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CONSENSUS AND TRAJECTORY TRACKING OF SISO LINEAR MULTI-AGENT SYSTEMS UNDER SWITCHING COMMUNICATION TOPOLOGIES AND FORMATION CHANGES

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The simultaneous problem of consensus and trajectory tracking of linear multi-agent systems is considered in this paper, where the dynamics of each agent is represented by a single-input single-output linear system. In order to solve this problem, a distributed control strategy is proposed in this work, where the trajectory and the formation of the agents are achieved asymptotically even in the presence of switching communication topologies and smooth formation changes, and ensuring the closed-loop stability of the multi-agent system. Moreover, the structure and dimension of the representation of the agent dynamics are not restricted to be the same, as usually assumed in the literature. A simulation example is provided in order to illustrate the main results.

Keywords: consensus, multi-agent systems, tracking, switching systems

Classification: 68T42, 93C30

1. INTRODUCTION

A multi-agent system (*MAS*) can be described as a group of dynamical systems which interact with each other by means of the state or the outputs of their neighbors, in order to achieve a common goal. Multi-agent systems typically execute common tasks with distributed control actions and local information. In recent years, multi-agent systems have attracted the attention of the scientific community for their capacity to perform cooperative and coordinated tasks under an individual control paradigm, and their multiple applications in the fields of autonomous vehicles, transportation systems, multi-processor computing and power systems, among others. Many topics such as consensus, formation, trajectory tracking and flocking have been widely studied, and other approaches as self organization, goal achievement with robustness to component failures and network evolution are becoming the objective of current research.

In this paper, we are interested in the output consensus and trajectory tracking problem of multi-agent systems. Output consensus deals with the problem of the agreement of the outputs of the agents composing a *MAS*, while output regulation deals with the problem of trajectory tracking of a signal generated by an exosystem. Addressing output consensus and output regulation simultaneously allows the interaction of different systems in a coordinated trajectory tracking using restricted information.

Regarding reported contributions in the literature related to consensus and/or trajectory tracking of *MAS*, the following results can be mentioned. A theoretical framework for the analysis of consensus algorithms in multi-agent systems is presented in [16], where an overview of methods for convergence and analysis of consensus algorithms is presented. Consensus algorithms, both for linear and nonlinear systems, are introduced in [17] and [14], and a Lyapunov function is used to analyze the convergence of the consensus algorithms. A general formulation of multi-agent formation is presented in [3], where the authors also address some issues such as how the information flow topology affects the system coordination stability and performance.

In [10], necessary and sufficient conditions for an appropriate distributed linear stabilizing feedback are established, and it is shown the relationship between the formation convergence rate and the Laplacian matrix (directed) eigenvalues. Optimal algorithms for the consensus of a MAS are proposed in [20] and [2], but trajectory tracking is not addressed.

Necessary and sufficient conditions to achieve agent consensus using distributed controllers, and the consensus of multi-agent systems with switching interaction topologies are studied in [22].

A feedback control law to achieve predefined formations for a MAS is proposed in [26], where virtual agents are used as leaders to ensure that the group of agents follows a desired trajectory.

In [18], the problem of MAS with switching communication topologies and communication time delays is analyzed. The stability analysis considering the MAS as a switched system is carried out using a common Lyapunov function. However, the analysis is focused only on single integrator systems.

A leader-follower consensus problem of multi-agents with a time-invariant communication topology and an observer-type consensus protocol based on output measurements is proposed in [11]. In [4, 25] and [15] a controller design to achieve formation and trajectory tracking is developed. These references consider the case of switching topology and input delays, however, the agent dynamics are restricted to singe integrators.

In [19], an overview of necessary and sufficient conditions for reaching consensus with fixed and switching communication topologies is presented. Also, control laws for consensus and trajectory tracking for single and double integrator dynamics are included.

Distributed control schemes for robust output regulation of a networked linear system with uncertainties are proposed in [24] and [23]. Even when the uncertainties in the dynamics can be considered as non homogeneous linear systems, the control scheme is limited to systems with the same dimension and the communication topology as well as the formation must be static.

An event-triggered technique for the control law of MAS is used in [6] in order to reduce the computational cost. Also, a tracking control law for leader-follower MAS with and without communication delays is designed and input-to-state stability is analyzed. However, the control law is restricted to double integrator dynamics.

In [21] and [1] the design of distributed control laws to achieve consensus and trajectory tracking for *MAS* is presented. Switching topologies and different dynamics of each agent are considered, however, the description of the systems must be of the same dimension and structure. Also, a minimum dwell time is required to ensure the convergence, which is not always possible to obtain.

Despite the previously mentioned contributions, most of the existing results related to *MAS* consensus are restricted to agents with relatively simple dynamics, i. e. single and double integrator, or a very restricted switching topology that depends on the speed of the system dynamics. In works where complex agents dynamics are considered, the problem of trajectory tracking is not addressed at all.

The present work focuses on the design of a distributed control law for *MAS*, where each agent dynamics is represented as a time-invariant single-input single-output (*SISO*) linear system. The proposed control law ensures simultaneously asymptotic tracking and group formation even if the communication topology is switching. This overcomes previous limitations in existing results, where the switching speed is limited by the system dynamics.

