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SOLID PARTICLES IN REFLECTION NEBULAE II. DIELECTRIC MODEL OF THE NEBULA NGC 7023.

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This paper is a continuation of the former paper by V. Vanýsek and the author (1964). Due to selective scattering of light some other physical parameters (mass, density, stability etc.) of the nebula NGC 7023 are derived.

1. INTRODUCTION

In the paper (V. VANÝSEK and J. SVATOŠ, 1964) colour excesses of some geometrical and physical models of reflection nebulae were theoretically investigated. By application of this results to the photoelectrical measurements of the nebula NGC 7023 some physical parameters of particles forming the nebula were derived. It was showed that (on assumption of dielectric nature of particles) the model having particles of radius 0,33 μ and refractive index 2 is most satisfactory.

In the present paper starting from the previous results and assumptions we try to derive some other physical parameters to get the compex physical constitution of this nebula.

2. DENSITY OF THE NEBULA

Number of particles per volume unity will be given by two independent ways to get the possibility of comparing and estimating the accuracy of the results.

A) Density computing on the basis of selective scattering.

We use the formula

$$N = \frac{\gamma(\lambda)}{\pi a^2 Q_{ext}(\lambda)} \tag{1}$$

where N denotes density, a = radius of the particle, γ = linear coefficient of absorption, λ = wave-lenght. Q_{ext} is the effective factor of extinction defined by

$$Q_{ext} = \frac{4}{x^2} \operatorname{Re}\{S(0)\} \tag{2}$$

where $x = \frac{2\pi a}{\lambda}$, $S(\vartheta)$ is the scattering function. $S(0)$ holds to

$$S(0) = \frac{1}{2} \sum_{n=1}^{\infty} (2n + 1) (a_n + b_n) \tag{3}$$

where a_n, b_n are the coefficients of MIE. As to this coefficients we refer to (V. VANÝSEK and J. SVATOŠ, 1964) or (VAN DE HULST, 1957).

The equations (1) for two wave-lengths form an homogeneous system and we cannot therefore determine the unknown values of N and γ as one-valued. We are to determine γ by help of complementary relation $\tau(\lambda) = \gamma(\lambda).r$, where τ is the optical depth and r denotes the radius of the nebula. For τ (4.100 Å) we use the value of 1 (V. VANÝSEK and J. SVATOŠ 1964) which necessitates in our case $\gamma = 0{,}442 . 10^{-18}\ cm^{-1}$ in CGS system. The values of Q were computed by IRVINE (1963) for a large interval of x in addition to the values of $\overline{\cos \vartheta}$ which will be used later. But for our case ($x = 5$) the corresponding value of Q is not listed in IRVINE's paper. The infinite series in (3) converge with relatively good speed allowing the author to take 7 elements in this evaluating. Then $Q_{ext} = 2{,}97$. It follows that $N = 4{,}46 . 10^{-11}\ cm^{-3}$.

B) Density computing on the basis of surface brightness.

The surface brightness of one square second of arc of this nebula at the distance 1,5′from the centre is approximately 23^m (V. VANÝSEK and J. SVATOŠ, 1964). The corresponding column of particles contributing to this brightness has a real volume of $78{,}3 . 10^{48}\ cm^3$. Let us consider in this column some „mean" particle at a mean distance 5,25′from the central star. The real value of this distance is $1{,}31 . 10^{18}\ cm$. Because of the ratio of radiative energy of the Sun to the star of a given spectral type, we can estimate the radiative energy of the illuminating star per solid angle as $1{,}2 . 10^{36}$ erg.sec^{-1}. It follows that our mean particle becomes an amount of energy $24{,}4 . 10^{-10}\ erg.\ sec^{-1}$, which is scattered into all directions. Intensity of this particle is given by

$$I = I_0 Q_{ext} \pi a^2 \tag{4}$$

Since in our case the scattering angle $\neq 0$, we cannot use the value of Q_{ext} [see eq. (2)]. Therefore we transform the formula (4) as follows:

$$I = I_0 \pi a^2 . \frac{|\bar{S}|^2}{x^2} \tag{4a}$$

Here $\bar{S}$ is the mean value of scattering function. The author computed $|\bar{S}|^2$ by numerical integration of corresponding values which are listed in (van de Hulst, 1957). x^2 in our case $= 13{,}4$ and $|\bar{S}|^2 = 9{,}45$. Formula (4a) then gives $I = 17.10^{-10}$ *erg. sec.*$^{-1}$ i. e. the intensity of the mean particle. Due to the known magnitude of the illuminating star and its intensity we can compute the intensity of the mentioned column surface. This is given by

