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ON DIRECT LIMIT CLASSES OF ALGEBRAS

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ABSTRACT. We investigate classes of algebras which can be obtained by a direct limit construction from an algebra. We generalize some results from monounary algebras.

Let $\mathcal{K}$ be a nonempty class of algebras of the same type. We denote by $\mathbf{L}\mathcal{K}$ the class of all algebras which are isomorphic copies of direct limits of algebras belonging to $\mathcal{K}$.

If $\mathcal{K} = \{\mathcal{A}\}$, where $\mathcal{A}$ is finite, then $\mathbf{L}\mathcal{K}$ consists precisely of retracts of the algebra $\mathcal{A}$, c.f. [4]. The class of direct limits of a cyclically ordered group was studied in [7]. The class of direct limits of a monounary algebra was studied in [2], [3]. Monounary algebras $\mathcal{A}$ with the property that the class $\mathbf{L}\{\mathcal{A}\}$ has exactly two nonisomorphic algebras were characterized in [5].

Let $\mathcal{S}$ be the class of all algebras $\mathcal{A}$ such that every surjective or injective endomorphism of $\mathcal{A}$ is an automorphism. (Note that every simple algebra has this property.) The aim of this paper is to generalize some results of [5] to the case of algebras from $\mathcal{S}$, c.f. Theorem 1 and Corollary 1 in the paper.

Preliminaries

For monounary algebras we will use the terminology as in [8].

Let A, B, C be sets. Let $B \subseteq A$. If ψ is a mapping from A into C, then $\psi|_B$ denotes the restriction ψ onto B. Denote by $\psi(B) = \{\psi(b) : b \in B\}$.

Denote by $\mathbb{N}$ the set of all positive integers.

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For $n \in \mathbb{N}$ and $\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_n$ algebras, denote by $[\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_n]$ the class of all isomorphic copies of $\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_n$.

If $\mathcal{K}$ is a class of algebras, then

$$[\mathcal{K}] = \bigcup_{\mathcal{B} \in \mathcal{K}} [\mathcal{B}].$$

For the notion of a direct limit, c.f. e.g. Grätzer [1, §21].

Let $\langle P, \leq \rangle$ be a directed partially ordered set. For each $p \in P$, let $\mathcal{A}_p = (A_p, F)$ be an algebra of some fixed type. Assume that if $p, q \in P$, $p \neq q$, then $A_p \cap A_q = \emptyset$. Suppose that for each pair of elements p and q in P with $p \leq q$, we have a homomorphism φ_{pq} of $\mathcal{A}_p$ into $\mathcal{A}_q$ such that $p \leq q \leq s$ implies that $\varphi_{ps} = \varphi_{pq} \circ \varphi_{qs}$. For each $p \in P$, suppose that φ_{pp} is the identity on A_p. The family $\{P, \mathcal{A}_p, \varphi_{pq}\}$ is said to be *direct.*

Assume that $p, q \in P$ and $x \in A_p$, $y \in A_q$. Put $x \equiv y$ if there exists $s \in P$ with $p \leq s$, $q \leq s$ such that $\varphi_{ps}(x) = \varphi_{qs}(y)$. For each $z \in \bigcup_{p \in P} A_p$ put $\overline{z} = \left\{ t \in \bigcup_{p \in P} A_p :\ z \equiv t \right\}$. Denote $\overline{A} = \left\{ \overline{z} :\ z \in \bigcup_{p \in P} A_p \right\}$.