Another important advantage of our results with respect to previous ones, is that we consider the consensus and trajectory tracking of *MAS*, where the agents can have different dynamics and even different state space dimension.

This work is organized as follows. Section 2 presents the notation and some preliminaries about switched linear systems, consensus and stability. In Section 3, the problem statement and the control law design for the consensus and trajectory tracking are presented. In Section 4, a simulation example is shown in order to illustrate the application of the proposed control strategy. Finally, in Section 5 the conclusions and future work are presented.

2. NOTATION AND PRELIMINARIES

This section presents notation and preliminaries about communication topologies, consensus, switched linear systems and stability.

2.1. Communication topology

Formation of a group of N mobile agents is commonly described by a graph, $G = (\vartheta, \xi, \mathcal{A}(t))$, where $\vartheta = \{1, 2, \dots, N\}$ is a set of nodes, $\xi \in \vartheta \times \vartheta$ is a set of edges connecting nodes (self edges are not allowed) and $\mathcal{A}(t)$, for simplicity $\mathcal{A}$, is the graph adjacency matrix containing positive weights, where $\mathcal{A} = [\alpha_{i,j}] \in \mathbb{R}^{N \times N}$. An edge $(\nu_i, \nu_j) \in \xi$ means that node ν_j can get information from node ν_i. If an edge (ν_i, ν_j) is contained in ξ, then $\alpha_{j,i} > 0$. We suppose in this work a bidirectional communication, i. e. that $(\nu_i, \nu_j) \in \xi \Leftrightarrow (\nu_j, \nu_i) \in \xi$, except for the reference node ν_0, for which $(\nu_0, \nu_i) \in \xi$ and $(\nu_i, \nu_0) \notin \xi$, $\forall i$. The set of neighbors of node i at time t will be denoted by $\mathcal{N}_i(t)$, i. e. $\mathcal{N}_i(t) = \{\nu_j : (\nu_j, \nu_i) \in \xi,\ j = 1, \dots, N\}$.

The topology switches over time as the agents evolve or the communication fails, therefore the adjacency matrix $\mathcal{A}$ changes according to these situations.

A *spanning tree*, assuming G strongly connected, is a subgraph of G where some arcs of G are removed in such a way that every node can get information from only one node, except for the one called the root (in our approach the root is the node ν_0). The root node does not receive information from any node. A graph has a spanning tree if it is strongly connected, i.e. if there is a direct path from every node to every other node. In the present work, every formation is represented by a strongly connected graph containing a spanning tree. This assumption is fulfilled since it will be considered that every node is directly connected to the reference node (the virtual agent), $\nu_0 \in \mathcal{N}_i(t), i = 1, \ldots, N, \forall t$.

The entries of the adjacency matrix $\mathcal{A}$ are defined as $\alpha_{ii} = 0$, $\alpha_{ij} = 1$ if $(\nu_i, \nu_j) \in \xi$ and 0 otherwise. Entries of the Laplacian matrix $\mathcal{L}$ are defined as $\ell_{ii} = -\Sigma_{j=1}^{N} \alpha_{ij}$ and $\ell_{ij} = \alpha_{ij}$.

Agent dynamics S_i is considered to be represented by a time-invariant *SISO* linear system given by

$$\begin{aligned} \dot{x}_i(t) &= A_i x_i(t) + B_i u_i(t), \\ y_i(t) &= C_i x_i(t), \qquad\qquad i = 1, \ldots, N, \end{aligned} \tag{1}$$

where $x_i(t) \in \mathbb{R}^{n_i}$, $u_i(t) \in \mathbb{R}$, $y_i(t) \in \mathbb{R}$, are respectively the state, input and output variables, and A_i, B_i and C_i are constant matrices and vectors of appropriate dimensions. In this work, we restrict ourselves to consider systems of the form (1) having the same relative degree ρ, i.e. the difference between the degrees of the denominator and numerator polynomials of the transfer function of the systems are equal to ρ. This implies that the input u_i appears explicitly for the first time at the ρth derivative of y_i for all the systems. The reason for this restriction will become evident while deriving the control law to be applied. Also, we suppose that the systems are stabilizable, i.e. if there exist uncontrollable dynamics, then they are stable. Additionally, in order to ensure the stability of the closed-loop system under the proposed control law, for simplicity the dimension of the controllability subspace of each agent dynamics given by $rank \begin{bmatrix} B_i & A_i B_i & \cdots & A_i^{n_i - 1} B_i \end{bmatrix}$ is supposed to be equal to the relative degree ρ. If this assumption does not hold, then the stability of the closed-loop system would depend on the zeros of the system (for instance closed-loop stability would also be guaranteed for minimum-phase zeros systems).

The communication topology of the *MAS* switches as the system evolves, thus the *MAS* becomes a Switched Linear System (*SLS*).

