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SHORT-TERM CHANGES IN THE COLOUR INDEX OF THE HEAD AND TAIL OF THE COMET AREND—ROLAND 1956h

KRÁTKODOBÉ ZMĚNY BAREVNÉHO INDEXU HLAVY A CHVOSTU KOMETY AREND—ROLAND 1956h

КОРОТКОВРЕМЕННЫЕ ИЗМЕНЕНИЯ ПОКАЗАТЕЛЯ ЦВЕТА В ГОЛОВЕ И ХВОСТЕ КОМЕТЫ АРЕНДА—РОЛАНА 1956h

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1. PROBLEM.

One of the most valuable measurements of the comet 1956h carried out at our scientific institutes was the determination of the photoelectric magnitudes of its head (diaphragm 4′) and tail at the distance of about 30′ from the nucleus. This determination resulted from observations performed at the University Observatory in Brno, and the obtained results were published by Vanýsek and Tremko [1]. The measurements were made in three different effective wave-lengths included in Table 1.

Table 1

Spectral ranges used for photoelectric photometry of the comet

magnitude	λ_{eff}	filter
B	4420 Å	BG12 (1 mm) + + GG13 (2 mm)
V	5510 Å	GG11 (2 mm)
P	4800 Å	without filter

The observations extended over the period from April 27th till May 30th, 1957. For our purposes were used 10 measurements of the brightness of the head, and 3 measurements of that of the tail, from which all three colour indices $(B - P)$, $(P - V)$ and $(B - V)$ were computed (however, only two of them are independent). The indices are listed in Table 2. If plotted one against the other they give the relations presented in Fig. 1. The full circles stand for values related to the cometary head, the open ones for those related to the tail. The number at each circle indicates the date of observation. The

Table 2
List of colour-index measurements

Date	(B — P)	((P — V)	(B — V)	region
	m	m	m	
1957 IV. 27.819	+0.74	+0.07	+0.81	head
27.837	+0.86	—0.14	+0.72	head
27.844	+0.78	—0.08	+0.70	tail
29.860	+0.47	—0.26	+0.21	head
29.872	+0.32	+0.06	+0.38	head
29.879	+0.41	—0.04	+0.37	head
V. 2.930	—0.02	+0.06	+0.04	head
2.936	+0.13	—0.38	—0.25	tail
2.960	—0.15	+0.34	+0.19	head
4.854	+0.47	+0.39	+0.86	head
4.863	+0.41	+0.45	+0.86	head
25.923	+1.02	—0.37	+0.65	head
25.948	+0.42	+0.42	+0.84	tail

