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A STOCHASTIC PROGRAMMING APPROACH TO MANAGING LIQUID ASSET PORTFOLIOS

Helgard Raubenheimer and Machiel F. Kruger

Maintaining liquid asset portfolios involves a high carry cost and is mandatory by law for most financial institutions. Taking this into account a financial institution's aim is to manage a liquid asset portfolio in an "optimal" way, such that it keeps the minimum required liquid assets to comply with regulations. In this paper we propose a multi-stage dynamic stochastic programming model for liquid asset portfolio management. The model allows for portfolio rebalancing decisions over a multi-period horizon, as well as for flexible risk management decisions, such as reinvesting coupons, at intermediate time steps. We show how our problem closely relates to insurance products with guarantees and utilize this in the formulation.

We will discuss our formulation and implementation of a multi-stage stochastic programming model that minimizes the down-side risk of these portfolios. The model is back-tested on real market data over a period of two years

Keywords: stochastic programming, portfolio optimization, liquid assets

Classification: 90C15, 97M30, 91G80

1. INTRODUCTION

Banks are deposit-taking institutes and when a run on the bank occurs, due to any adverse movements of a risk factor, the bank needs to have enough liquid assets to meet public demand. The South African Banks Act (see [9]) and Regulations Relating to Banks (see [10]) protects the public by requiring banks to keep a minimum amount in liquid assets. Liquid assets are assets which are easily redeemable for cash, and are defined in Section 1 of the Banks Act (e. g. Gold coin and bullion; Treasury bills of the RSA; Government bonds; and Securities of the SARB).

Section 72 of the Banks Act (see [9]) stipulate, that a bank shall hold liquid assets with respect to the value of its liabilities as may be specified by regulations. Regulation 20 of the Regulation Relating to banks (see [10]), requires a bank to hold over a period of one month an average daily amount of liquid assets equal to no less than 5 % of its reduced liabilities. For this purpose a bank needs to keep a *statutory portfolio*, also called a *liquid asset portfolio.*

The liquid assets that are available for inclusion into the liquid asset portfolio are interest rate sensitive, low (credit) risk financial instruments. Having a low risk implies having a small return, thus maintaining a liquid asset portfolio involves a high carry cost, and makes it a expensive portfolio to hold. However, the portfolio is mandatory and taking into account the high carry cost of the portfolio, the bank's aim is to manage the liquid asset portfolio in an "optimal" way, such that it keeps the minimum required liquid assets to comply with regulations, whilst maximizing the portfolio return to cover at least the carry cost.

To manage this portfolio in an "optimal" way the bank will need to rebalance or change the composition of the portfolio on a regular basis. Changing the portfolio composition will depend on certain aspects such as expert views on risk factors movements, legislation and regulations. With these legislation and regulations to adhere to and uncertainties to consider, the liquid asset portfolio management problem can be described as a multi-stage decision problem in which portfolio rebalancing actions are taken at successive future discrete time points. At each decision period the portfolio manager needs to decide which assets to buy, which to sell and which to hold.

The aim of the manager of the liquid asset portfolio, is to identify and minimize risks by analyzing the market, legislation, portfolio data, and many other factors. This paper will investigate the use of stochastic programming in addressing all of these aspects in a realistic way. Examples of the use of dynamic portfolio optimization models for asset and liability management are [6] and [7]. A large variety of compilations on the application and implementation of dynamic stochastic programming in the area of finance are available in [11, 12] and [4] amongst others. [8] presents the newest results in stochastic programming theory and its applications.

We will discuss the formulation and implementation of the multi-stage stochastic programming model. The model is back-tested on real market data over a period of two years.

2. SCENARIO OPTIMIZATION FRAMEWORK

In this section we discuss the formulation of a multi-stage stochastic programming model that minimizes the down-side risk of liquid asset portfolios. The liquid assets that are available for inclusion into the liquid asset portfolio are interest rate sensitive, low (credit) risk financial instruments. Having a low risk implies having a small return, so keeping the portfolio is mostly unprofitable. The portfolio is funded by a pool of funds with a cost equivalent to the bank's interdivisional borrowing rate, i. e. the portfolio is funded internally by the bank. Thus it can be seen that the department responsible for keeping the liquid asset portfolio borrows funds from other divisions in the bank in order to fund the portfolio. The capital that may have been invested in other securities can now only be invested in liquid assets. Thus maintaining this portfolio involves a high carry cost.