2.2. Switched linear systems

A *SLS*, denoted as $\langle F, \sigma \rangle$, is a switched linear dynamical system where $F = \{S_1, \ldots, S_k\}$ is a collection of time-invariant linear systems

$$S_\sigma : \begin{cases} \dot{\hat{x}}(t) = A_\sigma \hat{x}(t) + B_\sigma \hat{u}(t), \quad \hat{x}(t_0) = \hat{x}_0 \\ \hat{y}(t) = C_\sigma \hat{x}(t) \end{cases} \tag{2}$$

and $\sigma : [t_0, \infty) \to \{1, \ldots, k\}$ is a switching signal that determines the evolving linear system (*LS*), $\hat{x}(t) \in \mathbb{R}^n$, $\hat{u}(t) \in \mathbb{R}^m$, $\hat{y} \in \mathbb{R}^p$, are respectively the state, input and output variables, and A_σ, B_σ and C_σ are constant matrices of appropriate dimensions. This work supposes an observable *SLS* [5], because the switching signal, represented by the communication topology of the *MAS*, is known. With the knowledge of which *LS* is currently evolving, it is possible to design a control law for each *LS*, or in this case for each communication topology.

2.3. Stability of *SLS*

The following definitions and results about stability of *SLS* will be used in the next section to show the closed-loop stability of the *MAS* under the proposed control law that achieves consensus and trajectory tracking.

A *SLS* is said to be *uniformly asymptotically stable* if there exists a positive constant δ and a $\mathcal{KL}$ class [9] function γ such that for any switching sequence the solution of the autonomous *SLS* with $|\hat{x}(0)| < \delta$ satisfies the inequality

$$|\hat{x}(t)| \leq \gamma(|\hat{x}(0)|, t), \quad \forall t > 0. \tag{3}$$

If the function γ is of the form $\gamma(r, s) = cre^{-\lambda s}$ for some $c, \lambda > 0$, then we have that

$$|\hat{x}(t)| \leq c|\hat{x}(0)|e^{-\lambda t}, \quad \forall t > 0 \tag{4}$$

and then the autonomous *SLS* is said to be *uniformly exponentially stable*.

If (3) and (4) are valid for all switching signals and any initial condition, then the *SLS* is called *global uniform asymptotically stable* (*GUAS*) and *global uniform exponentially stable* (*GUES*), respectively.

The following result, presented in [12], establishes the conditions for a *SLS* to exhibit *GUES*.

Theorem 2.1. (Liberzon [12]) The *SLS* (2) is *GUES* if and only if it is locally attractive for every switching signal.

The equivalence between local attractivity and global exponential stability is presented in [12], where the problem of stability in *SLS* is solved obtaining a common *Lyapunov function* P, such that

$$A_\sigma^T P + PA_\sigma < 0 \quad \forall\, \sigma. \tag{5}$$

If the linear matrix inequality (5) holds, we have a *quadratic common Lyapunov function* and the *SLS* is *GUES*.

3. PROBLEM STATEMENT AND MAIN RESULTS

In this section, a control scheme to achieve consensus and trajectory tracking of a *MAS* is proposed, where each agent has a *SISO* linear system representation.

For a *MAS* the consensus is achieved when all states of the agents are equal, i. e. $\lim_{t\to\infty} |x_i - x_j| = 0, \forall i, j = 1, 2, \ldots, N$. This problem is particularly challenging because communication usually has limited range and can fail easily. Additionally, considering also trajectory tracking, all the agents of the *MAS* should track a predefined signal reference.

In [13] the consensus is achieved for the output's agents only, i. e.

$$\lim_{t\to\infty} |y_i - y_j| = 0, \quad \forall\, i, j = 1, 2, \ldots, N. \tag{6}$$

The advantage of considering only the output of the agents is that the amount of information transmitted between elements of the net is reduced, however in [13] the dynamics of the agents are restricted to be the same. Now, we will also consider the consensus of the output in order to reach consensus and trajectory tracking among agents with non homogeneous dynamics.

Let F be a finite family of *SISO* linear systems (1), $F = \{S_1, \ldots, S_N\}$, in which every linear system fulfills the constraints of relative degree (ρ) mentioned above, the uncontrollable dynamics being stable. Each S_i is considered to be observable to ensure that any state feedback can be applied, and let S_0 be a virtual agent which provides the reference signal, and consider a given communication topology. Then, the consensus error is defined as

$$e_i = \sum_{j=0}^{N} \alpha_{i,j} \left[(y_i - \Delta_i) - (y_j - \Delta_j) \right], \qquad i = 1, \ldots, N, \tag{7}$$

where y_0 is a reference signal that must be followed by the output y_i for each system of the set F. The adjacency matrix $\mathcal{A}$ has a switching time $\tau > 0$. The term Δ_i, with $i = 0, 1, \ldots, N$ and $\Delta_0 = 0$, is the required separation, not necessarily constant, between y_i and y_0, both y_0 and Δ_i are functions $C^{\rho-1}$ class. With this defined error (7), then the consensus problem consists on designing a control law u_i such that $\lim_{t\to\infty} e_i = 0\ \forall i$. It can be seen that the consensus and trajectory tracking is achieved when this condition holds because the error e_i includes both.

Note that the error (7) depends on the outputs of the agents (y_i), the reference (y_0), the formation (Δ_i) and the communication topology ($\alpha_{i,j}$). In this case, we consider a switching communication topology so that the error dynamics can be expressed as a *SLS* system as it will be shown later.

As in previous works, it is important to note that the element S_0 represents the virtual agent and that its output is always known for each agent, i. e. that $\alpha_{i,0} = 1$, $\forall t$ which ensures a *spanning tree* for G.

