1}$,

$$
\begin{aligned}
\widetilde{G}(x_1, v, w) &= \bar{G}(q(v, w) + x_1, v, w) - \bar{G}(q(v, w), v, w) \\
\widetilde{K}(x_1, v, w) &= \bar{K}(q(v, w) + x_1, v, w) - \bar{K}(q(v, w), v, w) \\
f_0(\bar{z}, \bar{\eta}, x_1, v, w) &= MN(v)b^{-1}(w)x_1 - N(v)b^{-1}(w)(\bar{H}(w)\bar{z} + \widetilde{K}(x_1, v, w)) \\
&\quad -N^{(1)}(v)b^{-1}(w)x_1 \\
f_1(\bar{z}, \bar{\eta}, x_1, v, w) &= \bar{H}(w)\bar{z} + \tilde{K}(x_1, v, w) + b(w)\Psi\Big(\bar{\eta} + N(v)b^{-1}(w)x_1\Big) \\
f_i(x_1, \ldots, x_i, \eta, v) &= -\frac{\partial \beta_{i-1}(\eta, v)}{\partial \eta} N(v)\bar{\xi}_1 - \lambda_{i-1}\bar{\xi}_{i-1}, \quad i = 2, \ldots, r.
\end{aligned}
$$

Denote $Z = (\bar{z}, \bar{\eta})^\top$, then the system (17) can be rewritten as

$$
\begin{aligned}
\dot{Z} &= F(Z, x_1, v, w) \\
\dot{x}_1 &= f_1(Z, x_1, v, w) + b(w)x_2 \\
\dot{x}_i &= f_i(x_1, \ldots, x_i, \eta, v) + x_{i+1}, \ i = 2, \ldots, r-1 \\
\dot{x}_r &= f_r(x_1, \ldots, x_r, \eta, v) + \bar{u}
\end{aligned} \tag{18}
$$

where $F(Z, x_1, v, w) = \begin{bmatrix} \bar{F}(w)\bar{z} + \tilde{G}(x_1, v, w) \\ M\bar{\eta} + f_0(\bar{z}, \bar{\eta}, x_1, v, w) \end{bmatrix}$.

Now, the solution of the stabilization problem with constraint on the output x_1 for the system (18) directly determines the solution of the original constrained robust output regulation problem for the system (12). It can be seen that the system (18) is actually in the form of the system (1) with the output x_1 except that the functions $f_i(x_1, \ldots, x_i, v, \eta), i = 2, 3, \ldots, r$ are associated with v and η. But it has no impact on employing the proposed control design in Section 3 due to the availability of v and η. Also, since $F(w)$ is Hurwitz for all w and M is also a Hurwitz matrix, by Lemma 3.1 of [24], Assumption 2.2 is satisfied with $\alpha(\|Z\|) = \|Z\|^2$, some known smooth positive definite function $\gamma_0(x_1)$ and some positive constant δ_0 depending on w and $v(0)$.

Thus, applying Theorem 3.1 to the system (18) gives the following theorem.

Theorem 4.1. Under Assumption 4.1 and Assumption 4.2, the feedback control law composed of (8) and (16) solves the constrained robust output regulation problem for the system (14) with the nonlinear exosystem (13).

5. AN EXAMPLE

In order to illustrate its effectiveness, the proposed control design is applied to solve the constrained robust output regulation problem for the following system.

$$\begin{aligned} \dot{z} &= -z + 2wv_1y + v_2^2 - 2(1+w)v_1v_2 \\ \dot{y} &= wz - v_1y + b(w)\xi_1 + v_1v_2 - v_1 - wv_2^2 - v_2 \\ \dot{\xi}_1 &= -\xi_1 + u \\ e &= y - v_2 \end{aligned} \tag{19}$$

where w represents the uncertain parameter and $b(w)$ represents the unknown control coefficient. The exogenous signal is generated by

$$\begin{aligned} \dot{v}_1 &= v_1 - \frac{v_1^3}{3} + v_2 \\ \dot{v}_2 &= -v_1. \end{aligned} \tag{20}$$