Furthermore the value of the liquid asset portfolio needs to be at least the value of the liquid asset requirement. The liquid asset requirement as mentioned in the introduction depends on the liabilities of the bank. This requirement grows over time as the bank's business grows, i. e. as the bank takes on more clients with deposit facilities, the amount of liabilities increase and thus the liquid asset requirement increases. Thus the funds invested by the bank for the purpose of meeting the liquid asset requirement, needs to have a growth rate of at least the rate at which the bank's liabilities are growing.

In this light our liquid asset problem can be seen as an minimum guarantee problem. Where the bank can be seen as a client investing an upfront amount of money, equal to the liquid asset requirement, into a guarantee fund, where the guaranteed rate of return is equal to the growth rate of the liabilities of the bank. [3] propose such an asset and liability management framework for a simple example of a closed-end guaranteed fund where no contributions are allowed after the initial cash outlay. They demonstrate the design of investment products with a guaranteed minimum rate of return focusing on the liability side of the product. We propose a similar framework for the asset and liability management of the liquid asset portfolio problem. Other than [3] we will assume that any shortfall in the portfolio will be funded by the bank, and that the funds provided by the bank for the purpose of the liquid asset requirement will carry some sort of liability payment equal to the interdivisional borrowing rate.

In the next subsections we will discuss the model features, the model variables and parameters and its dynamics together with its constraints. We will also discuss the objective function.

2.1. Model features

We investigate the optimal asset allocation of a bank's liquid asset portfolio. It is assumed that the portfolio will be funded upfront by the bank and this will be assumed to be a liability for the portfolio. This liability will grow at the maximum of the one month zero-rate, (assumed to be the interdivisional borrowing rate) and the minimum liquid asset growth rate, throughout the lifetime of the portfolio and is payable at the end of the horizon. By doing so and by minimizing the shortfall, which is defined to be the difference between the liabilities and the assets, we will ensure that the assets will cover at least the maximum of the minimum liquid requirement or the liabilities at the cost of the interdivisional borrowing rate. For budgeting purposes, banks will prefer to rebalance the portfolio quarterly or longer time period. But to keep the model flexible and tractable, coupon payments will be taken into account on a monthly basis. Any shortfall in the portfolio, at rebalancing, will be funded by the bank. This is to ensure that the minimum liquid requirement is achieved. The shortfall payments will accrue to a shortfall fund separate from the liabilities, which will grow at the one month zero-rate. The time horizon of the portfolio is T years and we will use three asset classes as liquid assets namely, (semi-annual) coupon bearing government bonds, gold and three and six month treasury bills.

We simulate the future yield curves to construct a scenario tree. A scenario tree is a discrete approximation of the joint distribution of random factors (yield curve and gold prices). To facilitate the mathematical formulation of the optimization problem, we represent the scenario tree in terms of states (nodes) $s_t^{v(t)}$, where time $t = 0, \frac{1}{12}, \frac{2}{12}, \ldots, T$ and $v(t) = 0, 1, 2, \ldots, N_t$ the numbers of the states at time t. The set of states at time t are denoted by $\Sigma_t = \left\{ s_t^{v(t)} | v(t) = 0, 1, \ldots, N_t \right\}$, where

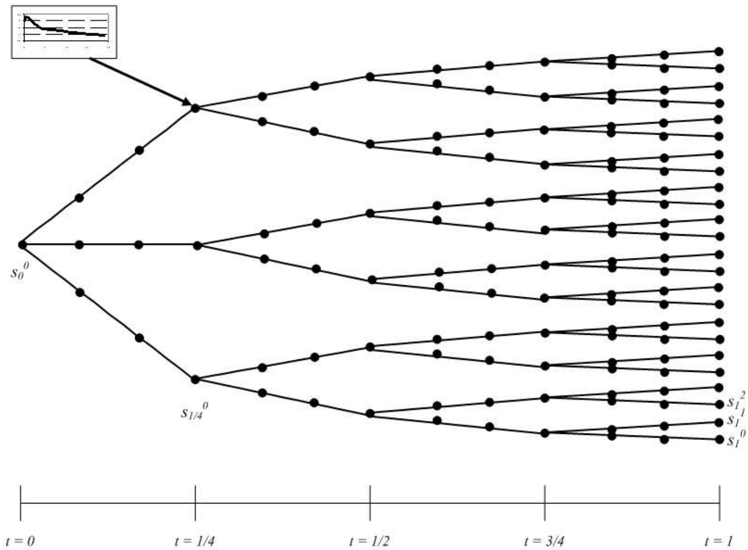

Fig. 1. Graphical representation of a yield curve scenario tree.