If we consider that each agent can transmit its output and error (7) to their neighbors, it is enough to have a *spanning tree* in the net to calculate the feedback control. But, in order to obtain the error of the neighbors of an agent using only the output of the agents, we propose the following communication topology

$$\nu_i \in \mathcal{N}_j \text{ and } \nu_j \in \mathcal{N}_k \Rightarrow \nu_k \in \mathcal{N}_i. \tag{8}$$

The previous topology means that if the agent i is connected with the agent j and the agent j is connected with the agent k then the agent i and the agent k are connected too, where all communications are bidirectional.

A control input u_i can be calculated by the ith agent without knowing the inputs of the other agents, i. e. the consensus and tracking control law is distributed. This makes the *MAS* more flexible and provides many advantages in computational resources for processing the control law and transmission of information.

In this work, the control law is designed to achieve the consensus of a *MAS* with respect to the output of each agent, which reduces the quantity of information to be transmitted by each agent, since it is enough to know the output of the neighboring agents to compute the control action, i. e. it is not necessary to know the whole state of the other agents. This modification allows achieving the consensus among agents of different state space dimension, since only the output is considered.

The control scheme to achieve the consensus and trajectory tracking of a *MAS* is presented in the next result.

Theorem 3.1. Let F be a *MAS* system with the characteristics mentioned before and the communication topology given by (8). Then, a distributed control scheme where

$$u_i = \frac{1}{\beta_{i,\rho}}\left[-\Gamma_{i,\rho}x_i + \overset{(\rho)}{\Delta}_i + \overset{(\rho)}{y}_0 - k_p\Upsilon_i - \sum_{j=1}^{N} k_{i,j}e_j\right] \tag{9}$$

is the input to be applied to the i-th agent, achieves consensus and trajectory tracking with error dynamics *GUES* for the *MAS*, where $\beta_{i,\rho} = C_iA_i^{\rho-1}B_i$, $\Gamma_{i,m} = C_iA_i^m$, $\Upsilon_i = [\Upsilon_{i,1} \ \dots \ \Upsilon_{i,\rho-1}]^T$, $\Upsilon_{i,m} = \Gamma_{i,m}x_i - \Delta_i^{(m)} - y_0^{(m)}$, $m \in [1, ..., \rho]$, $k_{i,j}$ is the j-th entry of the vector

$$k_i = \frac{q}{-\ell_{i,i}+1}\left[\begin{array}{ccccccc} \alpha_{i,1} & \cdots & \alpha_{i,i-1} & 2 & \alpha_{i,i+1} & \cdots & \alpha_{i,N} \end{array}\right],$$

and the scalar q and the vector k_p of dimension $\rho - 1$ are the entries belonging to the vector $[q \ k_p]$ which determines the characteristic polynomial of the Hurwitz matrix associated to the error dynamics.

Proof. The defined consensus error (7) can be written as

$$e_i = \sum_{j=0}^{N} \alpha_{i,j}\left(y_i - \Delta_i - y_0\right) - \sum_{j=1}^{N} \alpha_{i,j}\left(y_j - \Delta_j - y_0\right), \tag{10}$$

whose derivative is given by

$$\dot{e}_i = \sum_{j=0}^{N} \alpha_{i,j}\left(C_iA_ix_i + C_iB_iu_i - \dot{\Delta}_i - \dot{y}_0\right) - \sum_{j=1}^{N} \alpha_{i,j}\left(C_jA_jx_j + C_jB_ju_j - \dot{\Delta}_j - \dot{y}_0\right). \tag{11}$$

Rewriting the first ρ derivatives of (11) we obtain

$$\begin{aligned} \dot{e}_i &= \sum_{j=0}^{N} \alpha_{i,j}\left(\Gamma_{i,1}x_i + \beta_{i,1}u_i - \dot{\Delta}_i - \dot{y}_0\right) - \sum_{j=1}^{N} \alpha_{i,j}\left(\Gamma_{j,1}x_j + \beta_{j,1}u_j - \dot{\Delta}_j - \dot{y}_0\right), \\ &\vdots \\ \overset{(\rho)}{e}_i &= \sum_{j=0}^{N} \alpha_{i,j}\left(\Gamma_{i,\rho}x_i + \beta_{i,\rho}u_i - \overset{(\rho)}{\Delta}_i - \overset{(\rho)}{y}_0\right) - \sum_{j=1}^{N} \alpha_{i,j}\left(\Gamma_{j,\rho}x_j + \beta_{j,\rho}u_j - \overset{(\rho)}{\Delta}_j - \overset{(\rho)}{y}_0\right). \end{aligned} \tag{12}$$

Now, we define $\zeta_{i,1} = e_1$ and introducing a change of coordinates

$$\begin{aligned} \dot{\zeta}_{i,1} &= \zeta_{i,2} \\ &\vdots \\ \dot{\zeta}_{i,\rho-1} &= \zeta_{i,\rho} \end{aligned}$$

then, the equation (12) is represented as

$$\begin{aligned}\dot{\zeta}_{i,m} = \zeta_{i,m+1} = \sum_{j=0}^{N} \alpha_{i,j}\left(\Gamma_{i,m}x_i + \beta_{i,m}u_i - \overset{(m)}{\Delta}_i - \overset{(m)}{y_0}\right)\\ - \sum_{j=1}^{N} \alpha_{i,j}\left(\Gamma_{j,m}x_j + \beta_{j,m}u_j - \overset{(m)}{\Delta}_j - \overset{(m)}{y_0}\right).\end{aligned} \tag{13}$$