Let us first show that the system composed of (19) and (20) satisfies Assumption 4.1. In fact, it can be verified that the solution of the regular equations is $\mathbf{z}(v,w) = v_2^2$, $\mathbf{y}(v,w) = v_2$, $\mathbf{\Xi}_1(v,w) = \frac{1}{b(w)}v_2$ and $\mathbf{u}(v,w) = \frac{1}{b(w)}(v_2 - v_1)$. Also, since $\frac{d^2\mathbf{\Xi}_1(v,w)}{dt^2} = -\mathbf{\Xi}_1(v,w) + (1 - \frac{v_1^2}{3})\frac{d\mathbf{\Xi}_1(v,w)}{dt}$, Assumption 4.2 is satisfied with $\Gamma = [1 \;\; 0]$ and $\Phi(v) = \begin{bmatrix} 1 - \frac{v_1^2}{3} & 1 \\ -1 & 0 \end{bmatrix}$. Choosing the controllable pair (M,N) as $M = \begin{bmatrix} -2 & 1 \\ -1 & 0 \end{bmatrix}$, $N = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and solving the Sylvester equation $T\Phi_b - MT = N\Gamma$ gives $T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and the following internal model

$$\dot{\eta} = \begin{bmatrix} -2 & 1 \\ -1 & 0 \end{bmatrix}\eta + \begin{bmatrix} 3 - \frac{v_1^2}{3} \\ 0 \end{bmatrix}\xi_1. \tag{21}$$

With the above T, we can get $\beta_1(\eta) = \eta_1$, $\beta_2(\eta,v) = (2 - \frac{v_1^2}{3})\eta_1 + \eta_2$. Then by performing the coordinate and input transformation

$$\begin{aligned} \bar{z} &= z - \mathbf{z}(v,w) \\ e &= y - v_2 \\ \bar{\xi}_1 &= \xi_1 - \beta_1(\eta) \\ \bar{\eta} &= \eta - \theta(v,w) - N(v)b^{-1}(w)e \\ \bar{u} &= u - \beta_2(\eta,v) \end{aligned}$$

on the system composed of (19) and (21) and letting $Z = (\bar{z},\bar{\eta})^\top$, $x = (x_1,x_2)^\top = (e,\bar{\xi}_1)^\top$, it yields the following lower triangular system in the form of (18)

$$\begin{aligned} \dot{Z} &= F(Z,x_1,v,w) \\ \dot{x}_1 &= f_1(Z,x_1,v,w) + b(w)x_2 \\ \dot{x}_2 &= f_2(x_1,x_2,v) + \bar{u} \end{aligned}$$

where

$$\begin{aligned}
F(Z, x_1, v, w) &= \begin{bmatrix} 2wv_1 \\ (-\frac{5}{9}v_1^3 + \frac{4}{3}v_1^2 + \frac{2}{3}v_1v_2 + 3v_1 - 6)b^{-1}(w) \\ (\frac{1}{3}v_1^2 - 3)b^{-1}(w) \end{bmatrix} x_1 \\
&\quad + \begin{bmatrix} -1 & 0 & 0 \\ (-\frac{1}{3}v_1^2 + 3)b^{-1}(w) & -2 & 1 \\ 0 & -1 & 0 \end{bmatrix} Z \\
f_1(Z, x_1, v, w) &= (-\frac{1}{3}v_1^2 - v_1 + 3)x_1 + [w \;\; b(w) \;\; 0]\, Z \\
f_2(x_1, x_2, v) &= -x_2 - (3 - \frac{v_1^2}{3})x_1.
\end{aligned}$$

According to Theorem 4.1, a specific controller for the system composed of (19) and (20) is

$$\begin{aligned}
u &= \alpha_2 + \beta_2(\eta, v) \\
\dot{k} &= \rho_{x_1}\tilde{x}_1^2 \\
\dot{\hat{b}} &= -\frac{\partial \alpha_1}{\partial \tilde{x}_1}\tilde{x}_2 x_2 \\
\dot{\eta} &= \begin{bmatrix} -2 & 1 \\ -1 & 0 \end{bmatrix}\eta + \begin{bmatrix} 3 - \frac{v_1^2}{3} \\ 0 \end{bmatrix}\xi_1 \\
N(k) &= k^2\cos(k), \quad \rho_{x_1} = 40\Big(1 + \frac{1}{(k_{b_1}^2 - \tilde{x}_1^2)^2}\Big) \\
\alpha_1 &= N(k)(k_{b_1}^2 - \tilde{x}_1^2)\rho_{x_1}\tilde{x}_1 \\
\alpha_2 &= x_2 + (3 - \frac{v_1^2}{3})x_1 - \tilde{x}_2 + \frac{\partial \alpha_1}{\partial k}\dot{k} + \frac{\partial \alpha_1}{\partial k_{b_1}}\dot{k}_{b_1} + \hat{b}\frac{\partial \alpha_1}{\partial \tilde{x}_1}x_2 - \frac{1}{4}(\frac{\partial \alpha_1}{\partial \tilde{x}_1})^2\tilde{x}_2 \\
x_1 &= e, \quad x_2 = \xi_1 - \beta_1(\eta) \\
\tilde{x}_1 &= e, \quad \tilde{x}_2 = \xi_1 - \beta_1(\eta) - \alpha_1.
\end{aligned}$$