$\Sigma_0 = \left\{s_0^0\right\}$. The set of all states in the scenario tree is denoted by $\Sigma = \cup_{t=0}^{T} \Sigma_t$. Links $\varepsilon \in \Sigma \times \Sigma$, indicate the possible transitions between states. To enforce non-anticipativity, i. e. to prevent foresight of uncertain future events, we order the elements of ε in pairs $\left(s_t^{v(t)}, s_{t+1}^{v(t+1)}\right)$ where the dependence of the index $v\,(t)$ on t is explicitly indicated. The order of the states indicates that state $s_{t+1}^{v(t+1)}$ at time $t+1$ can be reached from state $s_t^{v(t)}$ at time t. $s_{t+1}^{v(t+1)}$ is the successor state and $s_t^{v(t)}$ the predecessor state. By using the superscript "+" to denote the successor states, and the superscript "-" to denote the predecessors, we have $s_t^{v(t)+} = s_{t+1}^{v(t+1)}$ and $s_{t+1}^{v(t+1)-} = s_t^{v(t)}$. Each state $s_t^{v(t)}$ has an associated probability p_t^s, for $s \in \Sigma_t$, such that $\sum_{s \in \Sigma_t} p_t^s = 1$.

Certain times $t_d = 0, \frac{1}{4}, \frac{1}{2}, \ldots, T - \frac{1}{4}$ correspond to the quarterly *decision times* at which the portfolio will trade to rebalance. We represent the branching of the tree structure with a tree-string, which is a string of integers specifying for each decision time t_d the number of branches for each node in states Σ_d. This specification gives rise to a balanced scenario tree where each sub-tree in the same period has the same number of branches. Figure 1 gives an example of a scenario tree with a (3,2,2,2) tree-string, giving a total of 24 scenarios.

2.2. Model variables and parameters

The following notation will be used for variables and parameters of the model, where time is index t takes values over the times $t = 0, \frac{1}{12}, \frac{2}{12}, \ldots, T$, and states index s from the set $\Sigma_t = \left\{s_t^{v(t)} | v\,(t) = 0, 1, \ldots, N_t\right\}$ at time t:

Time sets

$T^{\text{total}} = \left\{0, \frac{1}{12}, \frac{2}{12}, \ldots, T\right\}$: set of all times considered;
$T^{d} = \left\{0, \frac{1}{4}, \frac{1}{2}, \ldots, T - \frac{1}{4}\right\}$: set of decision times;
$T^{i} = T^{\text{total}} \backslash T^{d}$: set of intermediate times;

Index sets

$B = \{B_\tau\}$: set of government bonds with maturity denoted by τ;
GZ : set of gold prices;
$M3$: set of 3 month treasury bills;
$M6 = \{M6_{3M}, M6_{6M}\}$: set of 6 month treasury bills with time to maturity;
$I = B \cup GZ \cup M3 \cup M6$: set of all instruments;

Parameters

F_{B_τ} : face value of a government bond with maturity τ;
$F_{M3 \cup M6}$: face value of a treasury bills;
$r^s_{t,\tau}$: zero-rate with maturity τ at time t in state s;
g^s_t : minimum liquid asset growth rate at time t in state s;
$mg^s_t = \max\left(g^s_t, r^s_{t,\frac{1}{12}}\right)$: liability growth rate at time t in state s;
$P^{a,s}_{t,i} / P^{b,s}_{t,i}$: ask or bid price of asset $i \in I$ at time t in state s;
f_a / f_b : proportional transaction costs on ask or bid transactions;
p^s_t : probability of state s at time t;
L_0 : initial liability at the root node;
$z_0 = \{z_{0,i}\}_{i \in I}$: initial quantities of assets at the root node;

