Acta Universitatis Carolinae. Mathematica et Physica

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*Acta Universitatis Carolinae. Mathematica et Physica*, Vol. 18 (1977), No. 2, 3--12

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1977 ACTA UNIVERSITATIS CAROLINAE – MATHEMATICA ET PHYSICA VOL. 18, NO. 2, Pag. 3–12

# Space Charge Effects in the Omegatron

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*Received 22 December 1975*

On the basis of a two-dimensional model of the omegatron, space charge effects inside and outside the electron beam are treated theoretically. In the inside of the electron beam resonanions are partly retained, but a shift of frequency eliminates this effect. The corresponding formula is derived. Outside the beam the space charge causes prolongation of trajectories, which results in higher losses of resonant ions. This effect can also be eliminated by a corresponding shift of frequency. A simple formula is given based on an approximate procedure. As an example a practical case is evaluated.

Влияния пространственного заряда в омегатроне. На основе плоской модели омегатрона объясняется влияние пространственного заряда электронного пучка на сдвиг резонансной частоты. Пространственный заряд электронов вызывает внутри пучка в случае резонанса частичное задержание ионов, что понижает ток колектором. В случае когда частота внешнего поля высше циклотронной частоты это явление устранится. Для сдвига частоты выведена соответствующая формула (11). Вне пучка приводит пространственный заряд удлинение траекторий, и тем повышает потеры резонансных ионов. Также это явление можно устранить повышением внешней частоты, для которой была выведена формула (18) на основе определения деформации зоны группировки и компенсации этой деформации под влиянием внешнего поля. В случае обыкновенных условий являются для ионов средней массы решающими явления внутри электронного пучка, так-как они вызывают сдвиг резонансной частоты высшего порядка чем возмущения траекторий вне пучка.

Vliv prostorového náboje v omegatronu. Na základě rovinného modelu omegatronu byl objasněn vliv prostorového náboje elektronového svazku na posuv rezonanční frekvence. Uvnitř svazku způsobí prostorový náboj elektronů při rezonanci částečné zadržování iontů, což vede ke snížení kolektorového proudu. Zvýšením frekvence vnějšího pole nad cyklotronovou frekvenci příslušného iontu se tento jev dá odstranit. Pro posuv frekvence jest odvozena příslušná formule (11). Vně svazku způsobí prostorový náboj prodloužení trajektorií což vede ke zvýšení ztrát rezonančních iontů. Tento jev lze též odstraniti zvýšením vnější frekvence, pro něž byla odvozena formule (18) na základě odhadu deformace shlukovacího pásu a její kompensace vnějším polem.

Při běžných pracovních podmínkách jsou pro středně těžké ionty rozhodující jevy uvnitř elektronového svazku, neboť způsobují řádově větší posuv rezonanční frekvence, než poruchy trajektorií vně svazku.

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## 1. Introduction

Papers on the theory of two-dimensional omegatron ([1]—[6]) give no attention to the influence of space charge created by the ionizing electron beam. The aim of this work is to show that the influence of this space charge cannot in general be omitted as it causes a shift of the measured resonant frequency, i.e. the frequency corresponding to the maximum collector current in a peak.

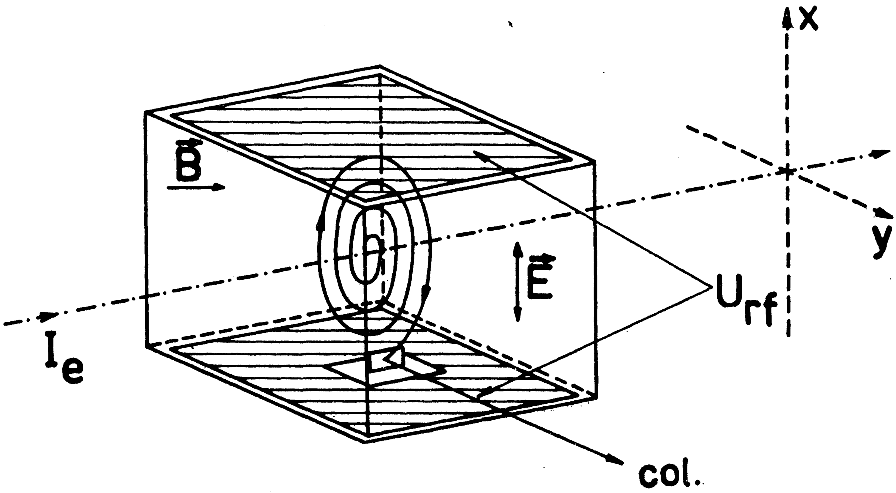


Fig. 1. Schematic diagram of the omegatron.