Let $f \in F$ be an n-ary operation. Let $x_j \in A_{p_j}$, $1 \leq j \leq n$ and let s be an upper bound of p_j. Define $f(\overline{x_1}, \dots, \overline{x_n}) = \overline{f(\varphi_{p_1 s}(x_1), \dots, \varphi_{p_n s}(x_n))}$. Then the algebra $\overline{\mathcal{A}} = (\overline{A}, F)$ is said to be the *direct limit of the direct family* $\{P, \mathcal{A}_p, \varphi_{pq}\}$. We express this situation as follows

$$\{P, \mathcal{A}_p, \varphi_{pq}\} \longrightarrow \overline{\mathcal{A}}. \tag{1}$$

Let $\mathcal{A} = (A, F)$ be an algebra and (1) hold. Let $\mathcal{A}_p \cong \mathcal{A}$ for every $p \in P$. Then we say that $\overline{\mathcal{A}}$ is obtained by a direct limit from $\mathcal{A}$ or that $\overline{\mathcal{A}}$ is a direct limit of $\mathcal{A}$.

We denote by $\mathbf{L}\mathcal{A}$ the class of all algebras which are isomorphic to some algebra obtained by direct limits from $\mathcal{A}$.

It is obvious that if every endomorphism of $\mathcal{A}$ is an automorphism, then

$$\mathbf{L}\mathcal{A} = [\mathcal{A}].$$

For monounary algebras the opposite implication holds, c.f. [6, Theorem 2.2].

Algebras $\mathcal{A}$ with $\mathbf{L}\mathcal{A} = \mathbf{L}\mathcal{B} \cup [A]$

Denote by $\mathbf{E}\mathcal{A}$ the set of all algebras which are endomorphic images of $\mathcal{A}$.

It is easy to see that, in general, $[\mathbf{E}\mathcal{A}] \subseteq \mathbf{L}\mathcal{A}$ need not hold, c.f. e.g. [5, Lemma 4].

Denote by $\mathcal{S}$ the class of all algebras $\mathcal{A}$ such that every surjective or injective endomorphism of $\mathcal{A}$ is an automorphism.

Theorem 1. *Let $\mathcal{A}, \mathcal{B} \in \mathcal{S}$ such that $[\mathbf{E}\mathcal{A}] = [\mathcal{A}, \mathcal{B}]$ and $\mathbf{E}\mathcal{B} = \{\mathcal{B}\}$. Then* $\mathbf{L}\mathcal{A} - [\mathcal{A}, \mathcal{B}]$.

Proof. Let $\mathcal{A}$, $\mathcal{B}$ be nonisomorphic algebras, $\mathcal{A} = (A, F)$ and $\mathcal{B} = (B, F)$.

We will prove that $\mathcal{B} \in \mathbf{L}\mathcal{A}$.

Without loss of generality we will suppose that $\mathcal{B} \in \mathbf{E}\mathcal{A}$. Assume that φ is an endomorphism from $\mathcal{A}$ onto $\mathcal{B}$. Since $\mathbf{E}\mathcal{B} = \{\mathcal{B}\}$ and $\mathcal{B} \in \mathcal{S}$, we have that $\varphi|_B$ is an automorphism of $\mathcal{B}$.

Let $\leq$ be the natural ordering of the set $\mathbb{N}$. For every $n \in \mathbb{N}$, let $\mathcal{A}_n = (A_n, F)$ be an algebra isomorphic to $\mathcal{A}$ and $A_n \cap A_m = \emptyset$ whenever $m \neq n$. Let ψ_n be an isomorphism from $\mathcal{A}$ onto $\mathcal{A}_n$. Let $\xi_{n,n}$ be the identity mapping of A_n. We define $\xi_{n,n+1} = \psi_n^{-1} \circ \varphi \circ \psi_{n+1}$ and $\xi_{n,n+m} = \xi_{n,n+1} \circ \xi_{n+1,n+2} \circ \cdots \circ \xi_{n+m-1,n+m}$. Then $\{\mathbb{N}, \mathcal{A}_n, \xi_{n,k}\}$ is a direct family. Denote by $\mathcal{D} = (D, F)$ the direct limit of this direct family.

For every $b \in B$ we put $\Phi(b) = \overline{\psi_1(b)}$. We will show that Φ is an isomorphism from $\mathcal{B}$ onto $\mathcal{D}$.