Decision variables

$x^s_t = \left\{x^s_{t,i}\right\}_{i \in I}$: quantities of assets *bought* at time t in state s;
$y^s_t = \left\{y^s_{t,i}\right\}_{i \in I}$: quantities of assets *sold* at time t in state s;
$z^s_t = \left\{z^s_{t,i}\right\}_{i \in I}$: quantities of assets *held* at time t in state s from time t to $t + \frac{1}{12}$;
W^s_t : value of portfolio wealth at time t in state s;
L^s_t : value of liability account at time t in state s;
EF^s_t : value of the shortfall fund at time t in state s;
c^s_t : amount of extra cash provided at time t in state s;
SF^s_t : amount of shortfall at time t in state s;

2.3. Variable dynamics and constraints

The variable dynamics and constraints for the minimum liquid asset problem are:

Cash balance constraints. We assume all bonds to pay semi-annual coupons at rate δ_{B_τ} at coupon payment times. $D^s_{t,B_\tau} = 1$, if a coupon is due at time t at state s and $D^s_{t,B\tau} = 0$ otherwise. The cash balance constraints ensure that the amount of cash that is received, from selling assets, coupon payments at decision times and extra cash supplied for shortfall, is equal to the amount of assets bought:

$$\sum\nolimits_{i \in I} P^{b,s}_{t,i} y^s_{t,i} (1 - f_b) + \sum\nolimits_{i \in B} D^s_{t,i} \frac{1}{2} \delta_i F_i z_{0,i} + c^s_t = \sum\nolimits_{i \in I} P^{a,s}_{t,i} x^s_{t,i} (1 + f_a),$$

$$\text{for } t \in \{0\} \text{ and } s \in \Sigma_t,$$

$$\sum\nolimits_{i\in I} P^{b,s}_{t,i} y^s_{t,i} (1-f_b) + \sum\nolimits_{i\in B} D^s_{t,i}\frac{1}{2}\delta_i F_i z^{s-}_{t-\frac{1}{12},i} + c^s_t = \sum\nolimits_{i\in I} P^{a,s}_{t,i} x^s_{t,i} (1+f_a),$$
$$\text{for } t\in T^d \setminus\{0\} \text{ and } s\in\Sigma_t.$$

Short sale constraints. The short sale constraints eliminate the possibility of short-selling assets in each state at each time period:

$$x^s_{t,i} \ge 0,\ y^s_{t,i} \ge 0, z^s_{t,i} \ge 0,$$
$$\text{for all } i\in I,\ t\in T^{\text{total}}\setminus\{T\} \text{ and } s\in\Sigma_t,.$$

Inventory constraints. The inventory constraints give the quantity invested in each asset in each state at each time period. The inventory constraints for bonds, gold and the three month treasury bills are straight forward, as the quantity of assets that are held for the next time period equals the quantity that was held in the previous time period plus the quantity bought minus the quantity sold:

$$z^s_{t,i} = z_{0,i} + x^s_{t,i} - y^s_{t,i},\ \text{for } i\in I\setminus\{M6\},\ t\in\{0\} \text{ and } s\in\Sigma_t$$

$$z^s_{t,i} = z^{s-}_{t-\frac{1}{12},i} + x^s_{t,i} - y^s_{t,i},\ \text{for } i\in I\setminus\{M6\},\ t\in T^{\text{total}}\setminus\{0\} \text{ and } s\in\Sigma_{.t}$$

Treasury bills are held to maturity and we assume that only new treasury bills will be bought. Due to the quarterly rebalancing of the portfolio, the six month treasury bills are split into two sets. The first set contains six month treasury bills which have time to maturity of six months on the previous decision time and the second set contains six month treasury bills which have time to maturity of three months on the previous decision time.