The computer simulation is carried out with initial conditions $(z(0), y(0), \xi_1(0)) = (0.5, 3.5, 0)$, $(k(0), \hat{b}(0), \eta(0)) = 0$ and $v(0) = (3, 3)$, and the parameters $w = 1.5$ and $|b(w)| = 1$. The constraint barrier of the tracking error $e(t)$ is selected as $k_{b_1}(t) = 3.0e^{-0.06t} + 0.1$.

The simulation results under conditions where $b(w) = 1$ or $b(w) = -1$ are shown in Figures 2 to 6. The tracking performance of the system composed of (19) and (20) is shown in Figures 2 to 3, from which we can see that the tracking error $e(t)$ asymptotically approaches to zero while keeping in the prescribed constraint barrier $k_{b_1}(t)$ for all $t \geq 0$. Figure 4 shows the portraits of the Nussbaum function, while Figures 5 to 6 give the states of the closed loop system. It can be seen that the proposed control design has a satisfactory performance.

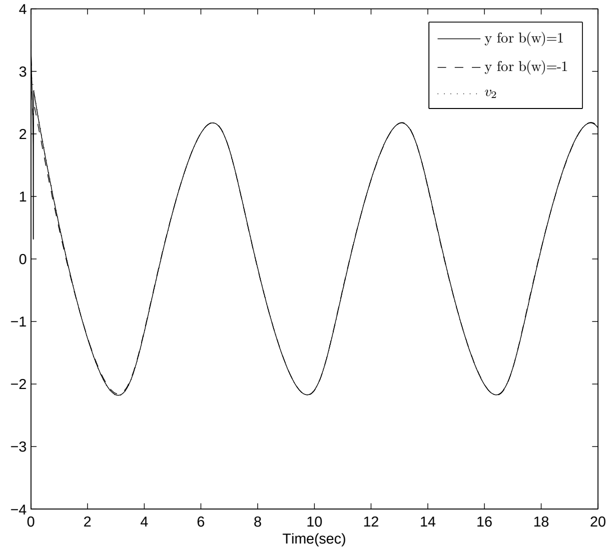

Fig. 2. Output of the system with $b(w) = 1$ and $b(w) = -1$.

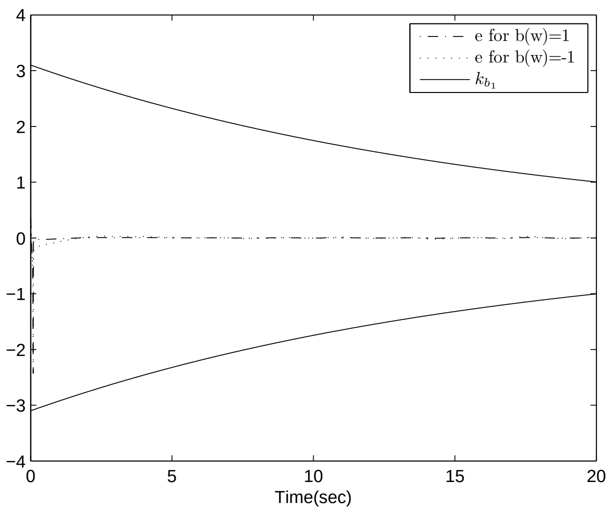

Fig. 3. Tracking error of the system with $b(w) = 1$ and $b(w) = -1$.

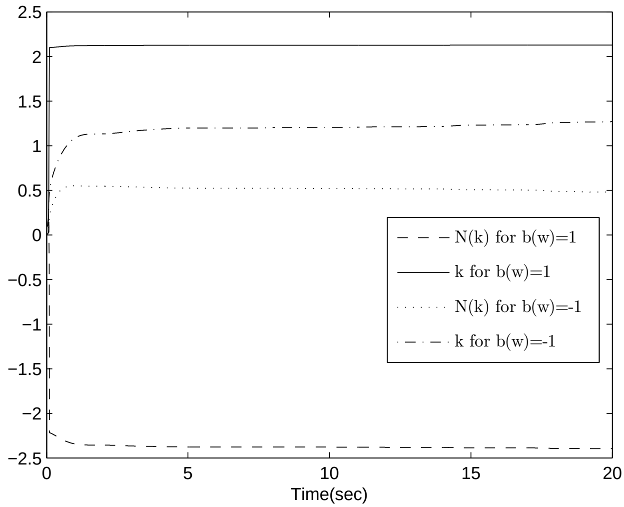

Fig. 4. Nussbaum gain with $b(w) = 1$ and $b(w) = -1$.