In the laboratory system $xy$ (see Fig. 1) perpendicular to the electron beam (and the magnetic field) an electron beam of circular cross-section of radius $R$ and space charge density $-\varrho$ ($\varrho$ positive) gives rise to a radial electric field $E_r$. Inside the beam it is given by $E_r = -(\varrho 2/\varepsilon_0)r$ and outside by $E_r = -(\varrho R^2/2\varepsilon_0 r)$, where $r$ is the radial coordinate and $\varepsilon_0$ is the permitivity. This field is added to the external radiofrequency electric field and changes the trajectories of ions so that maximum current is not drawn at the same frequency as in the case of no space charge.

As the behaviour of ions inside and outside the electron beam differs considerably the two cases will be treated separately.

## 2. Space Charge Effects Inside the Beam

### 2.1. RESONANCE

Equations of motion of a general ion can be written in the complex form (introducing $z = x + iy$)

$$\ddot{z} + i\omega\dot{z} + Kz = \eta E \sin \omega_0 t \tag{1}$$

where $\omega = (e/M)\,B = \eta B$ is the cyclotron frequency of ion ($M\ldots$ mass of ion, $B\ldots$ magnetic field), $K = \eta\varrho/2\varepsilon_0$ and $E$, $\omega_0$ is the amplitude, resp. the frequency of electric field.

This equation is valid for $t_0 \leq t \leq t_R$, $t_0$ is the moment when an ion originates and $t_R$ the moment it leaves the electron beam at radius $R$. First, resonant ions will be treated. Putting $\omega_0 = \omega$ (1) becomes

$$\ddot{z} + i\omega\dot{z} + Kz = \eta E \sin \omega t \tag{2}$$

Choosing the initial conditions $z(t_0) = z_0$ and $\dot{z}(t_0) = 0$ and going over from the laboratory system $xy$ to a rotating system

$$Z = z e^{i\omega t} \tag{3}$$

we find solution of (2) in this system

$$Z_{x_0,\,y_0} \approx Z_{0,0} + z_0 \frac{\delta}{\omega + 2\delta} e^{i\omega t_0} \,.\, e^{-i\delta(t-t_0)} + z_0 \frac{\omega + \delta}{\omega + 2\delta} e^{i\omega t_0} \,.\, e^{i(\omega+\delta)(t-t_0)} \tag{4}$$

where

$$\delta = \frac{\omega}{2}\left(\sqrt{1 + \frac{4K}{\omega^2}} - 1\right) \tag{5}$$

and

$$Z_{0,0} = i\,\frac{\eta E e^{2i\omega t_0}}{2(2\omega^2 - K)}\left[e^{2i\omega(t-t_0)} - \frac{2\omega + \delta}{\omega + 2\delta} e^{i(\omega+\delta)(t-t_0)} + \frac{\omega - \delta}{\omega + 2\delta} e^{-i\delta(t-t_0)}\right] +$$
$$+ i\,\frac{\eta E}{2K}\left[1 - \frac{\delta}{\omega + 2\delta} e^{i(\omega+\delta)(t-t_0)} - \frac{\omega + \delta}{\omega + 2\delta} e^{-i\delta(t-t_0)}\right]. \tag{6}$$

By inspection of (4) it is easy to see that there exists a maximum value of $Z_{x_0,y_0} \,.\, Z^*_{x_0,y_0}$ for which the relation

$$(Z_{x_0,y_0} \,.\, Z^*_{x_0,y_0}) < R^2 \tag{7}$$

could be valid.

It is evident that any resonant ion for which (7) holds cannot escape from the beam Thus the collector ion current could be reduced. The question is whether such an effect is practically possible. Expression (4) is too complicated for such consideration. It can be simplified for ions which are not too heavy under the following assumptions. At pressure less than $10^{-3}$ Pa $|\varrho_e| \gg |\varrho_i|$ ($\varrho_e$, $\varrho_i$ ... electron space charge density, resp. ion s.c.d.). Hence we can put $|\varrho| \doteq |\varrho_e|$. We have chosen following working conditions: magnetic field $B \sim 0.4$ T, accelerating voltage for the electron beam $U \sim 70$ V, electron current $I_e \sim 5\times10^{-6}$ A, radius of the electron beam $R \sim 5\times10^{-4}$ m.