The inventory constraints for the six month treasury bills with time to maturity of three months are:

$$z^s_{t,i} = z_{0,j},\ \text{for } i\in\{M6_{3M}\},\ j\in\{M6_{6M}\},\ t\in\{0\} \text{ and } s\in\Sigma_t,$$

$$z^s_{t,i} = z^{s-}_{t-\frac{1}{12},j},\ \text{for } i\in\{M6_{3M}\},\ j\in\{M6_{6M}\},\ t\in T^d\setminus\{0\} \text{ and } s\in\Sigma_t.$$

Furthermore the inventory constraints for the six month treasury bills with time to maturity of six months are:

$$z^s_{t,i} = x^s_{t,i},\ \text{for } i\in\{M6_{6M}\},\ t\in\{0\} \text{ and } s\in\Sigma_t,$$

$$z^s_{t,i} = z^{s-}_{t-\frac{1}{12},i},\ \text{for } i\in\{M6_{6M}\},\ t\in T^d\setminus\{0\} \text{ and } s\in\Sigma_t.$$

This enables us to distinguish between six month treasury bills that mature at the current decision time an those that only mature at the next decision time. For intermediate time periods the inventory constraints are the same as for the rest of the instruments:

$$z^s_{t,i} = z^{s-}_{t-\frac{1}{12},i} + x^s_{t,i} - y^s_{t,i},\ \text{for } i\in M6,\ t\in T^i \text{ and } s\in\Sigma_t.$$

Information constraints. As the portfolio is only rebalanced at decision times, the information constraints ensure that the portfolio can not be changed between decision times:

$$x^s_{t,i} = y^s_{t,i} = 0 \text{ for } i \in I \backslash \{B\}, \ t \in T^i \text{ and } s \in \Sigma_t.$$

The information constraints for bonds are included in the coupon reinvestment constraints.

Coupon reinvestment constraints. The coupon reinvestment constraints ensure that the coupons that are paid at the coupon times are reinvested in the same coupon bearing bonds:

$$x^s_{t,i} = \frac{D^s_{t,i}\frac{1}{2}\delta_i F_i z^{s-}_{t-\frac{1}{12},i}}{P^{a,s}_{t,i}(1+f_a)}, \text{ for } i \in \{B\}, \ t \in T^i \text{ and } s \in \Sigma_t,$$

$$y^s_{t,i} = 0, \text{ for } i \in \{B\}, \ t \in T^i \text{ and } s \in \Sigma_t,$$

Rollover constraints. Rollover constraints ensure that treasury bills are sold at maturity.

$$y^s_{t,i} = z_{0,i}, \text{ for } i \in \{M3, M6_{3M}\}, \ t \in \{0\} \text{ and } s \in \Sigma_t,$$

$$x^s_{t,i} = 0, \text{ for } i \in \{M6_{3M}\}, \ t \in T^d \text{ and } s \in \Sigma_t,$$

$$y^s_{t,i} = z^{s-}_{t-\frac{1}{12},i}, \text{ for } i \in \{M3, M6_{3M}\}, \ t \in T^d \backslash \{0\} \text{ and } s \in \Sigma_t,$$

$$y^s_{t,i} = 0, \text{ for } i \in \{M6_{6M}\}, \ t \in T^d \text{ and } s \in \Sigma_t.$$

Portfolio wealth constraints. The portfolio wealth constraints determine the value of the liquid asset portfolio in each state at each time period. The value of the portfolio wealth after rebalancing, i. e. any extra cash c^s_t that has been provided to fund shortfalls below the minimum liquid asset requirement, is taken into account by the cash balance constraints, are:

$$W^{as}_t = \sum\nolimits_{i \in I} P^{a,s}_{t,i} z^s_{t,i} (1+f_a), \text{ for } t \in T^{\text{total}} \backslash T \text{ and } s \in \Sigma_t.$$

The value of the portfolio wealth before rebalancing will be:

$$W^{bs}_t = \sum\nolimits_{i \in I} P^{b,s}_{t,i} z_{0,i} (1-f_b) + \sum\nolimits_{i \in \{B\}} D^s_{t,i} \frac{1}{2} \delta_i F_i z_{0,i},$$

$$\text{with } c^s_t \geq 0 \text{ for } t \in \{0\}, \text{ and } s \in \Sigma_t,$$

and

$$W^{bs}_t = \sum\nolimits_{i \in I} P^{b,s}_{t,i} z^{s-}_{t-\frac{1}{12},i} (1-f_b) + \sum\nolimits_{i \in \{B\}} D^s_{t,i} \frac{1}{2} \delta_i F_i z^{s-}_{t-\frac{1}{12},i},$$

$$\text{with } c^s_t \geq 0 \text{ for } t \in T^d \backslash \{0\}, \text{ and } s \in \Sigma_t.$$

