Kybernetika

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Kybernetika, Vol. 40 (2004), No. 2, [233]--256

Persistent URL: http://dml.cz/dmlcz/135591

KYBERNETIKA — VOLUME 40 (2004), NUMBER 2, PAGES 233–256

DESIGN OF REACHING PHASE FOR VARIABLE STRUCTURE CONTROLLER BASED ON SVD METHOD

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This paper considers a design of variable structure with sliding mode controller for a class of uncertain dynamic system based on Singular Value Decomposition (SVD) method. The proposed method reduces the number of switching gain vector components and performs satisfactorily while the external disturbance does not satisfy the matching conditions. Subsequently the stability of the global system is studied and furthermore, the design of switched gain matrix elements based on fuzzy logic approach provides useful results for smooth control actions and decreases the reaching phase time. The efficacy of the proposed method is demonstrated by considering an interconnected power system problem.

Keywords: reaching-phase, sliding mode, matching condition, singular value decomposition, fuzzy logic, Lyapunov function

AMS Subject Classification: 93B12, 93D30, 93C42, 93C95

1. INTRODUCTION

Variable structure control (VSC) owing to its insensitiveness to parametric uncertainties and disturbances has drawn wide attention in the literature over the past two decades [10]. It is powerful in controlling the system with bounded unknown disturbance and can provide very robust performance and transient performance [3]. The first step of VSC with sliding mode control is to select a sliding surface that models the desired closed-loop performance in state-space form. Then the control law is designed in such a way so that the system state trajectories are forced towards the sliding surface and stay on it. The system state trajectory in the period before reaching the sliding surface is known as the reaching phase in the control literature. The system trajectory sliding along the sliding surface to the origin is the sliding mode. The most salient feature of variable structure sliding mode control is that it is completely robust to matched uncertainties [1, 12]. It is certainly true that the many physical systems can be classified under these categories. However, there are even more systems which unfortunately are affected by mismatched uncertainties and do not enjoy nice matching condition. Thus the system behavior in the sliding mode is not invariant to the mismatched uncertainty; the system performance cannot be assured. Other remarkable advantages of sliding mode control approach are the simplicity of its implementation and the order-reduction of the closed loop system [4, 6]. Pole assignment or Linear Quadratic (LQ) techniques are often used as a component of sliding mode control. However, to be fair, one should also point out two foremost difficulties in the application of VSC with sliding mode control. One is the general necessity of full state vector measurements to implement the switching surface and the other is the possible occurrence of real sliding mode (chattering phenomena) instead of the ideal one. However, since a discontinuous control action is involved, chattering will take place and the steady-state performance of the system will be degraded. To overcome this problem, numerous schemes have been reported in the literature and one of the most common techniques to alleviate this drawback is to introduce a boundary layer about the sliding plane [8, 9]. The basic idea consists of introducing a boundary layer of the switching surface in which the control law is chosen to a continuous approximation of the discontinuous function when the system is inside the boundary layer. However, this approach provides no guarantee of convergence to the sliding mode and involves a trade off between chattering and robustness. Reduced chattering may be achieved without sacrificing robust performances by combining the attractive features of fuzzy control with sliding mode control [2]. The fuzzy sliding-mode controller takes the advantages to the both fuzzy and sliding mode controller characteristics and will result chattering in the system dynamic response.

In this paper, we shall discuss how to design a reaching phase based on Singular Value Decomposition (SVD) technique with static a state-feedback control law. The control law consists of linear feedback term plus a discontinuous term, which guarantee that the sliding mode exits and is globally reachable under a very mild restriction. This paper extends the work of White et al [11] in order to design a simple sliding mode with variable structure controller based on SVD method. This in turn reduces the number of switching gain vector components as compared to White et al [11] method and moreover, for the non-switched gain components no additional inequality constrains are required to drive the state trajectory into the sliding surface. The significant advantage of the proposed method is addressed for full/reduced switching control gains. A fuzzy logic approach is also adopted in order to avoid hard switching control gains and subsequently the corresponding control signals ensure the reaching conditions and decrease the reaching phase time.

This paper is organized as follows. In Section 2, a mathematical description of the problem is given. Reaching phase design technique, based on SVD method is developed in Section 3. Subsequently, the stability of the sliding mode state trajectories is studied in the same section. In Section 4, design of an equivalent switch gain matrix based fuzzy logic approach is considered. In Section 5, the effectiveness of the proposed VSC control scheme based on SVD technique is demonstrated by considering the load-frequency control problem of interconnected power systems. Section 6 provides a brief conclusion.