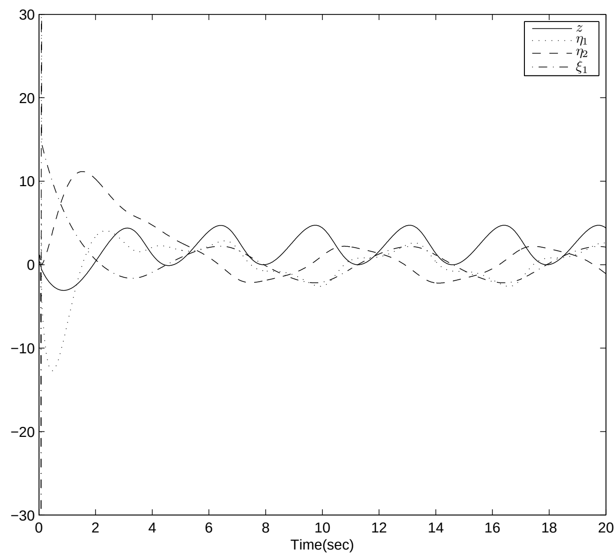

Fig. 5. States of the system with $b(w) = 1$.

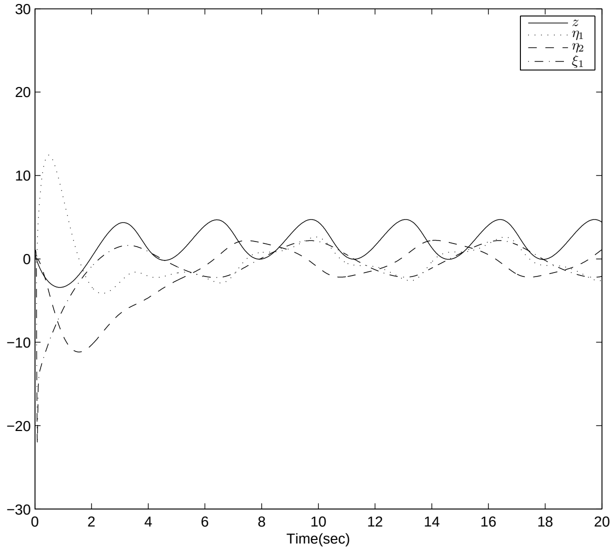

Fig. 6. States of the system with $b(w) = -1$.

6. CONCLUSION

In this paper, a feedback control design has been presented for the constrained robust adaptive stabilization problem for a class of lower triangular nonlinear systems without a prior knowledge of the control direction. Such a controlled problem arises from studying the constrained robust output regulation problem for a class of output feedback systems subject to a nonlinear exosystem and the unknown control direction. Thus an application of the main result leads to the solvability conditions of the output regulation problem.

ACKNOWLEDGEMENT

This work was supported by the National Natural Science Foundation of China under Grant No. 61374036 and partially supported by Guangdong Province Science and Technology Project (2012B091100106)

(Received May 21, 2013)

REFERENCES

[1] S. Čelikovský and J. Huang: Continuous feedback practical output regulation for a class of nonlinear systems having nonstabilizable linearization. In: Proc. IEEE Conference on Decision and Control 1999, Phoenix 1999.

[2] B. Castillo-Toledo, S. Čelikovský, and S. Digennaro: Generalized immersion and nonlinear robust output regulation problem. Kybernetika *40* (2004), 207–220.

[3] Z. Chen and J. Huang: Global robust servomechanism problem of lower triangular systems in the general case. Systems Control Lett. *52* (2004), 209–220.

[4] Z. Chen and J. Huang: Dissipativity, stabilization, and regulation of cascade-connected systems. IEEE Trans. Automat. Control *49* (2004), 635–650.

[5] Z. Chen and J. Huang: Robust output regulation with nonlinear exosystems. Automatica *41* (2005), 1447–1454.

[6] H. Chen, W. Sun, Z. Sun, and J. T. W. Yeow: Second order sliding mode control of a 2D torsional MEMS micromirror with sidewall electrodes. J. Micromech. Microengrg. *23* (2013), 1–9.