Under these conditions $\delta$ can be evaluated for $N_2^+$ and $O_2^+$ ions from (5) giving $\delta_{N_2+} = 0.12\ \omega_{N_2+}$ and $\delta_{O2+} = 0.14\ \omega_{O_2+}$. It can be assumed that in this case $\delta^2 \ll \omega^2$ and instead of (6) we can write

$$Z_{0,0} \approx \frac{i\eta E}{2\delta(\omega + \delta)} + \frac{i\eta E e^{2i\omega t_0}}{2\omega(2\omega - \delta)} e^{2i\omega(t-t_0)} -$$

$$- \frac{i\eta E\left(e^{2i\omega t_0} + \dfrac{\omega + \delta}{\omega + 3\delta}\right)}{2\omega(\omega + \delta)} e^{i(\omega+\delta)(t-t_0)} +$$

$$+ \frac{i\eta E}{2}\left(\frac{e^{2i\omega t_0}}{\omega(2\omega + 4\delta)} - \frac{1}{\delta(\omega + 2\delta)}\right) e^{-i\delta(t-t_0)} \tag{8}$$

The next step is to simplify (8) by considering the magnitudes of amplitudes at the exponentials and omitting those which are at laste by one order of magnitude smaller than the first righ-hand side member. Thus we obtain

$$Z_{0,0} \approx \frac{i\eta E}{2\delta(\omega + \delta)} - \frac{i\eta E}{2\delta(\omega + 2\delta)} e^{-i\delta(t-t_0)} +$$

$$- \frac{i\eta E}{2\omega(\omega + \delta)}\left[e^{2i\omega t_0} + \frac{\omega + \delta}{\omega + 3\delta}\right] e^{i(\omega+\delta)(t-t_0)} \tag{9}$$

Putting (9) into (4) and separating the real and imaginary parts gives

$$X_{x_0,y_0} \approx x_0 \frac{\delta}{\omega + 2\delta} \cos \varphi_1 - y_0 \frac{\delta}{\omega + 2\delta} \sin \varphi_1 - \frac{\eta E}{2\delta(\omega + 2\delta)} \sin \delta\,(t - t) +$$

$$+ x_0 \frac{\omega + \delta}{\omega + 2\delta} \cos \varphi_2 - y_0 \frac{\omega + \delta}{\omega + 2\delta} \sin \varphi_2 + \frac{\eta E}{2\omega(\omega + \delta)} \sin \varphi_3 +$$

$$+ \frac{\eta E}{2\omega(\omega + 3\delta)} \sin (\omega + \delta)(t - t_0)$$

$$Y_{x_0,y_0} \approx \frac{\eta E}{2\delta(\omega + \delta)} + x_0 \frac{\delta}{\omega + 2\delta} \sin \varphi_1 + y_0 \frac{\delta}{\omega + 2\delta} \cos \varphi_1 -$$

$$- \frac{\eta E}{2\delta(\omega + 2\delta)} \cos \delta(t - t_0) + x_0 \frac{\omega + \delta}{\omega + 2\delta} \sin \varphi_2 + y_0 \frac{\omega + \delta}{\omega + 2\delta} \cos \varphi^2 -$$

$$- \frac{\eta E}{2\omega(\omega + \delta)} \cos \varphi_3 - \frac{\eta E}{2\omega(\omega + 3\delta} \cos (\omega + \delta)(t - t_0)$$

where

$$\varphi_1 = \omega t_0 - \delta(t - t_0) \qquad \varphi_3 = 2\omega t_0 + (\omega + \delta)(t - t_0)$$
$$\varphi_2 = \omega t_0 + (\omega + \delta)(t - t_0)$$

A majorant to the expression $X^2_{x_0,y_0} + \left(Y_{x_0,y_0} - \dfrac{\eta E}{2\delta(\omega + \delta)}\right)^2$

under the same assumption $\delta^2 \ll \omega^2$ is given by

$$\left[X^2_{x_0,y_0} + \left(Y_{x_0,y_0} - \frac{\eta E}{2\delta(\omega + \delta)}\right)^2\right]_{maj} \approx \left(r_0 + \frac{\eta E}{2\omega\delta}\right)^2$$

where

$$r_0^2 = x_0^2 + y_0^2$$

Fig. 2 is a graphical representation of this expression. $N_2^+$ and $O_2^+$ (and all lighter ions) can move from the centre of the beam only to the distance

$$\frac{\eta E}{2\delta(\omega + \delta)} + r_0 + \frac{\eta E}{2\omega\delta}$$

When the radius of the electron bream $R$ is larger than this distance the ions cannot leave the region of the beam. This results in a decresae of the collector current in relation of the corresponding cross-sections (see Fig. 2):

$$\frac{S_2}{S_1 + S_2} = 1 - \left(\frac{r_0}{R}\right)^2 \qquad r_0 = R - \frac{\eta E}{2\omega\delta} - \frac{\eta E}{2\delta(\omega + \delta)}$$