2. PROBLEM FORMULATION

$$\dot{X}(t) = AX(t) + BU(t) \quad (1)$$
$$Y(t) = CX(t) \quad (2)$$

where $X(t)$ = state vector $\in \Re^{n\times 1}$, $U(t)$ = input vector $\in \Re^{m\times 1}$ and $Y(t)$ = output vector $\in \Re^{p\times 1}$. It is assumed that the system is observable and controllable. All the states are directly measurable and the linear system is assumed to be in regular form and the state equation (1) explicitly is described by following pair of equations:

$$\dot{X}_1(t) = A_{11}X_1(t) + A_{12}X_2(t) \quad (3a)$$
$$\dot{X}_2(t) = A_{21}X_1(t) + A_{22}X_2(t) + B_2U(t) \quad (3b)$$

where $X_1(t) \in \Re^{(n-m)\times 1}$, $X_2(t) \in \Re^{m\times 1}$, $B = \begin{bmatrix} 0 & B_2 \end{bmatrix}^T$ and $B_2 \in \Re^{m\times m}$. If the original system is not in a form of equation (3), it is required to transform the system (1) into a regular form by using a linear transformation matrix [4].

Before we propose the new VSC based on SVD method, a brief discussion on sliding surface is given below.

$$\sigma = SX(t) \quad (4)$$

which with no loss of generality, we can rewrite the equation (4) in more explicit form

$$\begin{aligned}\sigma_{m\times 1}(t) &= S_1X_1(t) + S_2X_2(t)\\ &= S_1X_1(t) + X_2(t)\end{aligned} \quad (5)$$

where $S_1(t) \in \Re^{m\times(n-m)}$, $S_2 \in \Re^{m\times m}$ with $S_2 = I_{m\times m}$. If the system state trajectory is on the sliding surface,

$$\sigma(t) = S_1X_1(t) + X_2(t) = 0$$

and, thus

$$X_2(t) = -S_1X_1(t). \quad (6)$$

Substituting equation (6) into equation (3), we get

$$\dot{X}_1(t) = (A_{11} - A_{12}S_1)X_1(t). \quad (7)$$

It can be noted that the reduced order dynamics of equation (7) on the sliding surface is independent of control input $U(t)$ and exhibits a state feedback structure where S_1 and A_{12} represent a 'state feedback' matrix and an 'input' matrix, respectively.

If the system (A_{11}, A_{12}) is stabilizable, it is possible to find the optimal control law, a 'feedback' control gain S_1, such that the control law minimizes performance index

$$J = \int_0^\infty \left[X_1^TQX_1 + X_2^TRX_2\right] \mathrm{d}t \quad (8)$$

where the lower limit of the integration refers to the initiation of sliding, $Q \geq 0$ and $R > 0$. This optimal gain S_1 minimizes index J and asymptotically stabilizes $X_1(t)$. It is needless to state that the system exhibits desirable dynamical behaviour when its trajectories are confined to the sliding surface ($\sigma = SX = 0$). A necessary condition for the system state trajectory to remain on the sliding surface $\sigma = 0$ is $\dot{\sigma}(X, t) = 0$ and the equivalent control for the nominal system has the form

$$U_{eq} = -(SB)^{-1}SAX(t) = -K_{eq}X(t) \tag{9}$$

Then equivalent control gain 'K_{eq}' can then be obtained from the above equation and the closed-loop system $(A - B\,K_{eq})$ having same $(n - m)$ eigenvalues as that of reduced order system (7) and remaining 'm' eigenvalues are at equilibrium point.

For the system (3), it is assumed that the control law

$$\begin{aligned} U(t) &= U_f(t) + U_s(t) \\ &= -K_f X(t) - \Delta K_s X(t) \end{aligned} \tag{10}$$

is employed with the choice of fixed control gain K_f (with $\Delta K_s = 0_{m\times n}$) such that the closed-loop system has $(n-m)$ eigenvectors lying with in the null space of S and the remaining eigenvectors will exhibit the range-space dynamics of S. On the other hand, the role of switched dynamically gain vector ΔK_s is to maintain a switching function σ as close to zero as possible and also to drive the state vector into the null space of S.

Consider a linear uncertain dynamic system described by the following state space form

$$\dot{X}(t) = (A + \Delta A)\,X(t) + (B + \Delta B)\,U(t) + \Gamma\,d(t) \tag{11}$$

$$Y(t) = CX(t) = \begin{bmatrix} C_1 & C_2 \end{bmatrix} \; X(t) \tag{12}$$

where $X(t) \in \Re^{n\times 1}$ is the measured current value of the state, $U(t) \in \Re^{m\times 1}$ is the control function, $Y(t) \in \Re^{p\times 1}$ is the output of the system, $d \in \Re^{r\times 1}$ is the external unknown constant disturbance vector bounded by $||d|| \leq d_{\max}$, A, B, Γ, C are constant matrices with appropriate dimensions, with B of full rank, and the matrices ΔA, ΔB represents uncertainty of the system matrix and input matrix, respectively.