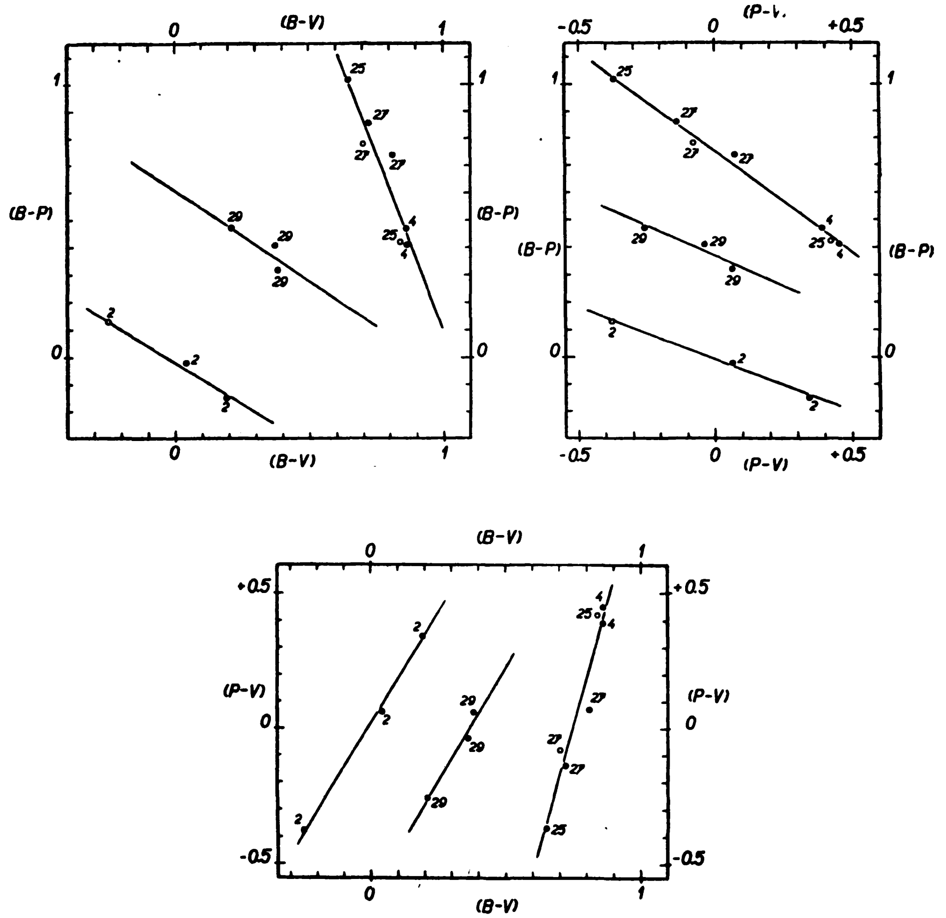

Fig. 1. Relations between three colour indices of the head and tail of the comet 1956h.

values of the indices in Fig. 1 are accumulated along three straight lines, the first of which corresponds to April 27th, May 4th and May 25th, the second to April 29th and the third to May 2nd. Furthermore, the interpretation is given of the relations ascertained between the individual indices.

2. INTERPRETATION OF THE VARIATION IN THE COLOUR INDICES $(B - P)$, $(P - V)$ AND $(B - V)$

Let us proceed from the conception that the dust-gas model of a comet applies to each of the three spectral ranges, i. e. let us assume that both molecular radiation and reflection on the dust particles of the coma and tail take place in each of studied ranges. This assumption is supported by polarization measurements in the integral light [2], and by the results of spectral analysis in the individual spectral regions [3].

If we denote the brightness of the comet in the unit of geocentric distance as I_Δ, further,

$$B = \frac{L}{RT_o},$$

L and T_o are the well-known parameters of Levin's formula [4], I_{od} and I_{og} the absolute brightness of the dust- and gas radiation- constituent respectively, and $\eta_d = \eta_d(r)$ the exponent expressing the variation of the dust-part brightness, then the fundamental equation giving the course of the total brightness has the following form:

$$I_\Delta = I_{og} . r^{-1/4} e^{B(1 - \sqrt{r})} + I_{od} r^{-\eta_d}. \quad (1)$$

By inserting into (1)

$$k = \frac{I_{od}}{I_{og}} \quad (2)$$

as the expression for the ratio of the absolute brightnesses of both radiation constituents, and

$$\Psi(r) = k . r^{1/4 - \eta_d} . e^{B(\sqrt{r} - 1)} \quad (3)$$

as that for the ratio of the brightnesses of both constituents in a given heliocentric distance, and by transcribing (1) into the magnitude scale we obtain

$$H_\Delta = -2.5 \log I_{og} + 0.625 \log r + 1.086\ B\left(\sqrt{r} - 1\right) - 2.5 \log (1 + \Psi). \quad (4)$$

If this equation is written for two effective wave-lengths λ_i, λ_j, we obtain — after subtraction of the latter from the former — the colour index $(CI)_{ij} = H_\Delta^{(i)} - H_\Delta^{(j)}$:

$$(CI)_{ij} = 2.5 \log \frac{I_{og}^{(j)}}{I_{og}^{(i)}} + 1.086\left(1 - \sqrt{r}\right)[B^{(j)} - B^{(i)}] + 2.5 \log \frac{1 + \Psi^{(j)}}{1 + \Psi^{(i)}}. \quad (5)$$

An analogous equation may be written for the wave-lengths λ_k, λ_l:

$$(CI)_{kl} = 2.5 \log \frac{I_{og}^{(l)}}{I_{og}^{(k)}} + 1.086\left(1 - \sqrt{r}\right)[B^{(l)} - B^{(k)}] + 2.5 \log \frac{1 + \Psi^{(l)}}{1 + \Psi^{(k)}}. \quad (6)$$