Since we are supposing that all agents representations have the same relative degree ρ, then it follows that $C_iA_i^{m-1}B_i = 0$, $m = 0, 1, \ldots, \rho-1$, and $C_iA_i^{\rho-1}B_i \neq 0$, [8]. Defining

$$\Upsilon_{i,m} = \Gamma_{i,m}x_i - \overset{(m)}{\Delta}_i - \overset{(m)}{y_0}, \quad m = 1, 2, \ldots, \rho-1 \tag{14}$$

then (13) becomes

$$\zeta_{i,m+1} = \sum_{j=0}^{N} \alpha_{i,j}\left(\Upsilon_{i,m}\right) - \sum_{j=1}^{N} \alpha_{i,j}\left(\Upsilon_{j,m}\right). \tag{15}$$

Thus, with the derivate of $\zeta_{i,\rho}$ we obtain the following equation

$$\dot{\zeta}_{i,\rho} = \sum_{j=0}^{N} \alpha_{i,j}\left(\Gamma_{i,\rho}x_i + \beta_{i,\rho}u_i - \overset{(\rho)}{\Delta}_i - \overset{(\rho)}{y_0}\right) - \sum_{j=1}^{N} \alpha_{i,j}\left(\Gamma_{j,\rho}x_j + \beta_{j,\rho}u_j - \overset{(\rho)}{\Delta}_j - \overset{(\rho)}{y_0}\right). \tag{16}$$

In (16), the coefficients $\beta_{\bullet,\rho}$ are nonzero since all the systems of F are supposed to have the same relative degree ρ. Now, we can apply the input (9) to (16), and notice that the input u_i requires all the states that appear in Υ_i for the feedback. This is fulfilled because all the S_i are supposed to be observable and all necessary states can be determined. Considering that $\sum_{j\in\mathcal{N}_i} k_{i,j}e_j = k_ie$, where $e = \begin{bmatrix} \zeta_{1,1} & \zeta_{2,1} & \cdots & \zeta_{N,1} \end{bmatrix}^T$ and $k_i = \begin{bmatrix} k_{i,1} & k_{i,2} & \cdots & k_{i,N} \end{bmatrix}$, the following equation is obtained

$$\dot{\zeta}_{i,\rho} = \sum_{j=0}^{N} \alpha_{i,j}\left(k_p\Upsilon_i\right) - \sum_{j=1}^{N} \alpha_{i,j}\left(k_p\Upsilon_j\right) + \sum_{j=0}^{N} \alpha_{i,j}\left(k_ie\right) - \sum_{j=1}^{N} \alpha_{i,j}\left(k_je\right) \tag{17}$$

where $\Upsilon_i = [\Upsilon_{i,1}\,\Upsilon_{i,2}\,\ldots\,\Upsilon_{i,\rho-1}]^T$.

Note that k_p is a fixed vector, thus in order to obtain an equal error dynamics for each topology it is necessary to get a fixed gain q for $e_i = \zeta_{i,1}$, i.e. to solve the following equation for all the agents

$$\sum_{j=0}^{N} \alpha_{i,j}\left(k_ie\right) - \sum_{j=1}^{N} \alpha_{i,j}\left(k_je\right) = -q\zeta_{i,1}, \tag{18}$$

which can be rewritten as

$$[-\alpha_{i,1} \ldots \ell_{i,i} \cdots - \alpha_{i,N}]\, Qe = -q\,[0 \ldots 1 \ldots 0]\, e, \tag{19}$$

or equivalently, to solve the following matrix equation that includes all the agents

$$-\mathcal{L}_RQ = -q\mathbf{I}_N \tag{20}$$

where $\mathcal{L}_R$ is the Laplacian matrix without the column and row of the virtual agent, $Q = [k_1^T k_2^T \dots k_N^T]^T$ and $\mathbf{I}_N$ is the identity matrix.

In order to solve the previous equation (20), we first analyze the case where the communication net is *strongly connected* [19], i. e. each agent knows the output of all other agents, thus $\mathcal{L}_R$ takes the form

$$\mathcal{L}_R = \begin{bmatrix} -N & 1 & \cdots & 1 \\ 1 & -N & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & -N \end{bmatrix}.$$

The previous matrix is always nonsingular with $N \neq 0$, so matrix Q satisfying (20) is given by

$$Q = q\mathcal{L}_R^{-1} = \frac{-q}{N+1} \begin{bmatrix} 2 & 1 & \cdots & 1 \\ 1 & 2 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & 2 \end{bmatrix}.$$

In the previous case all the agents are supposed to be connected, but if the communication topology has the configuration (8) the problem can be solved too, because it is always possible that the matrix $\mathcal{L}_R$ assumes a block diagonal form by a similarity transformation, even if the communication topology is switching, and the solution is obtained in a similar way to the above case, i. e.