Assumption I. (i) Matching Conditions: There exists matrices of appropriate dimensions F and E such that [4, 13]

$$\Delta A = BF, \quad \Delta B = BE, \quad ||E|| \leq \mu < 1 \tag{13}$$

the above condition is satisfied, and then the sliding mode is invariant due to parameter perturbation. The physical meaning of (13) is that all parameter uncertainties enter the system through the control input matrix or channel. It is assumed that the external disturbance component does not satisfy the matching condition. The constraint imposed on E is to ensure that the level of the uncertainty ΔB is not so large.

(ii) The pair (A, B) is completely controllable.

Assume that a sliding mode control is employed for controlling the system under structural assumption, all uncertain elements can be lumped and the system (11) can be written as

$$\dot{X}(t) = AX(t) + BU(t) + B\eta_p(t) + f_d(t) \tag{14}$$

where $f_d(t) = \Gamma d(t)$ and $\eta_p \in \Re^{m\times 1}$ represents the system total uncertainty or total perturbation [8] and it is given by

$$\eta_p(t, X) = FX(t) + EU(t). \tag{15}$$

Solely based on the knowledge of the bound on the uncertainty, we consider the following assumption.

Assumption II. There are positive constants c_0 and c_1 such that [13]

$$\|\eta_p(t, X)\|_2 \le c_0 + c_1 \|X\|_2 = \rho(t, X) \quad \text{for all } (t, X) \tag{16}$$

where $\rho(t, X)$ is the upper bound of the norm $\|\eta_p(t, X)\|_2$ and c_0 and c_1 are estimated by solving a pair of differential equations and it is discussed later.

We now consider the system (14) with (15), (16) and the solution of $X(t)$ at time 't' is obtained when the system equation (14) is forced by the input $\{U(t), \eta_p(t), f_d(t)\}$. The basic stability condition question is to find a control strategy $U(t, X(t))$ such that the system has a sliding mode and the origin is uniformly asymptotically stable in the large.

SMC design is broken down into two phases. The first phase entails constructing a switching surface so that the system restricted to the switching surface produces a desired behaviour. For convenience, it is assumed that the system (14) is in regular form

$$\begin{aligned} \dot{X}_1(t) &= A_{11}X_1(t) + A_{12}X_2(t) + f_{d1} \\ \dot{X}_2(t) &= A_{21}X_1(t) + A_{22}X_2(t) + B_2(U + \eta_p) + f_{d2} \qquad (17) \\ Y(t) &= [C_1 \quad C_2] \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}, \qquad (18) \end{aligned}$$

where $f_d = \begin{bmatrix} f_{d1}^T & f_{d2}^T \end{bmatrix}^T$. It is to be noted that the first part of the external disturbance vector f_{d1} directly affects the states $X_1(t)$ even after the system states are on the sliding mode. This in turn drives the system states away from the sliding surface and finally system response deviates from the desired behaviour.

Associated control law. In this subsection, we present the new sliding surface as

$$\sigma_{m\times 1}(t) = SX(t) + W\int_0^t (Y(t) - Y_{\text{ref}}(t))\,\mathrm{d}t = SX(t) + WZ(t)$$

$$= [S_1 \quad S_2] \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + WZ(t) = \begin{bmatrix} S_1 & W & S_2 \end{bmatrix} \begin{bmatrix} X_1(t) \\ Z(t) \\ X_2(t) \end{bmatrix}$$
$$= S_a X_a(t) \qquad (19)$$

and the corresponding control law that drives the states on the sliding surface is given by

$$U(t) = -(K_{af} + \Delta K_{as}) X_a(t) + V_p^* + V_f^* \qquad (20)$$

where the choice of K_{af} is in such a way so that $(n+p-m)$ eigenvectors of augmented closed-loop system (using equation (20)) are in the null space of S_a. The switched gain matrix ΔK_{as} maintains switching vector σ as close to zero as possible. The terms V_p^* and V_f^* represent the nonlinear feedback control for suppression of the effect the uncertainty and external disturbance. In addition to the switching gain matrix ΔK_{as}, the terms V_p^* and V_f^* are also help to drive the system trajectories toward the switching surface until intersection occurs.