Assume that $a, b \in B$ are such that $\Phi(a) = \Phi(b)$. Then there exists $n \in \mathbb{N}$ such that $\xi_{1,n}(\psi_1(a)) = \xi_{1,n}(\psi_1(b))$. The restriction $(\psi_1 \circ \xi_{1,n})|_B$ is an injective mapping, because $\varphi|_B$ is an automorphism of $\mathcal{B}$ and ψ_n is an isomorphism from $\mathcal{B}$ onto $(\psi_n(B), F)$. Thus $a = b$.

Suppose that $z \in D$. Consider $n \in \mathbb{N}$ and $y \in A_n$ such that $y \in z$. Since $\xi_{n,n+1}$ is onto $\psi_{n+1}(B)$, we have $(\psi_n^{-1} \circ \varphi)(y) = (\xi_{n,n+1} \circ \psi_{n+1}^{-1})(y) \in B$. The mapping $\varphi|_B$ is an automorphism of $\mathcal{B}$ and so there is $w \in B$ such that $\varphi^n(w) = (\psi_n^{-1} \circ \varphi)(y)$. Put $x = \psi_1(w)$. We obtain $\xi_{1,n+1}(x) = \psi_{n+1}\big(\varphi^n(\psi_1^{-1}(x))\big) - \psi_{n+1}\big(\varphi(\psi_n^{-1}(y))\big) = \xi_{n,n+1}(y)$. Therefore $\Phi(w) = \overline{x} = \overline{y} = z$.

Let $f \in F$ be an n-ary operation, $b_1, \ldots, b_n \in B$. Then

$$\Phi(f(b_1, \ldots, b_n)) = \overline{\psi_1(f(b_1, \ldots, b_n))} = \overline{f(\psi_1(b_1), \ldots, \psi_1(b_n))} \tag{2}$$

$$- f\big(\overline{\psi_1(b_1)}, \ldots, \overline{\psi_1(b_n)}\big) = f(\Phi(b_1), \ldots, \Phi(b_n)). \tag{3}$$

We have $[\mathcal{A}, \mathcal{B}]$ is a subset of $\mathbf{L}\mathcal{A}$. To see the opposite inclusion let (1) hold and $\mathcal{A}_p \cong \mathcal{A}$ for every $p \in P$. Two cases can occur:

(i) There exists $p \in P$ such that $\varphi_{pq}(A_p) = A_q$ for every $p \leq q$.

(ii) For every $p \in P$ there exists $q \in P$ such that $p \leq q$ and $\varphi_{pq}(A_p) \neq A_q$.

We claim $\overline{\mathcal{A}} \cong \mathcal{A}$ in the first event and $\overline{\mathcal{A}} \cong \mathcal{B}$ in the second one.

Consider (i). Denote $Q \quad \{q \in P : \ p \leq q\}$. Let $q, s \in Q$ be such that $q < s$. We have $\varphi_{qs}(A_q) = \varphi_{qs}(\varphi_{pq}(A_p)) = \varphi_{ps}(A_p) - A_s$. Thus φ_{qs} is an isomorphism from $\mathcal{A}_q$ onto $\mathcal{A}_s$. Conclude $\overline{\mathcal{A}} \simeq \mathcal{A}$, because Q is cofinal with P.

Now consider (ii). Suppose that for every $p \in P$ there exists $q \in P$ such that $p \leq q$ and $\varphi_{pq}(A_p) \neq A_q$.

Choose $p \in P$. Take $q \in P$ such that $p \leq q$ and $\varphi_{pq}(A_p) \neq A_q$. Denote $R - \{r \in P : \ q \leq r\}$. Let $B_r = \varphi_{pr}(A_p)$ and $\mathcal{B}_r = (B_r, F)$ for every $r \in R$. Then $\mathcal{B}_r \cong \mathcal{B}$ and $\{R, \mathcal{B}_r, \varphi_{rs}|_{B_r}\}$ is a direct family. Let $\{R, \mathcal{B}_r, \varphi_{rs}|_{B_r}\} \to \overline{\mathcal{B}}$. We have $\overline{\mathcal{B}} \cong \mathcal{B}$.