$$\frac{12{,}2.10^{36}}{i_2} = 2{,}5^{(23-7{,}3)}$$

$$\text{i. e. } i_2 = 7.10^{30}\ erg.\ sec^{-1}.$$

To fulfill the given column, we need therefore $\frac{7.10^{30}}{17.10^{-10}} = 0{,}41.10^{-11}$ of particles. Since we know the volume of the column we get $N = 4{,}1.10^{-11}\ cm^{-3}$. Though this density computation is not as exact as the foregoing one, we have got such a good coincidence so that we can estimate the value of $N = 4{,}5.10^{-11}\ cm^{-3}$ as a very good approximation.

3. MASS OF THE NEBULA

The values we have got above enable us to get by elementary computing the number of particles (216.10^{43}) contained in the nebula. On assumption that the specific mass of the particle is $3g.\mathrm{cm}^{-3}$, we get the total mass of the nebula approximately $940{,}5.10^{30}\ g$, i. e. 0,47 of the mass of the Sun.

4. RADIATION PRESSURE AND STABILITY OF THE NEBULA

Impulse transferred by the light wave is defined by

$$impulse = \frac{energy}{c},$$

where c denotes the speed of the light. Impulse is proportional to C_{ext}. This value is so called efficient cross-section of extinction and its dimension is cm^2 in CGS system. Part of the impulse is proportional to C_{abs} (effective cross-section for absorption) and the other part is lowered by the forward component of scattered light. This component is proportional to $\overline{\cos\vartheta} Q_{\text{sca}}$. Thus, the cross-section for radiative pressure is $C_{\text{pres}} = C_{\text{ext}} - \overline{\cos\vartheta} C_{\text{sca}}$ and the corresponding force acting upon the particle is given by

$$F = \frac{Q_{\text{ext}} - \overline{\cos\vartheta} Q_{\text{sca}}}{c} . I_0 . \pi d^2, \tag{5}$$

where I_0 = intensity of the incident light having a dimension $erg.cm^{-2}.sec^{-1}$, $Q = \frac{C}{\pi a^2}$,

$$\overline{\cos \vartheta} Q_{sca} = \frac{4}{x^2} \sum_{n=1}^{\infty} \frac{n(n+2)}{(n+1)} \mathrm{Re}(a_n a_{n+1} + b_n b_{n+1}) + $$

$$+ \frac{4}{x^2} \sum_{n=1}^{\infty} \frac{(2n+1}{n(n+1)} \mathrm{Re}(a_n b_n) \tag{6}$$

As has already been stated the values of Q and $\overline{\cos \vartheta}$ are listed in (IRVINE, 1963). Since the maximum of the energy emitted by star lies in the neighbourhood of 2000 Å we are to take on evaluating the values Q and $\overline{\cos \vartheta}$ for $x = 10$. As to I_0 we use the above mentioned energy emitted by the star ($1{,}2.10^{36}$ erg. sec^{-1}. $ster^{-1}$). Consequently, the amount of energy intercepted by 1 cm^2 of the particle is $\frac{1{,}2.10^{36}}{r^2}$ $erg.\ cm^{-2}.\ sec^{-1}$. The relation for F is then given by

$$F = \frac{Q_{ext} - \overline{\cos \vartheta} Q_{sca}}{c} . \pi a^2 . \frac{1{,}2.10^{36}}{r^2} \tag{7}$$

Every particle is influenced by the attraction force of the central star (if we neglect the influence of other particles) and by repulsive force due to radiation pressure. Both of these forces are indirect proportional to the square of distance. To answer the question whether the nebula is stabil or not, we are simply to compare the absolute value of $\frac{Q_{ext} - \cos \vartheta Q_{sca}}{c} . \pi a^2 . 1{,}2.10^{36}$ with the value of $m.M.G$, where m = mass of the particle, M = mass of the star and G = = gravitation constant. In our case we get:

$$\frac{0{,}78}{3.10^{10}} . 1{,}2.10^{36} . 33{,}4.10^{-10} = 10{,}4.10^{16}$$

due to radiation pressure and

$$435.10^{-15} . 6{,}31.10^{33} . 6{,}6.10^{-8} = 36{,}2.10^{13}$$

due to attraction force, where 6,31 is the ratio $\frac{M}{M_\odot}$ (ALLEN, 1955), 2.10^{33} g = = $M_\odot$. We can see that the force due to radiation pressure is about 3 orders larger than the attractive force. Consequently, there is a great probability that NGC 7023 is an unstable object. The corresponding acceleration of escape is then about $4{,}6.10^{-8}$ *cm. sec.*$^{-2}$. Hence it follows that the nebula gets twice as great linear dimension in about 10^6 years.

