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A Hybrid Procedure for Network Multi-Criteria Systems*

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Abstract

Many of today's systems are characterized by a network structure and evaluation of alternatives is based on multiple criteria. Network systems contain both positive and negative feedbacks. In the paper, a hybrid procedure is proposed for operation in network environment. The procedure is based on a combination of DEMATEL, ANP, and PROMETHEE approaches. DEMATEL is a comprehensive method for building and analyzing a structural model involving causal relationships between complex factors. For the network seems to be very appropriate Analytic Network Process (ANP) approach. The ANP makes possible to deal systematically with all kinds of dependence and feedback in the system. PROMETHEE methods are methods for multi-criteria evaluation of alternatives, based on preference relations between alternatives, which may be expressed as a network. These methods have specific advantages in analyzing and evaluating network systems. The combination of these approaches gives a powerful instrument for analyzing network systems by multiple criteria.

Key words: network problems, multiple criteria, DEMATEL, ANP, PROMETHEE

2010 Mathematics Subject Classification: 90B10, 90B50

1 Introduction

The network economy is a term for today's global relationship among economic subjects characterized by massive connectivity. The central act of the new era is to connect everything to everything in deep web networks at many levels of

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mutually interdependent relations, where resources and activities are shared, markets are enlarged and costs of risk are reduced. Many of today's systems are characterized by a network structure and evaluation of alternatives is based on multiple criteria. Network systems contain both positive and negative feedbacks. Network economy drives and is driven by dramatic acceleration in technological innovation, in information and communication technologies especially. New technologies provide a permanent feedback that enables activity modifications and quick responses and therefore fundamentally change business models. The analysis of possible effects of network economy is an opportune topic for challenging scientific research (Fiala, 2007).

Applications of the network problems are found in transportation, telecommunications, network reliability settings, finance, knowledge and other applications. The unifying concept of global networks with associated methodologies allows exploring the interactions among such networks as transportation networks, telecommunication networks, as well as financial networks. The basic problems and their solutions can be combined for more complex situations with multiple decision-makers and multiple criteria. The main aim of the paper is to propose a methodological framework for analyzing network economy. Network systems are characterized by elements and relations. Elements of network systems may represent agents, criteria, resources, alternatives, prices, channels, etc. Relations can capture influence linkages between elements. Network systems can be represented by a graph, where elements of the system are represented by nodes and relations by edges of the graph.

In the paper, a hybrid procedure is proposed for operation in network environment. Selection of methods was influenced by graphical representation of the analyzed network systems. The procedure is based on a combination of DEMATEL, ANP, and PROMETHEE approaches. The DEMATEL (Decision Making Trial and Evaluation Laboratory) method (Gabus and Fontela, 1972), originated from the Geneva Research Centre of the Battelle Memorial Institute, is especially pragmatic to visualize the structure of complicated causal relationships. DEMATEL is a comprehensive method for building and analyzing a structural model involving causal relationships between complex factors. Analytic Hierarchy Process (AHP), developed by T. Saaty, (Saaty, 1996), is a very popular method for setting priorities in hierarchical systems. A variety of feedback processes create complex system behavior. Analytic Network Process (ANP) approach seems to be very appropriate for the network (Saaty, 2001). The ANP makes possible to deal systematically with all kinds of dependence and feedback in the system. PROMETHEE (Preference Ranking Organization METHod for Enrichment Evaluations) methods are methods for multicriteria evaluation of alternatives, based on preference relations. The methods PROMETHEE I (partial ranking) and PROMETHEE II (complete ranking) were developed by J. P. Brans and then further developed as a family of methods. A considerable number of successful applications has been treated by the PROMETHEE methodology in various fields. The success of the methodology is basically due to its mathematical properties and to its particular friendliness of use.

Selected methods are shortly summarized in the paper with regard to their use for analysis of network systems. These methods have specific advantages in analyzing and evaluating network multi-criteria systems. The combination of these approaches gives a powerful instrument for analyzing network systems by multiple criteria.