Consider the augmented system and it is described by using the equations (17) – (19)

$$\dot{X}_a(t) = \begin{bmatrix} A_{11} & 0 & A_{12} \\ C_1 & 0 & C_2 \\ A_{21} & 0 & A_{22} \end{bmatrix} X_a(t) + \begin{bmatrix} 0 \\ 0 \\ B_2 \end{bmatrix} U(t) + \begin{bmatrix} 0 \\ 0 \\ B_2 \end{bmatrix} \eta_p(t) + \begin{bmatrix} f_{d1} \\ 0 \\ f_{d2} \end{bmatrix}$$
$$= A_a X_a(t) + B_a U(t) + B_a \eta_p(t) + f_{ad} \qquad (21)$$

where $X_a(t) = \begin{bmatrix} X_1^T(t) & Z^T(t) & X_2^T \end{bmatrix}^T$ and $Y_{\text{ref}} = 0$. Using the expression (20) in equation (21) the dynamic model of the closed-loop system is

$$\dot{X}_a(t) = A_{ac} X_a(t) - B_a \Delta K_{as} X_a(t) + B_a V_p^* + B_a V_f^* + B_a \eta_p(t,\, X_a(t)) + f_{ad} \quad (22)$$

where $A_{ac} = (A_a - B_a K_{af})$, $B_a = \begin{bmatrix} 0 & 0 & B_2{}^T \end{bmatrix}^T$ and $f_{ad} = \begin{bmatrix} f_{d1}^T & 0 & f_{d2}^T \end{bmatrix}^T$. As we have mentioned earlier that the selection of K_{af} is made in such a way so that $(n+p-m)$ eigenvectors of A_{ac} are placed in the null space of S_a with $S_2 = I_{m\times m}$ and the matrix K_{af} can be calculated using the following expression

$$S_{ai} A_{ac} = \lambda_{ri} S_{ai}, \quad i = 1, 2, \ldots, m \qquad (23)$$

where $S_a = \begin{bmatrix} S_{a1}^T & S_{a2}^T & \cdots & S_{am}^T \end{bmatrix}_{m\times m}^T$ is assumed to be the left eigenvectors of the matrix A_{ac} corresponding to the eigenvalues $\lambda_{r1},\ \lambda_{r2}, \ldots, \lambda_{rm}$ respectively. Switching surface S_a is designed by following the steps as discussed in this section (from equations (4) – (8)). It can be noted that the matrix K_{af} can be determined in such away so that the range space $(n+p-m)$ eigenvalues of the system A_{ac} are placed at desired locations and the corresponding distinct left eigenvectors of A_{ac} are within the null space of S_a. So, for any state X(t) lying in the null space of S_a, $\dot{X}(t)$ will also lie in the null space.

3. REACHING PHASE DESIGN USING PROPOSED SVD METHOD

Singular Value Decomposition (SVD) technique is employed in equation (19) and the corresponding switching surface is written as

$$\begin{aligned} \sigma_{m\times 1}(t) &= WDV^T X_a(t) \\ &\Rightarrow W^T\sigma(t) = DV^T X_a(t) \\ &\Rightarrow \overline{\sigma}(t) = D\overline{X}_a(t) \end{aligned} \tag{24}$$

where $W_{m\times m}$ and $V_{(n+p)\times(n+p)}$ are the orthogonal matrices and $D_{m\times(n+p)}$ is the rectangular matrix with diagonal elements are the singular values ($\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_m \geq 0$) of S_a [7]. White et al [11] treated the problem of reachability in variable structure control for single input systems and developed inequality conditions on the switch gain components to ensure reaching the null space of sliding surface from anywhere in the state space domain. The main idea of using SVD technique is to obtain the reaching phase conditions in a simpler form for multi inputs systems by exploiting the structure and properties of D matrix that arises from the decomposition of the matrix S_a and moreover, it decreases the number of switching gain vector components.

Let us consider the transformed state $\overline{X}_a(t) = V^T X_a(t)$ and the corresponding augmented transformed system model is given by

$$\dot{\overline{X}}_a(t) = \overline{A}_{ac}\overline{X}_{ac}(t) - \overline{B}_a\overline{\Delta K}_{as}\overline{X}_{ac}(t) + \overline{B}_a V_p^* + \overline{B}_a V_f^* + \overline{B}_a\eta_p(t,\, \overline{X}_a) + \overline{f}_{ad} \tag{25}$$

where $\overline{A}_{ac} = V^T A_{ac} V$, $\overline{B}_a = V^T B_a$, $\overline{\Delta K}_{as} = \Delta K_{as} V$ and $\overline{f}_{ad} = V^T f_{ad}$. To guarantee the sliding condition $\overline{\sigma} = 0$ implies $\sigma = 0$, we differentiate the equation (24) and use equation (22) to get the following expressions