If factor 1.086 $(1-\sqrt{r})$ is eliminated, the relation between both indices can be written in the form:

$$(CI)_{ij} = \alpha_m + \beta_m (CI)_{kl}, \tag{7}$$

where

$$\alpha_m = 2.5\left[\log \frac{I_{og}^{(j)}(1+\Psi^{(j)})}{I_{og}^{(i)}(1+\Psi^{(i)})} - \beta_m \log \frac{I_{og}^{(l)}(1+\Psi^{(l)})}{I_{og}^{(k)}(1+\Psi^{(k)})}\right] \tag{8}$$

and

$$\beta_m = \frac{B^{(j)} - B^{(i)}}{B^{(l)} - B^{(k)}}. \tag{9}$$

Let us denote

$$(CI)_{12} \equiv (B-P), \quad (CI)_{13} \equiv (B-V), \quad (CI)_{23} \equiv (P-V),$$

so that the indices i, j, k, l pass through 1, 2, 3, and m is equal to that of the indices i, j, k, l, which occurs twice in relation (7).

3. RESULTS AND CONCLUSIONS

The parameters of the straight lines in Fig. 1 are listed in Tab. 3. Since the indices $(B-P)$, $(P-V)$ and $(B-V)$ are interrelated by three equations

Table 3

Parameters of the colour index relations

Date 1957	$(B-V) \sim (B-P)$		$(P-V) \sim (B-P)$		$(B-V) \sim (P-V)$	
	α_1	β_1	α_2	β_2	α_3	β_3
	m m		m m		m m	
IV. 27 + V. 4 + V. 25	$+2.70 \pm 0.24$	-2.61 ± 0.31	$+0.75 \pm 0.01$	-0.74 ± 0.02	-2.70 ± 0.24	$+3.61 \pm 0.31$
IV. 29	$+0.61 \pm 0.10$	-0.66 ± 0.30	$+0.37 \pm 0.02$	-0.44 ± 0.10	-0.61 ± 0.10	$+1.66 \pm 0.30$
V. 2	-0.02 ± 0.01	-0.62 ± 0.06	-0.01 ± 0.01	-0.38 ± 0.02	$+0.02 \pm 0.01$	$+1.62 \pm 0.06$

of the form (7), the following five relations must apply to α_m and β_m, if our interpretation is correct:

$$\left.\begin{aligned}
A_0(\beta_m) &= \beta_2\beta_3 - \beta_1 \equiv 0,\\
A_1(\beta_m) &= \frac{1}{\beta_3} - \beta_2 - 1 \equiv 0,\\
A_2(\beta_m) &= \beta_1 + \beta_3 - 1 \equiv 0,\\
A_3(\alpha_m) &= \alpha_1 + \alpha_3 \equiv 0,\\
A_4(\alpha_m, \beta_m) &= \alpha_1(1+\beta_2) - \alpha_2 \equiv 0.
\end{aligned}\right\} \tag{10}$$

The values listed in Table 4 were for the parameters (10) obtained from the material given in Table 3. It is obvious that all empirical values $A_0, \ldots, A_4$ range very closely about zero, and that all ascertained differences lie within the limits of errors.

Thus, the obtained results support the interpretation of the experimental relations given by equation (7). The variability of the colour index can be regarded as a manifestation of the fluctuations in the amount of gas and dust

Table 4

Empirical values of identities $A_p(\alpha_m, \beta_m) \equiv 0$

Date 1957	IV. 27+ V. 4+ V. 25	IV. 29	V. 2
A_0	—0.06 ± 0.39	—0.07 ± 0.37	0.00 ± 0.07
A_1	+0.02 ± 0.03	+0.04 ± 0.15	0.00 ± 0.03
A_2	0.00 ± 0.44	0.00 ± 0.42	0.00 ± 0.08
A_3	0.00 ± 0.34	0.00 ± 0.14	0.00 ± 0.01
A_4	—0.05 ± 0.08	—0.03 ± 0.08	0.00 ± 0.01

in the head and tail of the comet. These fluctuations are also reflected in the numerical values of the parameters α_m, β_m, which are not constant, not even during a single night. However, since the characteristics of both radiation constituents in the individual spectral ranges appear in the coefficients α_m, β_m in the form of a ratio, the above-mentioned fluctuations are minimum. Moreover, the changes in the colour index during one night may be also affected by the shift of the photometer diaphragm with respect to the cometary nucleus, if several measurements are carried out in succession. Owing to this effect, the physical conditions are registered in a few somewhat different parts of the coma, in which the momentary ratio of both radiation constituents may differ quite considerably.