2 The DEMATEL method

The DEMATEL method can be summarized in the following steps:

Step 1. Find the initial direct relation matrix.
Suppose we have m experts in this study and n elements to consider. Each expert is asked to indicate the degree to which he believes an element i affects an element j. These pairwise comparisons between any two elements are denoted by a_{ij} and are given an integer score ranging from 0, 1, 2, 3, and 4, representing:
0 no influence,
1 low influence,
2 medium influence,
3 high influence,
4 very high influence.
The scores by each expert will give us a (n, n) non-negative answer matrix $\mathbf{X}^k = [x_{ij}^k]$, with $1 \leq k \leq m$.

The diagonal elements of each answer matrix $\mathbf{X}^k$ are all set to zero. We can then compute the (n, n) average matrix $\mathbf{A}$ for all expert opinions by averaging the m experts' scores as follows:

$$a_{ij} = \frac{1}{m} \sum_{k=1}^{m} x_{ij}^k.$$

The average matrix $\mathbf{A} = [a_{ij}]$ is called the initial direct relation matrix. Matrix $\mathbf{A}$ shows the initial direct effects that an element exerts on and receives from other elements. Furthermore, we can map out the causal effect between each pair of elements in a system by drawing an influence map. DEMATEL can convert the structural relations among the elements of a system into an intelligible map of the system.

Step 2. Calculate the normalized initial direct-relation matrix.
The normalized initial direct-relation matrix $\mathbf{D}$ is obtained by normalizing the initial direct-relation matrix $\mathbf{A}$ in the following way: Let

$$s = \max \left(\max_{1 \leq i \leq n} \sum_{j=1}^{n} a_{ij}, \max_{1 \leq j \leq n} \sum_{i=1}^{n} a_{ij} \right),$$

then

$$\mathbf{D} = \frac{1}{s} \mathbf{A}.$$

Since the sum of each row j of matrix $\mathbf{A}$ represents the total direct effects that element i gives to the other elements, $\max_{1\le i\le n}\sum_{j=1}^{n} a_{ij}$ represents the total direct effects of the element with the most direct effects on others. Likewise, since the sum of each column i of matrix $\mathbf{A}$ represents the total direct effects received by element i, $\max_{1\le j\le n}\sum_{i=1}^{n} a_{ij}$ represents the total direct effects received of the element that receives the most direct effects from others. The positive scalar s takes the greater of the two as the upper bound, and the matrix $\mathbf{D}$ is obtained by dividing each element of $\mathbf{A}$ by the scalar s. Note that each element d_{ij} of matrix $\mathbf{D}$ is between zero and one.

Step 3. Compute the total relation matrix.
A continuous decrease of the indirect effects of problems along the powers of matrix $\mathbf{D}$ guarantees convergent solutions to the matrix inversion similar to an absorbing Markov chain matrix. Note that $\lim_{k\to\infty}\mathbf{D}^k = \mathbf{0}$ and

$$\lim_{k\to\infty}\left(\mathbf{I}+\mathbf{D}+\mathbf{D}^2+\cdots+\mathbf{D}^k\right) = (\mathbf{I}-\mathbf{D})^{-1},$$

where $\mathbf{0}$ is the (n,n) null matrix and $\mathbf{I}$ is the (n,n) identity matrix. The total relation matrix $\mathbf{T}$ is an (n,n) matrix $\mathbf{T}=[t_{ij}]$, $i,j=1,2,\ldots,n$, and is defined as follow:

$$\mathbf{T}_k = \mathbf{D}+\mathbf{D}^2+\cdots+\mathbf{D}^k = \mathbf{D}\left(\mathbf{I}+\mathbf{D}+\mathbf{D}^2+\cdots+\mathbf{D}^{k-1}\right),$$
$$\mathbf{T} = \lim_{k\to\infty}\mathbf{T}_k = \mathbf{D}(\mathbf{I}-\mathbf{D})^{-1}.$$