[7] J. Huang: Nonlinear Output Regulation: Theory and Applications. SIAM, Philadelphia 2004.

[8] J. Huang and Z. Chen: A general framework for tackling the output regulation problem. IEEE Trans. Automat. Control *49* (2004), 2203–2218.

[9] A. Isidori: Nonlinear Control Systems, Volume II. Springer-Verlag, New York 1999.

[10] Z. Jiang and I. Mareels: A small-gain control method for nonlinear cascaded systems with dynamic uncertainties. IEEE Trans. Automat. Control *42* (1997), 292–308.

[11] M. Krstić, I. Kanellakopoulos, and P. Kokotović: Nonlinear and Adaptive Control Design. Wiley, New York 1995.

[12] L. Liu and J. Huang: Global robust stabilization of cascade-connected systems with dynamic uncertainties without knowing the control direction. IEEE Trans. Automat. Control *51* (2006), 1693–1699.

[13] L. Liu and J. Huang: Global robust output regulation of output feedback systems with unknown high-frequency gain sign. IEEE Trans. Automat. Control *51* (2006), 625–631.

[14] R. Marino and P. Tomei: Nonlinear Control Design. Prentice-Hall International, 1995.

[15] Y. Ma, S. Islam, and Y. Pan: Electrostatic torsional micromirror with enhanced tilting angle using active control methods. IEEE Trans. Mechatronics *16* (2011), 635–650.

[16] R. D. Nussbaum: Some remaks on a conjecture in parameter adaptive control. Systems Control Lett. *3* (1983), 243–246.

[17] B. Rehák, S. Čelikovský, J. Ruiz, and J. Orozco-Mora: A comparison of two FEM-based methods for the solution of the nonlinear output regulation problem. Kybernetika *45* (2009), 427–444.

[18] E. D. Sontag and A. R. Teel: Changing supply functions in input/state stable systems. IEEE Trans. Automat. Control *40* (1995), 1476–1478.

[19] E. D. Sontag: Mathematical control theory. In Texts in Applied Mathematics: Vol. 6. Deterministic finite-dimensional systems (Second edition). Springer-Verlag, New York 1998.

[20] W. Sun, J. Huang, and Z. Sun: Global robust stabilization for a class of time-varying output feedback systems. Internat. J. Robotics and Automation *26* (2011), 93–99.

[21] W. Sun, J. T. W. Yeow, and Z. Sun: Robust adaptive control of a one degree of freedom electrostatic microelectromechanical systems model with output-error-constrained tracking. J. Control Theory Appl. *6* (2012), 111–119.

[22] W. Sun: Global robust output regulation of lower triangular systems with nonlinear exosystems and its application. J. Control Theory Appl. *11* (2013), 468–476.

[23] K. P. Tee, S. S. Ge, and E. H. Tay: Barrier Lyapunov Functions for the control of output-constrained nonlinear systems. Automatica *45* (2009), 918–927.

[24] D. Xu and J. Huang: Robust adaptive control of a class of nonlinear systems and its applications. IEEE Trans. Circuits and Systems I: Regular Papers *57* (2010), 691–702.

[25] X. Yang and J. Huang: New results on robust output regulation of nonlinear systems with a nonlinear exosystem. Internat. J. Robust and Nonlinear Control *22* (2012), 1703–1719.

[26] X. Ye and J. Jiang: Adaptive nonlinear design without a priori knowledge of control. IEEE Trans. Automat. Control *43* (1998), 1617–1621.

[27] X. Ye: Decentralized adaptive stabilization of large-scale nonlinear time-delay systems with unknown high-frequency-gain signs. IEEE Trans. Automat. Control *56* (2011), 1473–1478.

[28] B. Zhang and W. Lan: Improving transient performance for output regulation problem of linear systems with input saturation. Internat. J. Robust and Nonlinear Control *23* (2013), 1087–1098.

Jianglin Lan, College of Automation Science and Engineering, South China University of Technology, Wushan, Guangzhou 510640. P. R. China.
e-mail: lanjianglin@126.com

Weijie Sun, (Corresponding author) College of Automation Science and Engineering, South China University of Technology, Wushan, Guangzhou 510640. P. R. China.
e-mail: auwjsun@scut.edu.cn

Yunjian Peng, College of Automation Science and Engineering, South China University of Technology, Wushan, Guangzhou 510640. P. R. China.
e-mail: pengyj@scut.edu.cn