The terminal portfolio wealth is given by:

$$W^s_T = \sum\nolimits_{i \in I} P^{b,s}_{T,i} z^{s-}_{T-\frac{1}{12},i} (1-f_b) + \sum\nolimits_{i \in \{B\}} D^s_{t,i} \frac{1}{2} \delta_i F_i z^{s-}_{T-\frac{1}{12},i}, \text{ for } s \in \Sigma_T$$

Liability account constraints. The liability account constraints determine the value of the liability account in each state at each time period. The liabilities are assumed to grow at liability growth rate, $mg_t^s = \max\left(g_t^s, r_{t,\frac{1}{12}}^s\right)$, the maximum of the one month zero-rate and the minimum liquid asset growth rate:

$$L_t^s = L_0, \text{ for } t \in \{0\} \text{ and } s \in \Sigma_t,$$

$$L_t^s = L_{t-\frac{1}{12}}^{s-} e^{\frac{1}{12} mg_{t-\frac{1}{12}t}^{s-}}, \text{ for } t \in T^{\text{total}} \setminus \{0\} \text{ and } s \in \Sigma_t.$$

Shortfall fund constraints. The shortfall fund constraints determine the value of the shortfall fund at each state for each time period. The shortfall fund is assumed to grow at the one month zero-rate plus any extra cash c_t^s that has been provided to fund shortfalls below the liability:

$$EF_t^s = c_t^s, \text{ for } t \in \{0\} \text{ and } s \in \Sigma_t,$$

$$EF_t^s = EF_{t-\frac{1}{12}}^{s-} e^{\frac{1}{12} r_{t-\frac{1}{12}t}^{s-}} + c_t^s, \text{ for } t \in T^{\text{total}} \setminus \{T\} \text{ and } s \in \Sigma_t.$$

Shortfall constraints. The shortfall constraints determine the shortfall of the portfolio with respect to the liabilities and the shortfall fund at each state for each time period. By including the shortfall fund, when calculating the shortfall, and by minimizing the shortfall we will encourage growth in portfolio wealth to cover not only the liabilities but also extra costs. The shortfall is calculated by using the value of the portfolio wealth after transaction:

$$SF_t^s + W_t^{as} \geq L_t^s + EF_t^s, \text{ for } t \in T^{\text{total}} \text{ and } s \in \Sigma_t,$$

where $SF_t^s \geq 0$ for $t \in T^{\text{total}}$ and $s \in \Sigma_t$.

Shortfall funding constraints. The shortfall funding constraints determine the amount of extra cash needed at decision times to ensure no shortfall below the liabilities. The amount of extra cash is calculated by using the value of the portfolio wealth before transaction:

$$c_t^s + W_t^{bs} \geq L_t^s, \text{ for } t \in T^{\text{total}} \text{ and } s \in \Sigma_t.,$$

Portfolio composition constraints. Portfolio composition constraints are introduced in order to reduce concentration risk. The following portfolio constraints are taken into account:

$$\begin{aligned}
\frac{\sum_{i \in GZ} P_{t,i}^{a,s} z_{t,i}^s}{W_t^s} &< 0.5, \text{ for } t \in T^d \text{ and } s \in \Sigma_t, \\
\frac{\sum_{i \in M3 \cup M6} P_{t,i}^{a,s} z_{t,i}^s}{W_t^s} &< 0.7, \text{ for } t \in T^d \text{ and } s \in \Sigma_t, \\
\frac{\sum_{i \in M3 \cup M6} P_{t,i}^{a,s} z_{t,i}^s}{W_t^s} &> 0.3, \text{ for } t \in T^d \text{ and } s \in \Sigma_t.
\end{aligned}$$

The first constraint ensures that the amount held in gold is restricted to be no more than 50 % of the total portfolio wealth and the remaining two ensure that the amount held in treasury bills is between 30 % and 70 %.