Vectors $\mathbf{r}$ and $\mathbf{c}$ are defined representing the sum of rows and sum of columns of the total relation matrix $\mathbf{T}$ as follows:

$$\mathbf{r} = (r_i)$$

where r_i be the sum of i-th row in matrix $\mathbf{T}$. Then r_i shows the total effects, both direct and indirect, given by element i to the other elements, and

$$\mathbf{c} = (c_j)$$

where c_j denotes the sum of j-th column in matrix $\mathbf{T}$. Then c_j shows the total effects, both direct and indirect, received by element j from the other elements.

Thus when $j=i$, the sum (r_i+c_i) gives an index representing the total effects both given and received by element i. In other words, (r_i+c_i) shows the degree of importance that element i plays in the system. In addition, the difference (r_i-c_i) shows the net effect that element i contributes to the system. When (r_i-c_i) is positive, element i is a net causer, and when (r_i-c_i) is negative, element i is a net receiver (Tzeng et al., 2007).

Step 4. Set a threshold value and obtain the impact-relations-map.
In order to explain the structural relation among the elements while keeping the complexity of the system to a manageable level, it is necessary to set a threshold value p to filter out some negligible effects in matrix $\mathbf{T}$. While each element of matrix $\mathbf{T}$ provides information on how one element affects another,

the decision-maker must set a threshold value in order to reduce the complexity of the structural relation model implicit in matrix **T**. Only some elements, whose effect in matrix **T** is greater than the threshold value, should be chosen and shown in an impact-relations-map (Tzeng et al., 2007).

3 The ANP method

The Analytic Network Process (ANP) is the method that makes it possible to deal systematically with all kinds of dependence and feedback. The well-known AHP theory is a special case of the Analytic Network Process that can be very useful for incorporating linkages.

The structure of the ANP model is described by clusters of elements connected by their dependence on one another. A cluster groups elements that share a set of attributes. At least one element in each of these clusters is connected to some element in another cluster. These connections indicate the flow of influence between the elements (see Fig. 1).

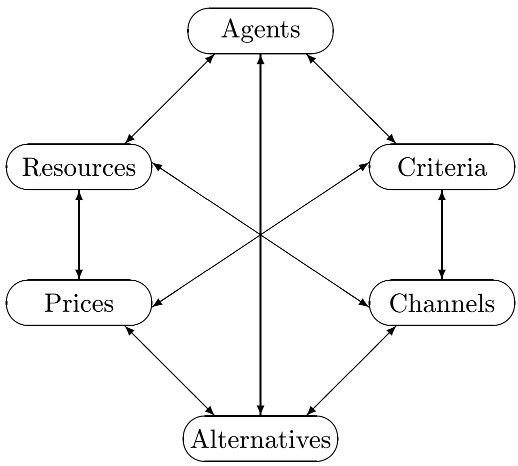

Fig. 1. Clusters and connections

Setting priorities by ANP can be summarized in the following steps:

Step 1. Supermatrix.
A supermatrix is a matrix of all elements by all elements. Paired comparisons are needed for all the connections in the model. The ANP derives ratio scale priorities by making paired comparisons of elements by using a 1 to 9 scale of absolute numbers. The weights from the paired comparisons are placed in the appropriate column of the supermatrix. The sum of each column corresponds to the number of comparison sets.

Step 2. Weighted supermatrix.
The weights in the column corresponding to the cluster are multiplied by the weight of the cluster. Each column of the weighted supermatrix sums to one and the matrix is column stochastic.

Step 3. Limited supermatrix.
Powers of weighted supermatrix can stabilize after some iterations to limited supermatrix. The columns of each block of the matrix are identical and we can read off the overall priority.