$$\begin{aligned} \dot{\overline{\sigma}}(t) &= D\dot{\underset{a}{\overline{X}}}(t) \\ &= DV^T\dot{X}_a(t) \\ &= DV^T\left[A_{ac}X_a(t) - B_a\Delta K_{as}X_a(t) + B_aV_p^* + B_aV_f^* + B_a\eta_p\left(t,\, X_a(t)\right) + f_{ad}\right] \\ &= W^TWDV^T\left[A_{ac}X_a(t) - B_a\Delta K_{as}X_a(t) + B_aV_p^* + B_aV_f^* + B_a\eta_p\left(t,\, X_a(t)\right) + f_{ad}\right] \\ &= W^T\operatorname{diag}\left[\lambda_{r1}, \lambda_{r2}, \ldots, \lambda_{rm}\right] \quad S_aX_a(t) - W^TB_2\Delta K_{as}X_a(t) + W^TB_2V_p^* \\ &\qquad + W^TB_2V_f^* + W^TB_2\eta_p(t, X_a(t)) + W^TS_af_{ad} \\ &\qquad\qquad [\text{note, } S_aA_{ac} = \operatorname{diag}[\lambda_{r1},\, \lambda_{r2},\, \ldots,\, \lambda_{rm}]\, S_a \\ &= W^T\operatorname{diag}\left[\begin{matrix}\lambda_{r1} & \lambda_{r2} & \cdots & \lambda_{rm}\end{matrix}\right] WDV^TX_a(t) - W^TB_2\overline{\Delta K}_{as}\overline{X}_a(t) \\ &\qquad + \overline{V}_p^* + \overline{V}_f^* + \overline{\eta}_p^* + \overline{f}_{ad}^* \\ &= \overline{W}D\overline{X}_a(t) - \overline{\Delta K}_{as}^*\overline{X}_a(t) + \overline{V}_p^* + \overline{V}_f^* + \overline{\eta}_p^* + \overline{f}_{ad}^* \end{aligned} \tag{26}$$

where $\overline{W} = W^T\operatorname{diag}\left[\begin{matrix}\lambda_{r1} & \lambda_{r2} & \cdots & \lambda_{rm}\end{matrix}\right]W$ = Symmetric matrix, $\overline{\Delta K}_{as}^* = W^TB_2\overline{\Delta K}_{as}$, $\overline{V}_p^* = W^TB_2V_p^*$, $\overline{V}_f^* = W^TB_2V_f^*$, $\overline{\eta}_p^* = W^TB_2\eta_p\left(t,\, X_a(t)\right)$ and $\overline{f}_{ad}^* = W^TS_af_{ad}$.

Equation (26) is written in different form

$$\dot{\bar{\sigma}}_i(t) = \sum_{k=1}^{m} (\alpha_k \bar{W}_{ik} - \overline{\Delta K}^*_{as,ik})\,\bar{x}_{a,k}(t) - \sum_{j=m+1}^{n+p} \overline{\Delta K}^*_{as,ij} \bar{x}_{a,j}(t) + \bar{V}^*_{p,i} + \bar{V}^*_{f,i} + \bar{\eta}^*_{p,i} + \bar{f}^*_{ad,i} \tag{27}$$

$i = 1, 2, \ldots, m$.

A sufficient condition for the existence of sliding mode is $\sigma^T(t)\dot{\sigma}(t) < 0$ for $\sigma \neq$ null vector. This condition can be also be written in the following form

$$\begin{aligned} \sigma^T W^T W \sigma < 0 \quad &\Rightarrow \quad (W\sigma)^T W\sigma < 0 \\ &\Rightarrow \quad \dot{\bar{\sigma}}^T \bar{\sigma} \Rightarrow \sum_{i=1}^{m} \bar{\sigma}_i \dot{\bar{\sigma}}_i < 0 \end{aligned} \tag{28}$$

and to meet the above condition, we need to consider the equation (27) and require to satisfy the following inequality conditions.

(i) $$\overline{\Delta K}^*_{as,\,ik} = \begin{cases} \alpha_1 \left|\bar{W}_{ik}\right| & \text{for} \quad \bar{\sigma}_i \bar{x}_{a,\,k}(t) > 0 \\ -\alpha_1 \left|\bar{W}_{ik}\right| & \text{for} \quad \bar{\sigma}_i \bar{x}_{a,\,k}(t) < 0 \end{cases},$$

$i \neq k$ and $\overline{\Delta K}^*_{as,ii} = \alpha_1 |\bar{W}_{ii}|$, $\bar{\sigma}_i \bar{x}_{a,ii} > 0$ and $\bar{\sigma}_i x_{a,ii} < 0$ does not exist $i = 1, 2, \ldots, m$, $k = 1, 2, \ldots, m$.

