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PERIODICAL CHANGES IN THE ACTIVITY OF THE ENCKE COMET

PERIODICKÉ ZMĚNY V AKTIVITĚ KOMETY ENCKEOVY

ПЕРИОДИЧЕСКИЕ ИЗМЕНЕНИЯ АКТИВНОСТИ КОМЕТЫ ЭНКЕ

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1. INTRODUCTION

In papers [1, 2] the problem was solved of the dependence of certain physical characteristics of comets on the intensity of incident solar radiation; the material consisted of 563 comets. The resulting curves were analyzed on the assumption that their forms were not affected by the actual differences between the individual comets of the set. The extent to which this assumption is correct can be verified only by studying the same curves of one comet in which the following basic conditions must be fulfilled:

(a) a large enough number of measurements of a given characteristic of the comet must be available over a long enough period of time;

(b) the effects associated with the development and "ageing" of the comet must be dependably eliminated.

Only the Encke comet satisfies both these conditions. Physical data of more than 40 returns are at our disposal at present, and the secular changes of this comet are roughly continuous. The influence of solar activity on the Encke comet will be investigated in the following characteristics:

(A) in the fluctuation of the absolute brightness of the comet;

(B) in the fluctuation of the observed brightness dispersion of the comet;

(C) in the fluctuation of the linear diameter of the cometary head.

2. VARIATION OF THE ABSOLUTE BRIGHTNESS

For the study of the brightness variation of the Encke comet its absolute magnitudes were used as ascertained in 41 returns during 1819—1954. These magnitudes are included in Vsekhsviatsky's "Catalogue of Absolute Magnitudes of Comets" [3, 4].

The continuity of the secular decrease of the absolute brightness of this comet is evident from Fig. 1; therefore, the dependence of the absolute magnitude H_{10} on time may be expressed in the form of a progression. Let us limit it to a quadratic term:

$$H_{10}(t) = a + b\,\Delta t + c(\Delta t)^2, \tag{1}$$

$\Delta t = t - t_0$, t is the moment of observation, $t_0 = 1900.0$. If Δt is expressed in thousands of years, the coefficients of equation (1) are as follows:

$$\left.\begin{aligned} a &= +\, 9^m.98 \pm 0^m.06, \\ b &= +\, 29.4 \pm 1.4, \\ c &= +\, 109 \pm 32. \end{aligned}\right\} \quad (2)$$

The absolute magnitudes H_{10} obtained for each t from the parameters (2) are included in column C of Tab. 1. For each return of the comet its phase-shift referred to the preceding minimum of sunspot numbers, Φ, the observed absolute magnitude O, weight (see [3, 4]) w, and residual $O—C$ are also presented in Tab. 1.

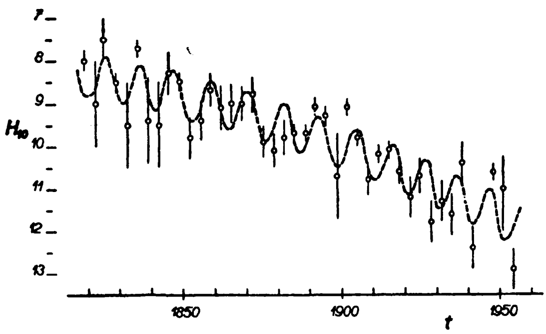

Fig. 1. Course of the absolute magnitude of the Encke comet.

In his paper Dobrovolsky [5] discusses the problem of the periodical fluctuation in the absolute magnitude of comets whose period of revolution is shorter than 10 years. He tries to prove that the ascertained changes are in no correlation either with the sunspot number R, or with Holetschek's criterion of the conditions of visibility of comets [6] ΔT, but that they are correlated with the so-called seasonal index which in principle characterizes the seasonal changes in the inclination of the zodiac to the horizon. Dobrovolsky introduced its values definitorically into a 0—6 scale; for comets with $q \leq 1$ A. U. its maximum value is in January, for those with $q > 1$ A. U., in July.

Dobrovolsky's study of the correlation mentioned above was based on the mere appearance of the curves so that it was rather subjective. Moreover, he did not eliminate the secular variations of the absolute magnitude, and he reffered the seasonal index to the moment of the perihelion passage instead of to the middle of the interval of observations. This caused the values of the seasonal index obtained by Dobrovolsky to be turned by $\pm$ 1 — 2 degrees, and in one case even by 5 degrees.