To get an idea what happens in a practical case let us use the values of parameters given above and for the radiofrequency electric field let us choose $E = 30\,\mathrm{Vm}^{-1}$. For both $N_2^+$ and $O_2^+$ when $\omega_0 = \omega_{N_2^+}$ (or $\omega_{O_2^+}$) the collector current is lower

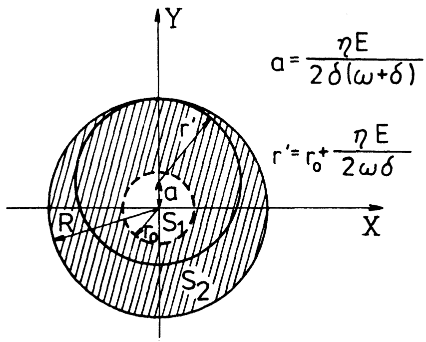


Fig. 2. Cross-section of the electron beam.

than 98 % of the originated one. Evidently the losses of ions retained in the electron beam are higher because our estimate was evaluated by means of a majorant and under the assumption of homogeneous space charge distribution over the cross section of the electron beam.

## 2.2. REAL RESONANCE. SHIFT OF THE RESONANT FREQUENCY

From the above considerations it follows that for equality of the radiofrequency and the cyclotron frequency, not all corresponding ions in general can leave the region of the electron beam. The trajectory given by (4) does not contain any term increasing with time. In the following it will be demonstrated that real resonance occurs when the radiofrequancy of the outer field $\omega_0$ equals either $\omega + \delta$ or $\delta$.

Let us suppose that $\omega_0 = \omega + \delta$, so that an ion of cyclotron frequency $\omega$

seems to be nonresonant. The equation of motion in the laborartoy system gives now

$$\ddot{z} + i\omega\dot{z} + Kz = \eta E \sin(\omega + \delta)\, t \tag{10}$$

For the sake of simplicity let the initial conditions be $z(t_0) = 0$ and $\dot{z}(t_0) = 0$. The solution of (10) in the rotating system (3) becomes

$$Z = -\frac{\eta E}{2(\omega + 2\delta)} e^{-i\delta t_0}(t - t_0)\, e^{-i\delta(t-t_0)} + \frac{i\eta E}{4\omega(\omega + \delta)} e^{i(2\omega+\delta)t_0} \,.\, e^{i(2\omega+\delta)(t-t_0)} +$$

$$-\frac{i\eta E}{2(\omega + 2\delta)} \left[\frac{1}{\omega} e^{i(2\omega+\delta)t_0} + \frac{1}{\omega + 2\delta} e^{-i\delta t_0}\right] e^{i(\omega+\delta)(t-t_0)} +$$

$$+ \frac{i\eta E}{2(\omega + 2\delta)} \left[\frac{1}{\omega + 2\delta} {}^{-i\delta t_0} + \frac{1}{2(\omega + \delta)} e^{i(2\omega+\delta)t_0}\right] e^{-i\delta(t-t_0)}$$

It follows that the first term increases linearly with time. That means a real resonance at the radiofrequency $\omega + \delta$, which gives for the shift of frequency

$$\Delta\omega = \delta = \frac{e}{2M} B \left[\sqrt{1 + \frac{2M\varrho}{e\varepsilon_0 B^2}} - 1\right] \tag{11}$$

### 3. Space Charge Effects Outside the Beam

The behaviour of ions after leaving the region of the electron beam will now be examined.