It can be observed that the quantity $\bar{\sigma}_i \bar{x}_{a,\,i}$ is always + we since $\alpha_i > 0$ [11] and $\bar{\sigma}_i \bar{x}_{a,\,i} = \alpha_i \bar{x}^2_{a,\,i}$.

(ii) $$\overline{\Delta K}^*_{as,\,ij} = \begin{cases} \geq 0 & \text{for} \quad \bar{\sigma}_i \bar{x}_{a,\,j}(t) > 0 \\ \leq 0 & \text{for} \quad \bar{\sigma}_i \bar{x}_{a,\,j}(t) < 0 \end{cases}$$

$i = 1, 2, \ldots, m$, $j = m+1, m+2, \ldots, n+p$.

(iii) $$\bar{V}^*_{p,\,i} = \begin{cases} -\left\|W^T B_2\right\| \left\|\eta_p(t,\,X_a)\right\|_2 & \text{for} \quad \bar{\sigma}_i > 0 \\ \left\|W^T B_2\right\| \left\|\eta_p(t,\,X_a)\right\|_2 & \text{for} \quad \bar{\sigma}_i < 0 \end{cases}$$

$i = 1, 2, \ldots, m$.

(iv) $$\bar{V}^*_{f,\,i} = \begin{cases} -\left|\bar{f}^*_{ad}\right|_\infty & \text{for} \quad \bar{\sigma}_i > 0 \\ \left|\bar{f}^*_{ad}\right|_\infty & \text{for} \quad \bar{\sigma}_i < 0 \end{cases}$$

$i = 1, 2, \ldots, m$ where $|g|_\infty = \max_i |g_i|$, $\|\eta_p(t,\,X_a)\|_2$ are the infinity and Euclidean norm of a vector respectively. $\left\|W^T B_2\right\|$ is the spectral norm of the matrix $W^T B_2$.

Consider the equations (15) and (16) and rewrite the upper bound of the norm $\|\eta_p(t,\,X_a)\|$ and is synthesized by

$$\|\eta_p(t,\,X_a)\| \leq \rho(t,\,X_a) = c_0(t,\,X_a) + c_1(t,\,X_a)\,\|X_a\| \tag{29}$$

where $c_0(t,\,X_a)$ and $c_1(t,\,X_a)$ are parameters. These parameters are computed using the following dynamic equations (see [8])

$$\dot{c}_0(t,\,X_a) = q_0 \left\|B_a^T S_a^T \sigma\right\| \tag{30}$$

$$\dot{c}_1(t,\ X_a) = q_1 \left\| B_a^T S_a^T \sigma \right\| \left\| X_a \right\| . \tag{31}$$

It may be noted that the matrix ΔK_{as} can be expressed in terms of transformed switching gain matrix $\Delta K_{as} = (WB_2)^{-1}\overline{\Delta K}_{as}^{*} V^T$.

It is important to note that the reachability condition for multi input system based on SVD technique can be obtained by adopting only $2^{m(j-1)}$ switching gain vector components and moreover it does not require any stringent condition need to be satisfied to ensure reachability. A detail comparative study on number of switching components and additional stringent conditions between the proposed method and White et al method [11] is given in the following table.

Table 1. Comparative study on number of Switching components and stringent condition.

Description	Proposed method (SVD)	White et al method
Number of state $= n$ Number of input $= m$ Number of switching gain vector components:		
(a) Full switching gain vector components $\overline{\Delta K}_{as}^{*}$	$2^{m(m-1)}$	2^{mn}
(b) Reduce switching gain vector components $\overline{\Delta K}_{as}^{*}$ (up to jth component, $j \geq m$)	$2^{m(j-1)}$ Inequality conditions need not to be satisfied.	2^{mj} Inequality conditions should be satisfied.

3.1. Sliding motion and equivalent control

Equivalent control determines the behavior of the system restricted to the switching surface and a necessary condition for the state trajectory to remain on the sliding surface σ is $\dot{\sigma}(t,\ X_a(t)) = 0$. The motion in the sliding mode may be determined by differentiating (19) with respect to time and inserting the value of $\dot{X}_a$ given in (21) gives

$$\dot{\sigma} = S_a \left[A_a X_a(t) + B_a U_{\text{EQ}}(t) + B_a \eta_p(t,\ X_a) + f_{ad} \right] = 0 \tag{32}$$