The correlation between the residuals $O - C$ on the one hand, and the sunspot number, the conditions of visibility and the seasonal index on the other, can be objectively established by determining the correlation coefficient ψ. If we denote the seasonal index for the perihelion passage and that for the middle of the observational interval dy n_p and n respectively, the following correlation coefficients are obtained from the data of Tab. 1 and those applying to the returns in 1786, 1795 and 1805:

$$\left.\begin{aligned} \psi[O—C, R] &= -0.39 \pm 0.09, \\ \psi\,[O—C, \Delta T] &= -0.36 \pm 0.09, \\ \psi[O—C, n_p] &= -0.18 \pm 0.10, \\ \psi[O—C, n] &= +0.08 \pm 0.10. \end{aligned}\right\} \quad (3)$$

Table 1

Residuals of the absolute magnitude of the comet Encke in individual returns

t	Φ	H_{10}				R	ΔT	n_p	n
		O	C	$O—C$	w				
		m	m	m					
1819.0	0.664	8.0	8.31	—0.31	2	28	+2	6	5
1822.5	0.943	9.0	8.36	+0.64	1	7	—6	2	1
1825.6	0.218	7.5	8.40	—0.90	2	22	+2.5	2	1
1828.9	0.525	8.5	8.44	+0.06	2	52	+4	6	4
1832.5	0.864	9.5	8.49	+1.01	1	27	—6	2	1
1835.6	0.174	7.7	8.54	—0.84	1	59	—2.5	1	0
1838.8	0.509	9.4	8.59	+0.81	2	81	+6	5	3
1842.3	0.870	9.5	8.65	+0.85	1	24	—3	3	4
1845.5	0.162	8.3	8.70	—0.40	2	31	—3	1	0
1848.8	0.421	8.5	8.76	—0.26	3	116	+5	4	3
1852.1	0.689	9.8	8.82	+0.98	1	68	0	4	5
1855.6	0.966	9.4	8.89	+0.51	2	2	—5	0	0
1858.7	0.240	8.7	8.95	—0.25	2	72	+4	3	2
1862.0	0.534	9.1	9.02	+0.08	2	66	+1	5	5
1865.3	0.832	9.0	9.09	—0.09	1	34	—6	2	3
1868.6	0.121	9.0	9.16	—0.16	2	33	+2	2	1
1871.8	0.396	8.8	9.24	—0.44	2	94	+6	5	3
1875.2	0.685	9.9	9.32	+0.58	2	26	—3	3	4
1878.6	0.978	10.1	9.40	+0.70	1	0	—4	0	1
1881.8	0.266	9.8	9.48	+0.32	2	58	+5	4	2
1885.1	0.582	9.7	9.56	+0.14	3	54	0	4	5
1888.6	0.906	9.7	9.66	+0.04	3	3	—5	1	1
1891.6	0.169	9.1	9.74	—0.64	3	49	+4	3	1
1895.0	0.442	9.3	9.83	—0.53	2	60	+1	5	5
1898.5	0.734	10.7	9.93	+0.77	1	18	—6	2	1
1901.6	0.995	9.1	10.03	—0.93	3	1	+2	2	1
1904.9	0.265	9.8	10.13	—0.33	3	46	+4	6	4
1908.4	0.565	10.8	10.23	+0.57	2	48	—6	2	1
1911.6	0.835	10.2	10.33	—0.13	3	4	—2	1	1
1914.8	0.118	10.1	10.44	—0.34	2	10	+6	5	3
1918.2	0.455	10.6	10.55	+0.05	2	78	—3	4	5
1921.6	0.800	11.2	10.66	+0.54	2	23	—5	0	1
1924.7	0.108	10.7	10.77	—0.07	2	23	+4.5	4	2
1928.1	0.437	11.8	10.89	+0.91	3	72	+1	5	6
1931.7	0.792	11.3	11.02	+0.28	2	16	—6	1	2
1934.6	0.076	11.6	11.13	+0.47	2	8	+2.5	2	1
1937.8	0.385	10.4	11.25	—0.85	2	100	+6	5	3
1941.2	0.711	12.4	11.38	+1.02	2	41	—3	3	4
1947.8	0.349	10.6	11.63	—1.03	3	159	+5	4	3
1951.1	0.675	11.0	11.77	—0.77	2	83	0	4	6
1954.1	0.973	12.9	11.89	+1.01	2	8	—5	0	5

The relations $O—C = f(R)$ and $O—C = f(\Delta T)$ are the only ones that come into consideration.