$$Q = \frac{-q}{-\ell_{R(i,i)} + 1} \begin{bmatrix} 2 & \alpha_{2,1} & \cdots & \alpha_{N,1} \\ \alpha_{1,2} & 2 & \cdots & \alpha_{N,2} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{1,N} & \alpha_{2,N} & \cdots & 2 \end{bmatrix}.$$

The above matrix yields the entries of the vector k_i, which switches as the communication topology does, and the feedback for the error dynamics is

$$\dot{\zeta}_{i,\rho} = -[q\ k_p]\zeta_i. \tag{21}$$

Thus, the error dynamics of the ith agent is described by

$$\dot{\zeta}_i = M\zeta_i$$

where M is a Hurwitz matrix whose eigenvalues are determined by the vector $[q\ k_p]$. The matrix M has the form

$$M = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & & & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -q & -k_{p_1} & -k_{p_2} & \cdots & -k_{p_{\rho-1}} \end{bmatrix}$$

and we obtain a block diagonal matrix considering all the $\zeta_{i,j}$ from the error equations

$$M' = diag\begin{pmatrix} M, & M, & M, & \cdots & M \end{pmatrix}$$

where each block M is stable and therefore M' is also stable. Furthermore, for any topology that satisfies (8) the matrix M' is the same. From Theorem 2.1, the system is locally attractive for every switching signal because M' is Hurwitz and is the same for every switching signal because the entries of M do not change due to the feedback. Also there exists a common Lyapunov function for $\dot{\zeta} = M'\zeta$, so any Lyapunov function is common. Therefore the error dynamics e is *GUES*, i. e. $\lim_{t\to\infty} e_i = 0$ and the consensus and trajectory tracking are asymptotically achieved for the MAS.

Moreover, it is possible to take the controllable part of the system S_i to the normal form [7] by a change of coordinates where the new coordinates will be denoted by z_i,

$$\begin{aligned} \dot{z}_{i,1} &= z_{i,2} \\ \dot{z}_{i,2} &= z_{i,3} \\ &\vdots \\ \dot{z}_{i,\rho-1} &= z_{i,\rho} \\ \dot{z}_{i,\rho} &= f_i(z) + \beta_i u_i. \end{aligned}$$

Furthermore, the uncontrollable dynamics are considered as a vanishing perturbation, because they are supposed to be stable. Based on the analysis of an *Asymptotic Output Tracking*, presented in [7], it can be seen that

$$\lim_{t\to\infty} \sum_{i=1}^{N} |z_{i,1} - \Delta_i - y_0| = 0$$

which for the ith agent implies that

$$\lim_{t\to\infty} |z_{i,1} - \Delta_i - y_0| = 0$$

and since the system is considered to be in normal form, it also holds that

$$\lim_{t\to\infty} \left| z_{i,m} - \overset{(m)}{\Delta}_i - \overset{(m)}{y_0} \right| = 0, \;\; m = 1, 2, \ldots, \rho$$

implying that the dynamics of each agent is also stable. □

4. EXAMPLE

The application of the previous results is illustrated in the following example, where we consider planar robot manipulator dynamics. Consider three agent dynamics, all of them different from each other. The first two agents correspond to a planar robot dynamics, given by

$$\begin{aligned} \dot{x}_{i,1} &= x_{i,2}, \\ \dot{x}_{i,2} &= \frac{1}{m_i l_{ci}^2 + I_{zi}} [\tau_i - \mu_i x_{i,2}], \\ y_i &= x_{i,1}; \qquad\qquad i = 1, 2 \end{aligned} \tag{22}$$

where $x_{i,1}$ is the angular position, $x_{i,2}$ is the angular velocity, y_i is the output, m_i is the mass of the link, l_{ci} is the center-of-mass length, I_{zi} is the moment of inertia, μ_i is the coefficient of friction and τ_i is the torque. For the simulation the parameters are taken as $m_1 = 1Kg$, $m_2 = 1.5Kg$, $l_{c1} = 0.2m$, $l_{c2} = 0.15m$, $I_{z1} = 0.05Kg \cdot m^2$, $I_{z2} = 0.08Kg \cdot m^2$, $\mu_1 = 0.005Kg/s$ and $\mu_1 = 0.01Kg/s$. Then, the dynamics for the simulation are given by

$$\begin{aligned} \begin{bmatrix} \dot{x}_{1,1} \\ \dot{x}_{1,2} \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ 0 & -0.0556 \end{bmatrix} \begin{bmatrix} x_{1,1} \\ x_{1,2} \end{bmatrix} + \begin{bmatrix} 0 \\ 11.1111 \end{bmatrix} u_1, \\ y_1 &= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_{1,1} \\ x_{1,2} \end{bmatrix} \end{aligned} \tag{23}$$

and

$$\begin{aligned} \begin{bmatrix} \dot{x}_{2,1} \\ \dot{x}_{2,2} \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ 0 & -0.0879 \end{bmatrix} \begin{bmatrix} x_{2,1} \\ x_{2,2} \end{bmatrix} + \begin{bmatrix} 0 \\ 8.7912 \end{bmatrix} u_2, \\ y_2 &= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_{2,1} \\ x_{2,2} \end{bmatrix}. \end{aligned} \tag{24}$$