Let $r \in R$ and let $s \in P$ be such that $q \leq s$ and $\varphi_{rs}(A_r) \neq A_s$. We have $(\varphi_{rs}(A_r), F) \sim \mathcal{B}$. Since $\mathcal{B}_s \cong \mathcal{B}$, there exists an isomorphism ψ from $(\varphi_{rs}(A_r), F)$ onto $\mathcal{B}_s$. In view of $B_s = \varphi_{ps}(A_p) = \varphi_{rs}(\varphi_{pr}(A_p)) - \varphi_{rs}(B_r)$ $\varphi_{rs}(A_r)$ we obtain that ψ is an endomorphism of $(\varphi_{rs}(A_r), F)$. Thus $\varphi_{rs}(A_r)$ - B_s according to $\mathbf{E}(\varphi_{rs}(A_r), F) = \{(\varphi_{rs}(A_r), F)\}$. That means that the direct limit of $\{R, A_r, \varphi_{rs}\}$ is isomorphic to $\overline{\mathcal{B}}$. We conclude $\overline{\mathcal{A}} \simeq \mathcal{B}$, because R is cofinal with P. □

Example. Let $\mathcal{B}$, $\mathcal{C}$ be monounary algebras such that $\mathcal{B}$ is a three-element cycle and $\mathcal{C}$ is a three-element cycle too. Let $\mathcal{A}$ be a disjoint union of $\mathcal{B}$ and $\mathcal{C}$.

We have $\mathbf{E}\mathcal{A} = \{\mathcal{A}, \mathcal{B}, \mathcal{C}\}$, $\mathbf{E}\mathcal{B} - \{\mathcal{B}\}$. So, $[\mathbf{E}\mathcal{A}] = [\mathcal{A}, \mathcal{B}]$ and $\mathcal{A}$, $\mathcal{B}$ satisfy assumptions. Thus $\mathbf{L}\mathcal{A} = [\mathcal{A}, \mathcal{B}]$.

Lemma 1. *Let $\mathcal{A} \in \mathcal{S}$. Further, let $\mathcal{B} = (B, F)$ be a subalgebra of $\mathcal{A}$ such that $\mathbf{E}\mathcal{A} - \mathbf{E}\mathcal{B} \cup \{\mathcal{A}\}$. If φ' is an endomorphism of $\mathcal{A}$, then $\varphi'(B) \subseteq B$.*

Proof. The assumption $\mathbf{E}\mathcal{A} \quad \mathbf{E}\mathcal{B} \cup \{\mathcal{A}\}$ yields $\varphi'(A) \subseteq B$ or $\varphi'(A) = A$. If $\varphi'(A) \subset B$ then obviously $\varphi'(B) \subseteq B$.

Suppose that $\varphi'(A) = A$. Denote $\mathcal{B}' = (\varphi'(B), F)$.

Let ψ be an endomorphism of $\mathcal{A}$ such that $\psi(A) - B$. Then $(\psi \circ \varphi')(A)$ $\varphi'(B)$. Therefore $\mathcal{B}' \in \mathbf{E}\mathcal{A}$. According to $\mathbf{E}\mathcal{A} = \mathbf{E}\mathcal{B} \cup \{\mathcal{A}\}$ we have $\varphi'(B) \subseteq B$ or $\varphi'(B) = A$. If $\varphi'(B) \subseteq B$, then the proof is finished.