Parameter β_m, which is a function of the heat of evaporation necessary to liberate a certain amount of gas, is probably related to the initial velocity v_0 in the cometary tail. A comparison of one of the parameters β_m with v_0 derived in paper [5] from the width of the tail (see Fig. 3 of [5]) is given in Table 5.

Table 5

Comparison of the variability of parameter β_2 with the initial particle velocity in the cometary tail

Date 1957	β_2	v_0 km/s
IV. 27.9	—0.74 ± 0.02	12.0 ± 0.5
IV. 29.9	—0.44 ± 0.10	6.8 ± 0.8
V. 2.9	—0.38 ± 0.02	7.3 ± 0.5
V. 4.9	—0.74 ± 0.02	>9.0

For want of comparable date this relation cannot be verified more in detail. Parameter α_m quantitatively demonstrates the amount of gas and dust ejected into the cometary head- and tail- regions. Besides the small number of measurements, it is a range of unknown quantities in the expression for α_m that prejudices the analysis of this problem. Thus, the results obtained so far are, as a rule, of a qualitative character. The main result of the present paper is the finding that problems of multicoloured cometary photometry may be solved on the basis of the dust-gas model of a comet.

REFERENCES

[1] V. Vanýsek, J. Tremko: Publ. Astr. Inst. Czech. Acad. Sci. No 34 (1958), 59.
[2] M. Blaha, A. Hruška, Z. Švestka, V. Vanýsek: Publ. Astr. Inst. Czech. Acad. Sci. No 34 (1958), 71.
[3] J. Rajchl: Publ. Astr. Inst. Czech. Acad. Sci. No 34 (1958), 83.
[4] B. J. Levin: DAN 38 (1943), 82; AJ UdSSR 20 (1943), 37.
[5] V. Vanýsek, J. Grygar, Z. Sekanina: BAC 10 (1959), 116.

Souhrn

V práci jsou zkoumány změny barevného indexu hlavy a chvostu komety Arend-Roland 1956h v období mezi 27. IV. a 25. V. 1957. Je použito fotoelektrických měření jasnosti ve třech spektrálních oborech s efektivními vlnovými délkami: *B* - 4420 Å, *P* - 4800 Å a *V* - 5510 Å. Z materiálu vyplývá lineární vztah mezi libovolnými dvěma z indexů (*B* — *P*), (*P* — *V*) a (*B* — *V*). Parametry těchto přímek se mění den ode dne. Jsou určovány numerické hodnoty těchto parametrů a na základě pracho-plynového modelu komety je podána i interpretace nalezeného vztahu. V závěru je poukázáno i na možnost souvislosti sklonu nalezených přímek s počáteční rychlostí částic prachu a molekul plynu vyvržených do oblasti chvostu komety.

Резюме

В работе изучаются изменения показателя цвета в голове и хвосте кометы Аренда-Ролана 1956h с 27 апреля 1957 до 25 мая 1957. Используются фотоэлектрические измерения блеска в трех областях спектра с ответствующими длинами волн: *B* — 4420 A, *P* — 4800 Å, *V* — 5510 Å. На основании обработки материала оказывается, что между любыми двумя из индексов (*B*—*P*), (*P*—*V*) и (*B*—*V*) существует линейная зависимость. Параметры этих прямых изменяются с каждым днем. Определяются численные значения этих параметров и на основании пыле-газовой модели кометы приведена интерпретация обнаруженного отношения. В заключение показывается на возможность связи наклона прямых с начальной скоростью пылевых частиц и газовых молекул изверглых в область хвоста кометы.