In paper [2] it has been shown that during the solar cycle the curves of the cometary characteristics of the second class reach the maximum values at $\Phi_0 = 0.25 \pm 0.01$ with the semiamplitude corresponding to about $0^m.5$. Thus, let us assume in the first approximation the following time course of the residuals $O—C$:

$$O—C = d.\cos 2\pi(\Phi - \Phi_0). \tag{4}$$

The amplitude of the fluctuations

$$d = -0^m.38 \pm 0^m.01$$

and the phase of the maximum

$$\Phi_0 = 0.248 \pm 0.005.$$

The corresponding correlation coefficient is

$$\psi[O-C, \cos 2\pi(\Phi - 0.25)] = -0.55 \pm 0.07. \qquad (5)$$

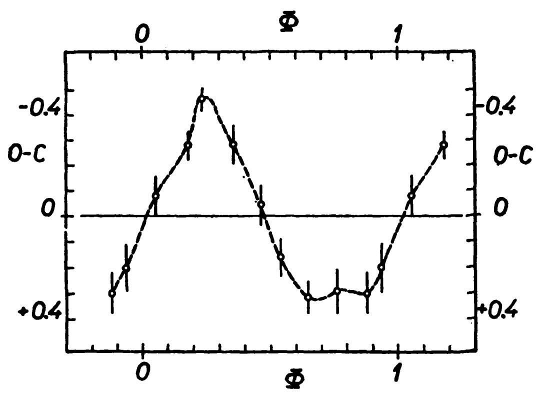

Fig. 2. Residuals $O-C$ as related to the phase of the solar cycle.

The form of the smoothed-out curve of the residuals $O-C$ is shown in Fig. 2. These results prove beyond any doubt that the character of the residuals $O-C$ is the same as that of the characteristics of the second class. Thus, the ascertain ed brightness fluctuations of the Encke comet can be interpreted as a sign of physical processes within the comet [7].

The analysis of these fluctuations has even a certain prognostic bearing. The general formula describing the dependence of the absolute brightness of the Encke comet on time has the following form:

$$H_{10}(t) = a + b\Delta t + c(\Delta t)^2 + d.\cos 2\pi(\Phi - \Phi_0), \qquad (6)$$

where the constants are equal to (Δt is again expressed in thousands of years):

$$\left.\begin{aligned} a &= +10^m.03 \pm 0^m.05, \\ b &= +28.5 \pm 1.2, \\ c &= +87 \pm 27, \\ d &= -0^m.50 \pm 0^m.05, \\ \Phi_0 &= 0.250 \pm 0.016. \end{aligned}\right\} \qquad (7)$$

Since formula (6) is only approximate its applicability for prognostic purposes is limited to about 2000.

3. FLUCTUATIONS OF THE DISPERSION OF THE OBSERVED BRIGHTNESS ESTIMATES

Beyer [8], and later Dobrovolsky [9], considered the average absolute values of the departures in the brightness estimates from the smoothed-out photometrical curve, given by the well-known parameters H_0 and n, a parameter characteristic of the observed brightness of a comet. This method though correct in principle, has certain disadvantages which were discussed in paper [10].

Nevertheless, let us apply this method to the comet Encke. The material used consisted of 114 brightness estimates from 19 returns of the comet before 1915, collected by Holetschek [11], 43 estimates from 1937 till 1951 carried out by Beyer [8], and 23 observations made during 1947, taken over from several Copenhagen Circulars [12].

Table 2

Average dispersions Δm of the comet Encke

t	Φ	Δm	w	R
		m		
1805.8	0.610	0.26	1	35
1825.6	0.218	0.30	2	22
1828.9	0.525	0.07	2	52
1838.8	0.509	0.16	2	81
1848.8	0.421	0.48	2	116
1852.1	0.689	0.31	1	68
1858.7	0.240	0.53	1	72
1862.0	0.534	0.69	1	66
1868.6	0.121	0.12	1	33
1871.8	0.396	0.46	2	94
1878.6	0.978	0.26	2	0
1881.8	0.266	0.25	2	58
1891.6	0,169	0.20	2	49
1895.0	0.442	0.62	3	60
1898.5	0.734	0.03	1	18
1901.6	0.995	0.42	2	1
1904.9	0.265	0.47	3	46
1914.8	0.118	0.41	1	10
1937.8	0.385	0.11	2	100
1947.8	0.349	0.32	3	159
1947.8	0.349	0.66	3	159
1951.1	0.675	0.07	3	83

The numerical results are listed in Tab. 2; the individual columns give: the middle of the interval of observations t, the phase-shift referred to the preceding minimum of solar activity Φ, the average dispersion of the brightness estimates Δm, its weight w, and the corresponding average sunspot number R.