5. ALBEDO AND SHADOWING EFFECT

Due to radiation pressure shadowing effects cause an inverse — square-law attraction between the particles. This attraction force for two identical particles is given by

$$F = \frac{\pi^2 a^4}{4r^2} \int_0^\infty [1 - \gamma(\lambda)]\, Q^2(\lambda) U(\lambda)\, a\lambda\,, \tag{8}$$

where $Q(\lambda)$ is the ratio of the radiation scattered or absorbed to the radiation incident on the geometrical cross-section πa^2, $U(\lambda)$ denotes the energy density, γ = albedo, r = distance between the particles. A number of authors have been dealing with shadowing effects especially in connection with formation of stars. It is easily seen that albedo plays an important role in (8). Albedo, as judged by the brightness of reflection nebulae, seemed to be very high and the value of $[1-\gamma]$ in (8) becomes very small. For this reason, the idea that star formation could be caused by shadowing effects has fallen into disfavor. Recently Harwit (1962) re-examines the contribution of shadowing forces in star formation among others by stating that albedo for ultraviolet radiation is very likely low and the term $[1 - \gamma)$ in (8) is close to unity. This leads, after Harwit, to large shadowing forces and an increased likelihood to star formation. In general, shadowing effects could essentially influence the stability and geometrical configuration in our problem. We are therefore to extimate albedo for short-wave radiation (2000 Å). Albedo is given by

$$\gamma = \frac{\sum_{n=1}^{\infty} (2n + 1)\left(|a_n|^2 + |b_n|^2\right)}{\sum_{n=1}^{\infty} (2n + 1)\, \mathrm{Re}\, (a_n + b_n)} \tag{9}$$

From our calculations resulted (though the author used only 8 terms in (9)) that albedo practically is independent upon wave-lenght. Albedo for $\lambda = 2000$ Å is very near to unity (0,99) as well as for $\lambda = 4100$ Å. It seems in general that when dealing with dielectric particles, shadowing forces can be neglected.

To the end of this paragraph the basic data of NGC 7023 are listed:

NGC 7023

distance	280 ps (Bečvář 1959)
apparent radius	$9' \times 9'$
physical properties of particles	dielectric, refractive index = 2
radius of particles	$3{,}3 . 10^{-5}$ *cm*
number of particles in the nebula	$216 . 10^{43}$
density of the nebula	$4{,}5 . 10^{-11}$ cm^{-3}
mass of the neb.	$9{,}4 . 10^{32}$ *g*, i. e. 0,47 $M_\odot$
attraction force at a dist. 9′	$7{,}1 . 10^{-23}$ *dyn*
force due to radiation pressure	$2 . 10^{-20}$ *dyn*
acceleration of escape	$4{,}6 . 10^{-8}$ *cm*. sec^{-2}

6. CONCLUSION

The nebula NGC 7023 is practically the only suitable object of this kind enabling due to its geometrical configuration, surface-brightness etc. a more detailed investigations. Nevertheless a number of limiting assumptions are to be taken in account to treat the problem theoretically and to get at least an approximative idea about the physical constitution of particles in interstellar space.

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PEVNÉ ČÁSTICE V REFLEXNÍCH MLHOVINÁCH II.

Dielektrický model mlhoviny NGC 7023

(Souhrn)

Tato práce navazuje na předcházející práci V. Vanýska a autora (1964). Na základě selektivní vlastnosti rozptylu světla jsou v ní odvozeny další fyzikální parametry reflexní mlhoviny NGC 7023 (hustota, hmota, stabilita atd.) přispívající k celkové její fyzikální stavbě.

ТВЕРДЫЕ ЧАСТИЦЫ В ОТРАЖАТЕЛЬНЫХ ТУМАННОСТЯХ II.

(Резюме)

Эта работа является продолжением предыдущей работы (V. Vanýsek, J. Svatoš, 1964). На основании избирательных свойств рассеяния света были в работе получены дальнейшие физические параметры отражательной туманности NGC 7023 (масса, плотность, стабильность и. т. д.), которые определяют ее общую физическую структуру.