and equivalent control law in the sliding mode is obtained from (32) as

$$\begin{aligned} U_{\text{eq}}(t) &= -(S_a B_a)^{-1} \left[S_a A_a X_a(t) + S_a B_a \eta_p(t,\ X_a) + S_a f_{ad} \right] \\ &= -(S_a B_a)^{-1} \left[S_a A_a X_a + S_a B_a \left(F_a X_a(t) + E U_{\text{eq}}(t) \right) + S_a f_{ad} \right] \\ &\Rightarrow (I_m + E)\, U_{\text{eq}}(t) = -\left[(S_a B_a)^{-1} (S_a A_a X_a + S_a f_{ad}) + F_a X_a(t) \right] \\ &\Rightarrow U_{\text{eq}}(t) = -(I_m + E)^{-1} \left[(S_a B_a)^{-1} (S_a A_a X_a + S_a f_{ad}) + F_a X_a(t) \right] \end{aligned} \tag{33}$$

where $F_a = \begin{bmatrix} F_1^T & 0_{m \times p} & F_2^T \end{bmatrix}_{m \times (n+p)}^T$. Using the following relation [7]

$$\frac{1}{1 + \|E\|} \leq \left\| (I_m + E)^{-1} \right\| \leq \frac{1}{1 - \|E\|}$$

in equation (33), we obtained

$$U_{\text{eq}}(t) = -\tfrac{\gamma}{1-\mu}\left[(S_aB_a)^{-1}\left(S_aA_aX_a+S_a\begin{bmatrix}\text{sign}(f_{ad,1})|f_{ad,1}|_{\max}\\ \text{sign}(f_{ad,2})|f_{ad,2}|_{\max}\\ \vdots\\ \text{sign}(f_{ad,n+p})|f_{ad,n+p}|_{\max}\end{bmatrix}+F_aX_a(t)\right)\right] \tag{34}$$

where $\|E\|$ is the spectral norm of E, $|f_{ad,i}|_{\max}$ is the upper bound of $|f_{ad,i}|$ and $0 < \gamma < 1$. Here, we need to adjust γ in such away so that the control law $U_{\text{eq}}(t)$ will drive the states on the sliding surface and the corresponding control law (34) can then be expressed in terms of states $X(t)$ and rewritten in the following form

$$\begin{aligned}
U_{\text{eq}}(t) &= -\alpha\left\{\begin{bmatrix} L_1 & L_2\end{bmatrix}\begin{bmatrix}X_1(t)\\ X_2(t)\end{bmatrix}\right.\\
&\quad\left.+\sum_{i=1}^{n-m} S_{1,i}\,\text{sign}(f_{ad,i})\,|f_{ad,i}|_{\max}+\sum_{j=n-m+1}^{n} e_j\,\text{sign}(f_{ad,j})\,|f_{ad,j}|_{\max}\right\}\\
&= -\alpha\left\{LX(t)+\sum_{i=1}^{n-m} S_{1,i}\,\text{sign}(f_{ad,i})\,|f_{ad,i}|_{\max}+\sum_{j=n-m+1}^{n} e_j\,\text{sign}(f_{ad,j})\,|f_{ad,j}|_{\max}\right\}\\
&= U_1(t)-\left\{\sum_{i=1}^{n-m} S_{1,i}\,\text{sign}(f_{ad,i})\,|f_{ad,i}|_{\max}+\sum_{j=n-m+1}^{n} e_j\,\text{sign}(f_{ad,j})\,|f_{ad,j}|_{\max}\right\}
\end{aligned} \tag{35}$$

where $L_i = (B_2)^{-1}(S_1A_{1,i} + WC_i + A_{2,i}) + F_i$, $i = 1, 2$. e_j is the unit vector whose jth element is 1 and $\alpha = \frac{\gamma}{1-\mu}$.

3.2. Composite system stability study

Consider a Lyapunov function candidate $V(X(t)) = X^T(t)PX(t)$ of the system (14). Taking derivative of $V(X(t))$ along the sliding trajectories, using Assumptions I, II and combining with (35), we obtain

$$\begin{aligned}
\dot V(X(t)) &= X^T(t)(A^TP+PA)X(t)+2U^TB^TPX(t)+2\eta_p^TB^TPX(t)+2f_d^TPX(t)\\
&= X^T(t)(A^TP+PA)X(t)-2\alpha\,(LX(t)+M)^T\,B^TPX(t)+2\eta_p^TB^TPX(t)+2f_d^TPX(t)
\end{aligned}$$

where 'M' is assumed as equal to the second part of the right hand side of equation (35).

$$\begin{aligned}
\dot V(X(t)) &= X^T(t)(A^TP+PA)X(t)-2\alpha X^T(t)L^TB^TPX(t)\\
&\quad+2\left(\eta_p^T-\alpha M^T\right)B^TPX(t)+2f_d^T\sqrt{P}\,\sqrt{P}X(t)\\
&= X^T(t)(A^TP+PA)X(t)-2\alpha X^T(t)\,L^TB^TPX(t)\\
&\quad+2\bar\eta_p^TB^TPX(t)+2f_d^T\sqrt{P}\,\sqrt{P}X(t)
\end{aligned} \tag{36}$$

where $\bar n_p^T = (n_p - \alpha M)^T$.