Assuming the dependence $\Delta m = \Delta m(\Phi)$ in the form of

$$\Delta m = \overline{\Delta m} + A(\overline{\Delta m}) \cos 2\pi(\Phi - \Phi_0) \quad (8)$$

we obtain

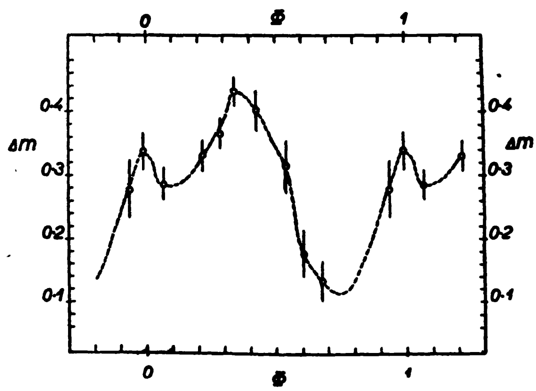

Fig. 3. Average dispersion Δm as related to the phase of the solar cycle.

$$\left.\begin{aligned} \overline{\Delta m} &= 0^m.29 \pm 0^m.02, \\ A(\Delta m) &= 0^m.14 \pm 0^m.03, \\ \Phi_0 &= 0.307 \pm 0.034, \end{aligned}\right\} \quad (9)$$

so that parameter Φ_0 is in good agreement with the parameters of the cometary characteristics of the second class. Nevertheless, the values (9) must be taken with great reserve owing to the defects of the method [10]. The curve $\Delta m = \Delta m(\Phi)$ is given in Fig. 3.

4. VARIATION OF THE COMETARY HEAD DIAMETER

On the basis of the material assembled by Bouška and Švestka [13], and supplemented by a few values taken by author from [4], the course of the coma diameter of the Encke comet is investigated, reduced to the unit of geocentric distance. From this material it follows that there takes place a gradual decrease of the coma diameter (Tab. 3) which may be assumed in the form analogous to (1):

$$D(t) = \alpha + \beta \Delta t + \gamma (\Delta t)^2, \quad (10)$$

where again $\Delta t = t - t_0$, $t_0 = 1900.0$. The individual coefficients are as follows (Δt is again expressed in thousands of years):

$$\left.\begin{aligned} \alpha &= +1'.90 \pm 0'.09, \\ \beta &= -13.0 \pm 2.4 \\ \gamma &= -30 \pm 56. \end{aligned}\right\} \quad (11)$$

The residuals are listed in Tab. 3. The individual columns give the designation of the Encke comet, the observed coma diameter, that computed according to (10), the residual $O—C$, and the number of observations N.

The correlation of the coma dimension residuals $O—C$ with the sunspot number, with Holetschek's criterion and Dobrovolsky's seasonal index is given by the following correlation coefficients:

$$\left.\begin{aligned} \psi[O—C, R] &= +0.25 \pm 0.11, \\ \psi[O—C, \Delta T] &= +0.27 \pm 0.11, \\ \psi[O—C, n] &= +0.19 \pm 0.12. \end{aligned}\right\} \quad (12)$$

Each of these coefficients is too low to indicate an actual degree of correlation; this fact is to a considerable extent due to the uncertainty of the coma dimension estimates. If the departures $O—C$ are again assumed in the form of a sine curve, the general expression of the course of the cometary head diameter

$$D(t) = \alpha + \beta \Delta t + \gamma (\Delta t)^2 + \delta \cos 2\pi(\Phi - \Phi_0), \quad (13)$$

where

$$\left.\begin{aligned} \alpha &= +1'.81 \pm 0'.09, \\ \beta &= -14.7 \pm 2.3, \\ \gamma &= -65 \pm 56, \\ \delta &= +0'.44 \pm 0'.10, \\ \Phi_0 &= \quad 0.323 \pm 0.034. \end{aligned}\right\} \quad (14)$$