The equation of motion comprising the space charge becomes

$$\ddot{z} + i\omega\dot{z} = \eta E \sin\omega_0 t + \frac{k^2}{z^*}, \tag{12}$$

$$k^2 = \frac{\eta\varrho R^2}{2\varepsilon_0}.$$

This equation holds only for $t \geq t_1$, where $t_1$ is the moment of departure from the boundary of the electron beam, i.e. $z \,.\, z^* \geq R^2$. Equation (12) represents a system of two nonlinear equations which for each trajectory could be computed numerically to obtain numerical or graphical results for the whole ensemble of trajectories in every special case. However the form of every separate trajectory is for our purpose not interesting. More interesting is the integral deformation of the bunching zone (see [4]). To estimate this deformation the following procedure has been developed: The principal part of trajectory which represents the bunching zone for $\omega_0 = \omega$ (resonance) in the system with no space charge (see [4]) is given by

$$z \approx -\frac{\eta E}{2\omega^2} [\omega(t - t_0)]\, e^{-i\omega t} \tag{13}$$

We approximate equation (12) by replacing $z^*$ in the second right-hand term with (13) to get an approximate linearized equation

$$\ddot{z} + i\omega\dot{z} \approx \eta E \sin \omega t + \frac{k^2 e^{-i\omega t}}{\frac{\eta E}{2\omega^2}\,\omega(t - t_0)} \tag{14}$$

valid for $t \geq t_1$, $t_1$ is given by

$$R^2 = \left(\frac{\eta E}{2\omega^2}\right)^2 [\omega(t_1 - t_0)]^2$$

The solution of the linearized equation (14) can be written as $z + \Delta z$, where $\Delta z$ represents the correction for space charge, where $z$ is the solution of (14) when the second term on the right-hand side is zero. After transformation to the rotating system $\Delta Z = \Delta z\, e^{i\omega t}$ the correction becomes

$$\Delta Z = \frac{ik^2}{\frac{\eta E}{2\omega}}\left[\int_{t_1}^{t} \frac{dt}{\omega(t - t_0)} - e^{i\omega t}\int_{t_1}^{t} \frac{e^{-i\omega t}}{\omega(t - t_0)}\,dt\right] \tag{15}$$

The time needed by a resonant ion to reach the collector can be evaluated from $\omega(t_{R_c} - t_0) \approx \frac{R_c}{\eta E/2\omega^2}$. Thus, at the distance $R_c$ where the inner edge of the collector is placed the correction can be evaluated giving

$$\Delta Z(R_c) = \frac{iR^2\varrho}{\varepsilon_0 E}\left\{\ln\frac{R_c}{R} - e^{i[R_c/(\eta E/2\omega^2)]}\left[Ci\,\frac{R_c}{\frac{\eta E}{2\omega^2}} - Ci\,\frac{R}{\frac{\eta E}{2\omega^2}} - \right.\right.$$

$$\left.\left. - i\left(Si\,\frac{R_c}{\frac{\eta E}{2\omega^2}} - Si\,\frac{R}{\frac{\eta E}{2\omega^2}}\right)\right]\right\}$$

where

$$Si\,\psi = \int_0^{\psi} \frac{\sin u}{u}\,du \quad \text{and} \quad Ci\,\psi = -\int_{\psi}^{\infty} \frac{\cos u}{u}\,du\,.$$

For the detuning of the resonant frequency the deviation in the $Y$-direction is decisive:

$$\Delta Y(R_c) = \frac{R^2\varrho}{\varepsilon_0 E}\left\{ln\,\frac{R_c}{R} - \left[Ci\,\frac{R_c}{\frac{\eta E}{2\omega^2}} - Ci\,\frac{R}{\frac{\eta E}{2\omega^2}}\right]\cos\frac{R_c}{\frac{\eta E}{2\omega^2}} + \right.$$

$$\left. - \left[Si\,\frac{R_c}{\frac{\eta E}{2\omega^2}} - Si\,\frac{R}{\frac{\eta E}{2\omega^2}}\right]\sin\frac{R_c}{\frac{\eta E}{2\omega^2}}\right\}$$

As in all practival cases $[R_c/(\eta E/2\omega^2)] \gg 1$ resp. $[R/(\eta E/2\omega^2)] \gg 1$ use can be made of the asymptotic formulas $Ciu \approx (\sin u/u)$ and $Siu \approx (\pi/2) - (\cos u/u)$.