Let $\varphi'(B) = A$. Then φ' is a surjective endomorphism of $\mathcal{A}$ and so, φ' is an automorphism. We obtain that $B - A$. □

Theorem 2. *Let $\mathcal{A} \in \mathcal{S}$ and $\mathcal{B}$ be such that $\mathbf{E}\mathcal{A} = \mathbf{E}\mathcal{B} \cup \{\mathcal{A}\}$. Then*

$$\mathbf{L}\mathcal{A} \subseteq \mathbf{L}\mathcal{B} \cup [A].$$

Proof. Let $\mathcal{B} = (B, F)$. Let (1) hold and $\mathcal{A}_p \cong \mathcal{A}$. Let ψ_p be an isomorphism of $\mathcal{A}$ into $\mathcal{A}_p$.

a) Suppose that there exists $p \in P$ such that the following implication is satisfied: if $q \in P$ and $p \leq q$, then $\varphi_{pq}(A_p) = A_q$. Then $\overline{\mathcal{A}} \cong \mathcal{A}$ analogously as in the case (i) of the proof of Theorem 1.

b) Suppose that for every $p \in P$ there exists $q \in P$ such that $p \leq q$ and $\varphi_{pq}(A_p) \neq A_q$.

The mapping $\varphi_{pq}|_{\psi_p}(B)$ is a homomorphism of the algebra $(\psi_p(B), F)$ into $(\psi_q(B), F)$ for every $p, q \in P$, $p \leq q$, according to Lemma 1.

So, $\{P, (\psi_p(B), F), \varphi_{pq}|\psi_p(B)\}$ is a direct family. Denote by $\overline{\mathcal{B}}$ the direct limit of this family. Since the algebra $(\psi_p(B), F) \cong \mathcal{B}$ for all $p \in P$, we have $\overline{\mathcal{B}} \in \mathbf{L}\mathcal{B}$.

Let $p \in P$. Choose $q \in P$ such that $p \leq q$ and $\varphi_{pq}(A_p) \neq A_q$. That means $\varphi_{pq}(A_p) \subseteq \psi_q(B)$. We obtain $\overline{\mathcal{A}} \cong \overline{\mathcal{B}}$. □

Example. Let $A = \{a, b, c\}$ and $f(a) = b$, $f(b) = f(c) = c$. Let $\mathcal{A} = (A, \{f\})$. Suppose that $\mathcal{B}$ is a subalgebra of $\mathcal{A}$ which is generated by b. Then $\mathcal{A}, \mathcal{B}$ satisfy assumptions. In view of [5, Theorem 1] the algebra $\mathcal{B}$ does not belong to $\mathbf{L}\mathcal{A}$. We have $\mathbf{L}\mathcal{A} \subset \mathbf{L}\mathcal{B} \cup [\mathcal{A}]$.

Corollary 1. *Let $\mathcal{A} \in \mathcal{S}$. Further, let $\mathcal{B}$ be a subalgebra of $\mathcal{A}$ such that $\mathbf{E}\mathcal{A} = \mathbf{E}\mathcal{B} \cup \{\mathcal{A}\}$. If $\mathbf{L}\mathcal{B} \subseteq \mathbf{L}\mathcal{A}$, then $\mathbf{L}\mathcal{A} = \mathbf{L}\mathcal{B} \cup [\mathcal{A}]$.*

On monounary algebras

In this section all monounary algebras which satisfy all assumptions of Theorem 1 will be described and we will see that there exists a countable system of types of monounary algebras which satisfy all assumptions of Corollary 1.

We will handle algebras from [5]. We use the same terminology, notation, and symbols as in [5].

We denote by $\mathcal{T}_1^*$, $\mathcal{T}_2^*$, $\mathcal{T}_4^*$ the following classes of monounary algebras:

Notation 1.

$\mathcal{T}_1^* = \{A \in \mathcal{U} :$ there exists $a \in A$ such that $A - \{a\} \in \mathcal{T}$ and $\{a\}$ fails to be a subalgebra of $A\}$,

$\mathcal{T}_2^* = \{A \in \mathcal{U} :$ there exist $B \in \mathcal{T}$ and $k, l \in \mathbb{N}$, $l \neq 1$, such that $A = B \cup C$, B contains a cycle of length k and C is a cycle of length $k \cdot l\}$;

$\mathcal{T}_4^* = \{A \in \mathcal{U} :$ A is connected and there exists $a \in A$ such that $A - \{a\} \sim \mathbb{Z}\}$.