Table 3

Residuals of the coma diameter of the comet Encke in individual returns

designation	D			N
	O	C	O—C	
	′	′	′	
1825 III	2.2	2.70	—0.50	2
1829	3.4	2.68	+0.72	7
1838	1.9	2.59	—0.69	8
1842 I	1.0	2.55	—1.55	1
1848 II	3.9	2.49	+1.41	5
1852 I	2.3	2.46	—0.16	1
1855 III	1.4	2.42	—1.02	6
1858 VIII	1.3	2.39	—1.09	4
1862 I	2.5	2.35	+0.15	20
1868 III	3.9	2.28	+1.62	5
1871 V	1.9	2.25	—0.35	2
1875 II	3.2	2.21	+0.99	3
1878 II	1.7	2.17	—0.47	2
1881 VII	2.9	2.13	+0.77	4
1885 I	1.5	2.09	—0.59	8
1888 II	1.3	2.05	—0.75	3
1891 III	1.7	2.01	—0.31	3
1895 I	2.6	1.97	+0.63	5
1898 III	1.2	1.92	—0.72	2
1901 II	2.0	1.88	+0.12	6
1905 I	2.3	1.84	+0.46	15
1908 I	1.0	1.79	—0.79	5
1914 VI	2.3	1.71	+0.59	4
1918 I	1.3	1.66	—0.36	3
1924 III	1.5	1.57	—0.07	2
1928 II	1.0	1.52	—0.52	5
1934 III	0.5	1.42	—0.92	1
1937 VI	1.7	1.37	+0.33	9
1941 V	0.9	1.32	—0.42	3
1947 XI	1.8	1.22	+0.58	3

The difference between the value of the phase-shift Φ_0 and the values derived in another way is not great enough to exclude the identity of the character of these fluctuations with the cometary characteristics of the second class.

5. CONCLUSIONS

(1) The time course of the absolute brightness of the comet Encke may be split into two superimposed curves: the secular decrease, and the periodical fluctuations of a period equal to the length of the eleven-year solar cycle. As to the amplitude and the phase-shift referred to the sunspot-number curve, the curve of the departures *O—C* perfectly agrees with the curves of the cometary characteristics of the second class [2]. There is no doubt that the character of both quantities is the same.

(2) In addition, a relatively high degree of correlation makes it possible to apply the general formula describing the absolute magnitude variation for prognostic purposes.

(3) Beyer's method of the observed brightness-estimation dispersion applied

to 21 returns of the comet gives a course of the Δm-dependence on the phase of the solar cycle that is very similar to that of the cometary characteristics of the second class.

(4) The variation of the coma diameter analogous to that given under (1) is only intimated; it is obvious that the ascertained form of the course of the comet-head dimensions is affected by the conditions of visibility, and by other effects resulting from the non-homogeneity of the material.

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Souhrn

V práci jsou sledovány periodické změny v jasnosti a rozměrech hlavy komety Enckeovy. Po eliminaci sekulárních změn se dospívá k závěru, že absolutní jasnost této komety vykazuje během slunečního cyklu velmi podobný chod jako tzv. kometární charakteristiky druhé třídy [2]. Podobně je tomu v případě lineárního průměru hlavy této komety, i když korelace není zdaleka tak výrazná. Aplikace Beyerovy metody odchylek pozorovaných jasností od proložené fotometrické křivky ukazuje, že charakter této disperse rovněž odpovídá kometárním charakteristikám druhé třídy, takže jí lze interpretovat podle [7].

Резюме

В настоящей работе исследованы периодические колебания блеска и размеров головы кометы Энке. После исключения секулярного падения приходится к заключению, что изменения абсолютного блеска этой кометы в течение одиннадцатилетнего солнечного цикла очень похожи на ход так называемых кометных характеристик второго класса [2]. Подобно тому также в случае линейного диаметра головы этой кометы, хотя корреляция уже не так высока. Применение метода отклонений оценок наблюдаемого блеска от фотометрической кривой Бейера показывает, что характер этой дисперсии в течение солнечного цикла также соответствует характеру кометных характеристик второго класса, так что она может быть интерпретирована согласно [7].