The third agent will be considered as a theoretical dynamics that has an uncontrollable part and is given by

$$\begin{aligned} \begin{bmatrix} \dot{x}_{3,1} \\ \dot{x}_{3,2} \\ \dot{x}_{3,3} \end{bmatrix} &= \begin{bmatrix} -2 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_{3,1} \\ x_{3,2} \\ x_{3,3} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u_3, \\ y_3 &= \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_{3,1} \\ x_{3,2} \\ x_{3,3} \end{bmatrix}, \end{aligned} \tag{25}$$

where we have that all these systems are stabilizable and observable. The entries of the adjacency matrix $\mathcal{A}(t)$ are given by

$$\alpha_{i,j} = \begin{cases} 1 & \text{if } \|y_i - y_j\| \leqslant r \text{ for } j \neq i, \\ 1 & \text{if } \alpha_{i,k} = 1 \text{ and } \alpha_{k,j} = 1, \\ 0 & \text{otherwise,} \end{cases} \tag{26}$$

where r is the action's radius, with $r = 0.35$.

Figure 1 shows how the communication topology switches while the agents are evolving.

We establish the reference function to be tracked as

$$y_0 = \sin(t)$$

which is always known by the agents, i. e. there is always a direct connection from each agent to the virtual agent.

The initial conditions of each agent are taken as $\mathbf{x}_1(0) = \begin{bmatrix} -0.1 & 1 \end{bmatrix}^T$, $\mathbf{x}_2(0) = \begin{bmatrix} 0.8 & -1 \end{bmatrix}^T$ and $\mathbf{x}_3(0) = \begin{bmatrix} -0.8 & -0.7 & 0.5 \end{bmatrix}^T$. The position of each agent in the desired formation is determined by the initial conditions in order to avoid collisions.

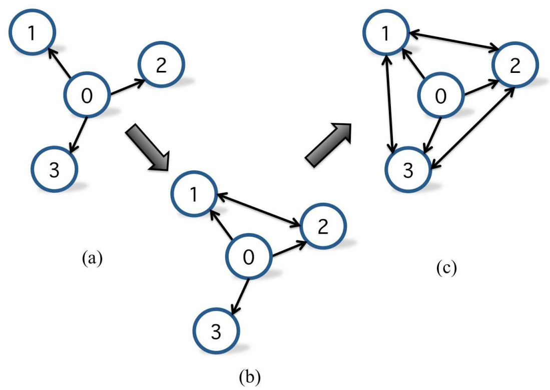

Fig. 1. (a) Initial topology, the agents have no communication between them (b) Topology in which only some of the elements communicate (c) Final topology, all agents communicate.

The required formation indicates a separation from each agent to the reference function y_0, given by $\Delta_2 = 0.2$, $\Delta_3 = -0.2$ and a change of formation given by

$$\Delta_1 = \begin{cases} 0; & t < 14 \\ 0.05t^2; & 14 <= t < 15 \\ -0.05t^2 + 0.1t + 0.05; & 15 <= t < 16 \\ 0.1; & t >= 16. \end{cases}$$

The objective of the above function, obtained by a *quadratic B-spline* interpolation, is the continuity in the derivatives of Δ_1.

We select the stable polynomial $s^2 + 3s + 1$ to impose the desired error dynamics, whose feedback vector is $[q\, k_p] = [1\ 3]$ and the vectors k_i, are as follows

$$\begin{aligned} k_1 &= \big(1/(-\ell_{R(1,1)} + 1)\big)\, [\ 2 \quad \alpha_{2,1} \quad \alpha_{3,1}\ ], \\ k_2 &= \big(1/(-\ell_{R(2,2)} + 1)\big)\, [\ \alpha_{1,2} \quad 2 \quad \alpha_{3,2}\ ], \\ k_3 &= \big(1/(-\ell_{R(3,3)} + 1)\big)\, [\ \alpha_{1,3} \quad \alpha_{2,3} \quad 2\ ]. \end{aligned}$$

The matrix M, with the values of q and k_p previously selected, is

$$M = \begin{bmatrix} 0 & 1 \\ -1 & -3 \end{bmatrix}$$

which can be easily seen to be stable, so the error dynamics is also stable, even if the communication topology switches.

With (23), (24), (25) and the indicated reference and separation functions, we obtain that $\Gamma_{1,2} = [0\ -0.0556]$, $\Gamma_{2,2} = [0\ -0.0879]$, $\Gamma_{3,2} = [1\ 0\ -1]$, $\dot{y_0} = cos(t)$, $\ddot{y}_0 = -\sin(t)$, $\beta_{1,2} = 11.1111$, $\beta_{2,2} = 8.7912$, $\beta_{3,2} = 1$. Then, the control can be designed as

$$
\begin{aligned}
u_1 &= \frac{1}{\beta_{1,2}}(-[0 \ \ -0.0556]x_1 + (-\sin(t)) + \ddot{\Delta}_1 - 3\Upsilon_1 - \sum_{j=1}^{3} k_{1,j}e_j),\\
u_2 &= \frac{1}{\beta_{2,2}}(-[0 \ \ -0.0879]x_2 + (-\sin(t)) + \ddot{\Delta}_2 - 3\Upsilon_2 - \sum_{j=1}^{3} k_{2,j}e_j),\\
u_3 &= (-[1 \ 0 \ \ -1]x_3 + (-\sin(t)) + \ddot{\Delta}_3 - 3\Upsilon_3 - \sum_{j=1}^{3} k_{3,j}e_j),
\end{aligned}
\tag{27}
$$

where $\Upsilon_1 = x_{1,2} - \dot{\Delta}_1 - cos(t)$, $\Upsilon_2 = x_{2,2} - \dot{\Delta}_2 - cos(t)$, $\Upsilon_1 = x_{3,3} - \dot{\Delta}_3 - cos(t)$ and $e_j = \sum_{i=0}^{3} \alpha_{j,i} \left[(y_j - \Delta_j) - (y_i - \Delta_i)\right]$.