Proposition 1. *Let $A \in \mathcal{U}$. The following two conditions are equivalent:*

(ii) *$A \in \mathcal{S}$ and there exists B such that $[\mathbf{E}A] = [A, B]$ and $\mathbf{E}B = \{B\}$;*

(i) $A \in \mathcal{T} \cup \mathcal{T}_1^* \cup \mathcal{T}_2 \cup \mathcal{T}_3 \cup \mathcal{T}_4^* \cup [\mathbb{Z}, \mathbb{Z} + \mathbb{Z}]$.

Proof. Assume that (i) is fulfilled. Then $A \in \mathcal{S}$ according to the construction of all homomorpfisms between two monounary algebras, c.f. [9].

If $A \in \mathcal{T} \cup \mathbb{Z}$, then let $B = A$. If $A \in \mathcal{T}_1^*$, then let $B = A - \{a\}$ where a is as in the definition of $\mathcal{T}_1^*$. If $A \in \mathcal{T}_2 \cup \mathcal{T}_3$, then let B be the algebra from the definition of $\mathcal{T}_2$, $\mathcal{T}_3$, respectively. If $A \sim \mathbb{Z} + \mathbb{Z}$ or $A \in \mathcal{T}_4^*$, then let B be a subalgebra of A which is isomorphic to $\mathbb{Z}$.

Now let the condition (ii) be satisfied. Then $\mathbf{L}A = [A, B]$ by [5, Lemma 4, Theorems 1, 2, 3]. If $A \cong B$, then $A \in \mathcal{T} \cup [\mathbb{Z}]$ according to [3, Theorem 1].

Suppose that A is not isomorphic to B. Then $A \in \mathcal{T}_1 \cup \mathcal{T}_2 \cup \mathcal{T}_3 \cup \mathcal{T}_4 \cup [\mathbb{Z} + \mathbb{Z}, \mathbb{N}$ in view of [5, Theorem 4].

Let $A \in \mathcal{T}_1$ or let $A \in \mathcal{T}_4$. Let R be the chain from the definition of $\mathcal{T}_1$ or $\mathcal{T}_4$, respectively. If R is finite and R contains at least two elements, then the equality $[\mathbf{E}A] = [A, B]$ is not fulfilled. If R is infinite, then $A \notin \mathcal{S}$. Thus $A \in \mathcal{T}_1^* \cup \mathcal{T}_4^*$.

If $A \simeq \mathbb{N}$, then B does not exist.

We conclude $A \in \mathcal{T}_1^* \cup \mathcal{T}_2 \cup \mathcal{T}_3 \cup \mathcal{T}_4^* \cup [\mathbb{Z} + \mathbb{Z}]$. □

PROPOSITION 2. *If $A \in \mathcal{T}_1^* \cup \mathcal{T}_2^* \cup \mathcal{T}_3 \cup \mathcal{T}_4^*$, then $A \in \mathcal{S}$ and there exists a subalgebra B of A such that $\mathbf{E}A = \mathbf{E}B \cup \{A\}$ and $\mathbf{L}B \subseteq \mathbf{L}A$.*

Proof. The algebra $A \in \mathcal{S}$ according to the construction of all homomorphisms between two monounary algebras, c.f. [9].

If $A \in \mathcal{T}_2^* \cup \mathcal{T}_3$, then we take B from the definition of $\mathcal{T}_2^*, \mathcal{T}_3$, respectively.

If $A \in \mathcal{T}_1^* \cup \mathcal{T}_4^*$, then let a be an element of A from the definition of $\mathcal{T}_1^*, \mathcal{T}_4^*$, respectively. We put $B = A - \{a\}$.

We have $\mathbf{E}B = \{B\}$ and $\mathbf{E}A = \{A, B\}$. Further, $\mathbf{L}B = \{B\}$ according to [3, Theorem 1] and $\mathbf{L}A = \{A, B\}$ according to [5, Lemma 4, Theorems 1, 2, 3]. Therefore, $\mathbf{L}B \subseteq \mathbf{L}A$. □

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