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On the Method of Least Squares on the Boundary

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The method of least squares on the boundary for solving problem (1), (2), based on minimalization of functional (4), is treated.

Let the biharmonic problem

$$\Delta^2 U = 0 \text{ in } G\,, \tag{1}$$

$$U = g_0(s)\,, \frac{\partial U}{\partial \nu} = g_1(s) \text{ on } \Gamma \tag{2}$$

be given, where G is a bounded simply connected region in E_2 with a Lipschitz boundary Γ, $g_0 \in W_2^{(1)}(\Gamma)$, $g_1 \in L_2(\Gamma)$. (These assumptions are sufficiently general to include regions and loadings which we meet most frequently in problems of plane elasticity, single loads including.) Many methods have been developed for solution or approximate solution of problem (1), (2) (classical variational methods, methods based on the theory of functions of a complex variable, the method of finite differences, the finite element method, etc.), having their specific preferances and disadvantages. The method of least squares on the boundary is closely related with the method given in [4], p. 285 and with variational methods explained in [2] or [3]. An approximate solution of the problem (1), (2) is assumed in the form

$$U_n(x, y) = \sum_{i=1}^{4n-2} a_{ni} z_i(x, y),\ n \geqq 2\,, \tag{3}$$

where $\{z_i(x, y)\}$ is the well known system of biharmonic polynomials (for details see [1]; in (3) just polynomials of degree $\leqq n$ are included) and the coefficients a_{ni} $(i = 1, \ldots, 4n - 2)$ are determined from the condition that the functional

$$F(V_n) = \int_0 \left[(V_n - g_0)^2 + \left(\frac{\partial V_n}{\partial s} - \frac{\mathrm{d}g_0}{\mathrm{d}s} \right)^2 + \left(\frac{\partial V_n}{\partial \nu} - g_1 \right)^2 \right] \mathrm{d}s \tag{4}$$

considered on the set of functions

$$V_n = \sum_{i=1}^{4n-2} b_{ni}\, z_i(x, y)$$

be minimal for the function (3). Thus for the function (3) the boundary conditions (2) are best approximated in the sense of least squares on the boundary.

The just mentioned condition leads to the solution of a system of $4n - 2$ linear algebraic equations for $4n - 2$ unknowns a_{ni}. In [1] it is shown that this system is uniquely solvable and that, for $n \to \infty$, the sequence $\{U_n(x, y)\}$ converges, in $L_2(G)$, to the so called very weak solution of the problem (1), (2). The proof of this assertion is not easy, because the density of traces of all linear combinations of functions $z_i(x, y)$, $i = 1, 2, \ldots$, in the space $W_2^{(1)}(\Gamma) \times L_2(\Gamma)$ is to be shown. In [1] also a numerical example is presented. The presence of the middle term in functional (4) is substantial both of theoretical and numerical reasons.

Applied to the solution of problem (1), (2), the method essentially takes use of the form of equation (1). As to the idea itself, this method, properly modified, can be applied also to the solution of other problems.

References

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