4 The PROMETHEE method

The PROMETHEE II method can be summarized in the following steps:

Step 1. Formulation of a criteria matrix and weights.
The multicriteria evaluation problem is defined by set of alternatives $A = \{a_1, a_2, \ldots, a_m\}$ and set of evaluation criteria $F = \{f_1, f_2, \ldots, f_k\}$. The evaluations of alternatives by criteria can be expressed in a criteria matrix. The importance of criteria can be expressed by a weight vector.

Step 2. Calculation of multicriteria preference indices.
PROMETHEE methods use so-called preference functions defined for all pairs of alternatives for each criterion

$$P\colon A \times A \to [0, 1].$$

Preference function values depend on the difference d evaluation of alternatives according to the criterion f

$$P(a_i, a_j) = P(f(a_i) - f(a_j)) = P(d).$$

To express preferences in both directions it is possible to define a function

$$\begin{aligned} H(d) &= P(a_i, a_j), \quad d \geq 0, \\ H(d) &= P(a_j, a_i), \quad d \leq 0. \end{aligned}$$

Preference function values are calculated for each criterion ($h = 1, 2, \ldots, k$) and all ordered pairs of alternatives ($i, j = 1, 2, \ldots, m$) $P_h(a_i, a_j)$. The so-called multicriteria preference index measures preference of the alternative a_i before the alternative a_j in terms of all the criteria

$$\pi(a_i, a_j) = \sum_{h=1}^{k} w_h P_h(a_i, a_j).$$

Step 3. Calculation of outranking flows.
The positive outranking flow is of the form:

$$F^+(a_i) = \sum_{a_j \in A} \pi(a_i, a_j).$$

The negative outranking flow is of the form:

$$F^-(a_i) = \sum_{a_j \in A} \pi(a_j, a_i).$$

The net outranking flow is applied and is of the form:

$$F(a_i) = F^+(a_i) - F^-(a_i).$$

The higher net outranking flow $F(a_i)$ follows the better alternative.

5 Combination of the methods

The presented methods have some specific advantages in analyzing and evaluating network multi-criteria systems:

DEMATEL

- Expert evaluations of relations in network system give initial compromise view of the structure of the network system.
- Introduction of a threshold value gives a filter of some parts in the model and brings reduction of the model size.

ANP

- Introduction of clusters specifies the structure of the network system.
- All kinds of dependence and feedback are included in the model.
- More precise evaluations of relations are provided.

PROMETHEE

- Introduction of preference functions provides a gentler expression of preferences among alternatives.
- Calculation of outranking flows brings the evaluation of alternatives in terms of relations with all other alternatives.

These advantages are used in the design of a hybrid procedure. The structure of the hybrid procedure for network multi-criteria systems can be as follows:

Step 1. Using of DEMATEL – to clarify initial relations of elements in the network system.

Step 2. Using step 1 of ANP – to form a supermatrix by pairwise comparisons.

Step 3. Using step 4 of DEMATEL and step 2 of ANP – the weighted supermatrix is obtained by multiplying the total-influence matrix (DEMATEL) with supermatrix (ANP) method.

Step 4. Using step 3 of ANP – to get the limited supermatrix with weighs.

Step 5. Using of PROMETHEE – evaluation of flows between alternatives with weights from the limited supermatrix (ANP).

6 Conclusions

Network systems are very popular in theory and practice because many of today's systems are characterized by a network structure. Preference elicitation is the key feature for network systems and it is a complex problem. The proposed hybrid procedure is based on a combination of DEMATEL, ANP, and PROMETHEE approaches because these methods complement each other in multi-criteria analysis of network systems. The combination of such approaches can give more complex views on network systems. The presented approach is very flexible. Modifications of the approach are possible according to other structures of combination, other methods, a dynamization of the approach, fuzzy evaluations, and others. Such hybrid approaches can be used in many applications (e.g. Ou Yang et al., 2008, Tzeng et al., 2007).

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