This results in

$$\varDelta Y(R_c) \approx \frac{R^2\varrho}{\varepsilon_0 E}\left[\ln\frac{R_c}{R} - \frac{\frac{\eta E}{2\omega^2}}{R}\sin\frac{R_c - R}{\frac{\eta E}{2\omega^2}}\right]$$

As the second right-hand term can be again omitted in all practical cases $[(R/(\eta E/2\omega^2)] \gg 1)$, the $Y$-deviation takes finally a very simple form

$$\varDelta Y(R_c) \approx \frac{R^2\varrho}{\varepsilon_0 E}\ln\frac{R_c}{R}\,. \tag{16}$$

This deviation which is caused by the space charge of the electron beam can be compared with the deviation caused by a shift of the resonant frequancy in the case of the zero space charge.

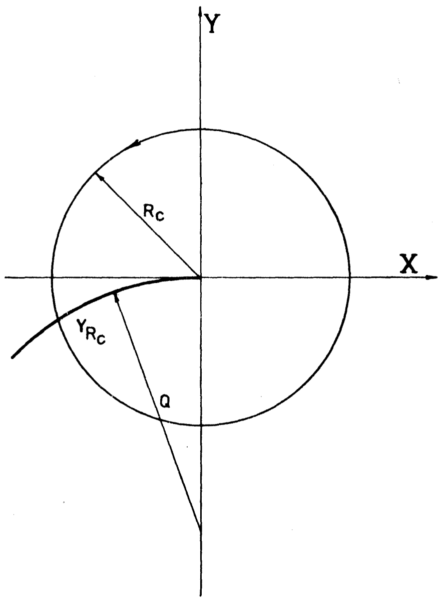


Fig. 3. Deviation of the p.p. of trajectory in the rotating system. $Q = \frac{\eta E}{2\omega\Delta\omega}$.

Suppose $\omega_0 = \omega + \varDelta\omega$, $\varDelta\omega > 0$ i.e. the frequancy of the signal is higher than the cyclotron frequency of the ion. The principal part of the trajectory of such a non-resonant ion is in the rotating system (see [4])

$$X \approx -\frac{\eta E}{2\omega\,.\,\varDelta\omega}\sin\varDelta\omega(t - t_0)$$

$$Y \approx -\frac{\eta E}{2\omega\,.\,\varDelta\omega}\left[1 - \cos\varDelta\omega(t - t_0)\right].$$

The corresponding deviation $Y_{R_c}$ at the distance of the collector one gets at the point where the principal part of the trajectory intercepts the collector rotating in this system of coordinates at a circular line of radius $R_c$ with centre at the origin. In other words solving the system of equations (see Fig. 3)

$$X^2 + \left(Y + \frac{\eta E}{2\omega \, . \, \Delta\omega}\right)^2 = \left(\frac{\eta E}{2\omega \, . \, \Delta\omega}\right)^2$$

$$X^2 + Y^2 = R_c^2$$

whence

$$Y_{R_c} = -\frac{R_c^2}{\eta E} \, \omega \, . \, \Delta\omega \, . \tag{17}$$

When increasing the outer frequency the deviation has a negative value, while the space charge correction makes it positive. It follows that the outer frequency can be increased in such a way as to balance the two deviations. Setting

$$\Delta Y(R_c) = |Y_{R_c}|$$

we obtain

$$\Delta\omega = \frac{\varrho}{\varepsilon_0 B} \cdot \frac{R^2}{R_c^2} \ln \frac{R_c}{R} \tag{18}$$

as the required shift of frequency. In this approximation it is independent of the chrage-to-mass ratio and is the same for all ions. It has been evaluated for the parameters given above and for $R_c = 8$ mm leading to $\Delta\omega \approx 4$ kHz. For $N_2^+$ ions this gives $\approx 0.4\,\%\ \omega_{N_2^+}$ .

## 4. Conclusion

It has been demonstrated that the space charge of the electron beam retains resonant ions inside the beam and this results in a decrease of the collector current. To eliminate this effect it is necessary to increase the frequency of the external electric field above the corresponding cyclotron frequency. The frequency shift depends on the charge-to-mass ratio. Evaluation for a practical case i.e. for ions of medium mass and under reasonable working conditions of the omegatron gives about 10 %.

Outside the electron beam its space charge causes deformation of the resonant bunching zone – represented in the approximate procedure by the principal part of the trajectory – which can be balanced by increasing again the frequency of the external field. The needed frequency shift is independant of the charge-to-mass ratio and in a practical case evaluated at the same conditions as above is of an order lower than the first one. It can be concluded that at current working conditions the processes inside the electron beam are decisive for the frequency behaviour of the omegatron.

Simple formulas (11), (18) have been derived for evaluation of both frequency shifts under quite general conditions.

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