2.4. Objective function

When managing a minimum liquid asset portfolio there are two main goals to take into account. The first aim is the management of the investment strategies of the fund in order to comply with the minimum liquid asset requirement and the liability. The second is to minimize the extra cost necessary to stay above the requirement. The objective we consider is the minimum average expected shortfall over all periods and the average expected extra cash that is needed for shortfall funding. [3] have shown that monitoring shortfall at intermediate nodes improve results. The objective function is given as:

$$\left\{ \begin{array}{c} \max \\ x_{t,i}^s, y_{t,i}^s, z_{t,i}^s \\ i \in I, t \in T^d \cup \{T\}, s \in \Sigma_t \end{array} \right\} \left\{ \alpha \sum_{t \in T^{\text{total}}} \sum_{s \in \Sigma_t} \frac{SF_t^s}{|T^{\text{total}}|} p_t^s + (1-\alpha) \sum_{t \in T^d} \sum_{s \in \Sigma_t} \frac{c_t^s}{|T^d|} p_t^s \right\}$$

where the value of $0 \leq \alpha < 1$ sets the level of importance to shortfall or cost. If the value of α is closer to 1, more importance is given to the shortfall and less given to the extra cost of the portfolio and visa versa. The value of α may not be equal to 1, as this will result in the problem to be infeasible, because there is no restriction on the amount of extra cash.

3. RESULTS

In this section we discuss the performance of the model. The first part explains the data and instruments that are used to generate scenario trees, which is the input to the mathematical optimization problem. In the second part we present back-testing results for the model for different levels of the minimum liquid asset requirements and different levels of alpha.

The model is implemented and solved in SAS/OR using PROC OPTMODEL.

3.1. Data and instruments

We use eight different assets, namely, (semi-annual) coupon bearing bonds with maturities 5, 7, 10, 15 and 19 years, gold and three and six month treasury bills. We use the Perfect Fit Bond Curves, one of the five BEASSA Zero Coupon Yield Curve series of yield curves (see [1]), with maturities 1, 2, 3, 6, 9, 12, 15, 18, 21, 24, 36, 48, 60, 72, 84, 96, 108, 120, 132, 144, 156, 168, 180, 192, 204, 216 and 228 months. The curves are derived from government bond data and the technical specifications are described in [2]. We use end-of-month data from August 1999 through to February 2009.

We use a moment-matching scenario generation approach, introduced by [5], to generate the input scenarios for the optimization problem. Parameters are fitted to the market data up to an initial decision time t and scenario trees are generated from time t to time T. The optimal first stage/root node decisions are then implemented at time t. The success of the portfolio strategy is measured by its performance with historical data up to time $t + \frac{3}{12}$. This whole procedure is rolled forward. At each

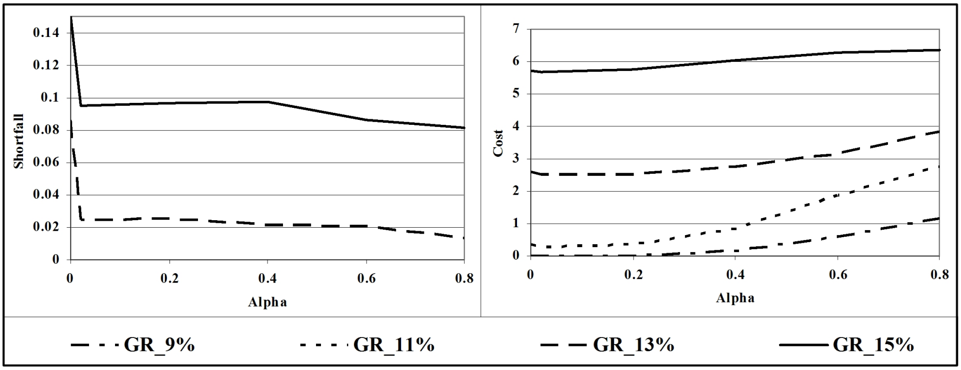

Fig. 2. Actual average shortfall and cost of the portfolio.

decision time t, the mean reversion parameters are re-estimated using the historical data up to and including time t.