The MAS is composed by the agents dynamics (23), (24) and the communication topology defined by (26).

Figure 2 shows the outputs of the agents when the control (27) is applied to the MAS system, where the topology shown in Figure 1 changes approximately at $0.60s$ and $4.00s$.

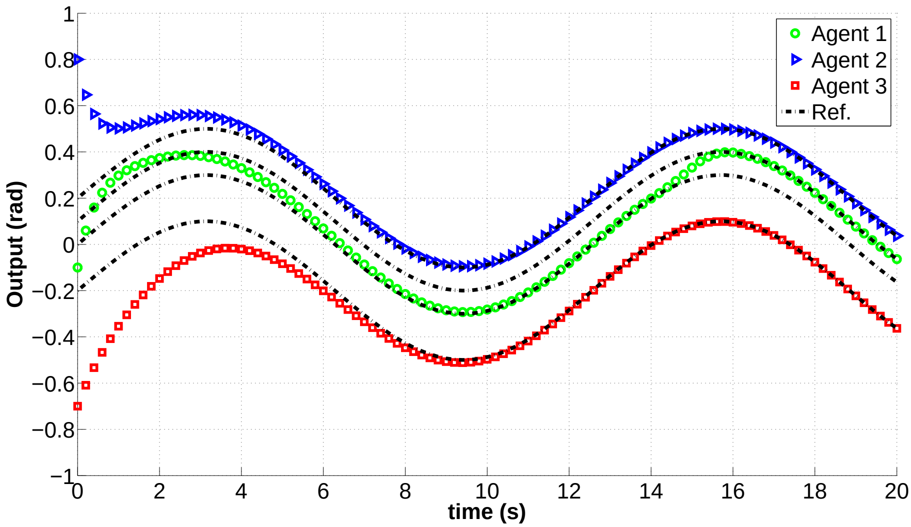

Fig. 2. Output path of each agent with respect to the reference signal y_0 in a topology that switches in time.

Note in Figure 2 that the output of each agent achieves the desired separation (Δ_i), while the reference function (y_0) is tracked, even if the communication topology and the formation change.

The consensus and trajectory tracking errors of the agents are shown in Figure 3, where it can be noticed that although the error value changes abruptly when the communication topology switches, it continues to decrease as the system evolves.

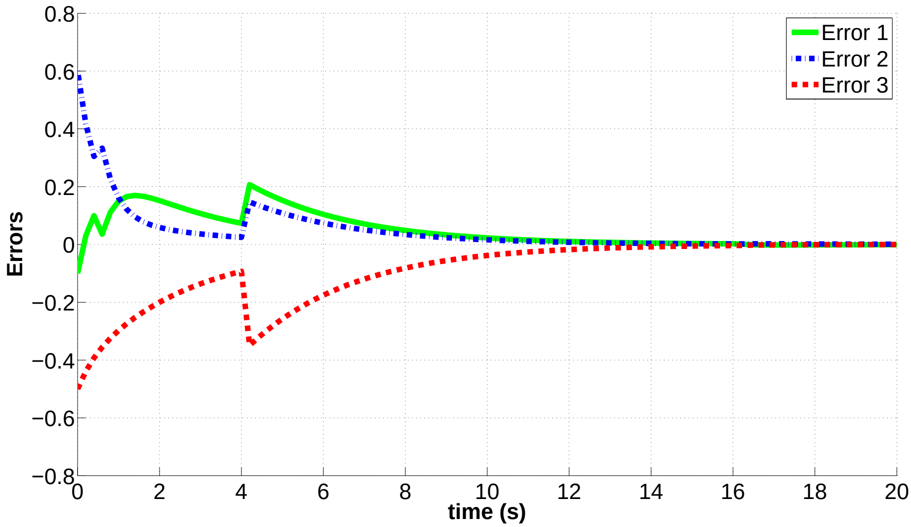

Fig. 3. Consensus and trajectory tracking error of reference of each agent with a switching communication topology.

5. CONCLUSIONS

In this paper, we presented the design of a control law for the formation and trajectory tracking of *MAS* with fixed and switching topologies, including formation changes. With the proposed control, the error dynamics is ensured to be *GUES*, even if the formation and the topology of the configuration change, ensuring the existence of a common Lyapunov function.

As future work, the results presented in this paper can be extended to consider the case of *MIMO* or nonlinear agent dynamics. Also, it can be addressed the study of other control techniques in order to relax the condition of relative degree, and discrete communication with delays can be considered.

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