3.2. Back-testing results

We perform back-tests over a period of two years, from February 2007 through to February 2009. We assume the growth rate for the minimum liquid asset requirement to be constant over the entire period, $g_t^s = g$. We back-test for different levels of minimum liquid asset requirement growth rate (GR) and for different levels of alpha. For each of these back-tests, at different levels, we report the average shortfall, taken to be

$$\sum\nolimits_{t=1}^{T} \frac{SF_t}{T}.$$

We also report the cost of the portfolio, taken to be present value of the final shortfall fund

$$\left(\frac{EF_T}{\prod_{t=1}^{T} \left(1 + r_{t,t+\frac{1}{12}} \right)} \right).$$

Figures 2 present the average shortfall and cost for different levels of alpha and the minimum liquid asset requirement growth rates (GR). As expected the average shortfall increases as the growth rate increases and decreases as alpha increases and more importance is given to shortfall in the objective. Recall that shortfall was defined to be the difference between the liabilities plus the shortfall fund and the portfolio wealth ($SF_t^s + W_t^s \geq L_t^s + EF_t^s$), and that the extra cash needed to stay above the minimum liquid asset requirement is calculated by using the shortfall of the portfolio wealth below the liabilities. Thus by minimizing only the extra cash needed stay above the minimum liquid asset requirement will result in a larger shortfall below liabilities including the shortfall fund. For growth rates of 11 % and below, no shortfall is reported and the shortfall increases as the growth rate increases from 11 %. It is also clear the cost of the portfolio increases as the requirement increases and as alpha increases and more importance is given to minimum shortfall. For

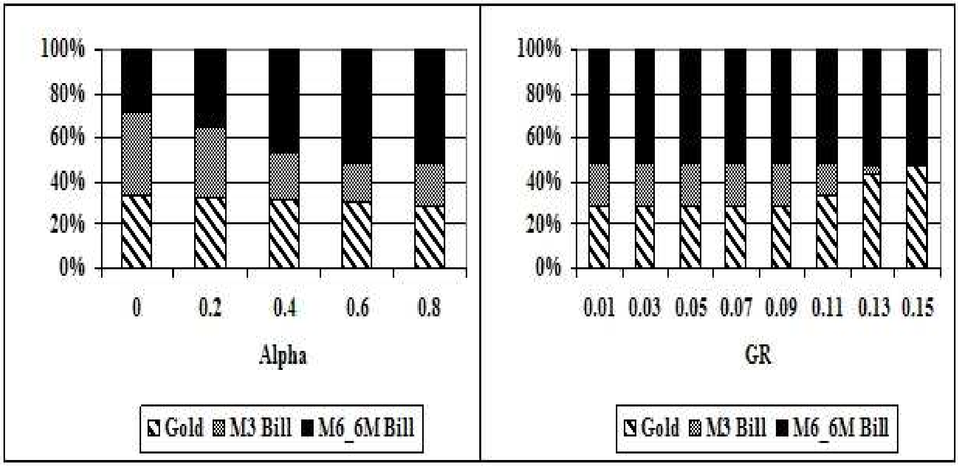

Fig. 3. First stage asset allocations.

growth rates of 9 % and above, extra costs are needed to stay above the requirement. This is largely due to the one month zero-rate, which is above 7 % resulting in the liability growth rate to be above 7 %.

In Figure 3 we present the first stage asset allocation for different levels of alpha at a minimum liquid asset requirement growth rate (GR) of 9 %. As expected less funds are allocated to gold (which is a more risky asset) as alpha increases and more importance is given to the minimum shortfall. Figure 3 also presents the asset allocation for different levels of the growth rates, at an alpha of 0.6. More funds are allocated to gold as the growth rates increases.

Although limited results are displayed, it is clear that the proposed framework handles the risks associated with the minimum liquid asset requirement in a controlled manner.

4. CONCLUSION

This paper presented a multi-stage dynamic stochastic programming model for the integrated asset and liability management of minimum liquid asset portfolios. The model allows for portfolio rebalancing decisions over a multi-period horizon, as well as for flexible risk management decisions, such as reinvesting coupons, at intermediate time steps. We have shown that our problem is related to insurance products with guarantees and utilized this in the formulation.

We have shown the model features at different levels of alpha (importance of minimum expected average shortfall) and minimum liquid asset requirement growth rates. The model performs as expected with average shortfall decreasing and cost increasing as alpha increases and average shortfall and cost increasing as the minimum liquid asset requirement growth rates increase. This model can also be used when analyzing the investment decision made by the financial institution and when considering different levels of minimum liquid asset requirement growth rates.

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