We need to use the following useful lemma to obtain a simplified expression.

Lemma. (See [14].) For any matrices or vectors 'V'and 'W' with appropriate dimensions, we have

$$V^T W + W^T V \leq \beta V^T V + \frac{1}{\beta} W^T W \tag{37}$$

for any positive constant β. Using equation (37) in equation (36), we obtain

$$\begin{aligned}
\dot{V}\,(X(t)) &\leq X^T(A^T P + PA - \beta_1 PBB^T P + \beta_2 P + \alpha Q)X + \frac{1}{\beta_1}\bar{\eta}_p^T \bar{\eta}_p \\
&\quad + 2\beta_1 X^T PBB^T PX - 2\alpha X^T L^T B^T PX + \frac{1}{\beta_2} f_d^T P f_d - X^T \alpha Q X, \\
&\qquad\qquad \text{where } Q > 0 \\
&\leq X^T(t)\left[\left(A + \frac{I\beta_2}{2}\right)^T P + P\left(A + \frac{I\beta_2}{2}\right) - \beta_1 PBB^T P + \alpha Q\right] X(t) \\
&\quad - X^T(t)\alpha Q X(t) + 2X^T(t)\left(\beta_1 PBB^T P - \alpha L^T B^T P\right) X(t) \\
&\quad + \frac{1}{\beta_1}\bar{\eta}_p^T \bar{\eta}_p + \frac{1}{\beta_2} f_d^T P f_d \\
&\leq X^T(t)\left[\left(A + \frac{I\beta_2}{2}\right)^T P + P\left(A + \frac{I\beta_2}{2}\right) - \beta_1 PBB^T P + \alpha Q\right] X(t) \\
&\quad + 2\left[\lambda_{\max}(\beta_1 PBB^T P) - \frac{1}{2}\lambda_{\min}(\alpha(L^T B^T P + (\alpha - 1)Q))\right] \|X\|_2^2 \\
&\quad - \lambda_{\min}(Q)\|X\|_2^2 + \frac{1}{\beta_2}\lambda_{\max}(P)\|f_d\|_2^2 + \frac{1}{\beta_1}\left\|\bar{\eta}_p\right\|_2^2
\end{aligned} \tag{38}$$

where β_1 and $\beta_2$are the positive constants and α is the tuning parameter of the control law (35). Examination of equation (38) revels that sufficient conditions for $\dot{V} < 0$ are

$$\left[\left(A + \frac{I\beta_2}{2}\right)^T P + P\left(A + \frac{I\beta_2}{2}\right) - \beta_1 PBB^T P + \alpha Q\right] = 0 \tag{39}$$

$$\frac{1}{2}\lambda_{\min}\left(L^T B^T P + PBL + (\alpha - 1)Q\right) \geq \lambda_{\max}(\beta_1 PBB^T P) \tag{40}$$

$$\lambda_{\min}(Q) \geq \frac{1}{\beta_2}\lambda_{\max}(P)\left(\frac{\|f_d\|_2}{\|X\|_2}\right)^2 + \frac{1}{\beta_1}\left(\frac{\|\bar{\eta}_p\|_2}{\|X\|_2}\right)^2. \tag{41}$$

It can be noted that the solution of Riccati equation (39) along with the above two additional conditions ensure $\dot{V} < 0$. Thus, we conclude that the sliding mode state trajectories of the uncertain system (14) and (15) under the equivalent control action (35) are robustly asymptotically stable in the large. Thus, we have successfully developed a new constructive reaching phase design based on SVD method and subsequently the stability condition for completely uncertain system is established.

4. DESIGN OF SWITCH GAIN COMPONENTS ($\overline{\Delta K}^*_{as,ij}$) BASED ON FUZZY LOGIC APPROACH

It is well known that each control method always has its advantages and drawbacks, or we can say that all control techniques have their individual characteristic features. Combining several control theories to design a new controller may have possibly better system performance than one based on single control theory only. In this section, the design of switch gain control components based on fuzzy logic approach is proposed with a view to achieve good dynamic system response, smooth control actions and to decrease the reaching phase time. Here we recall the reaching phase control law (20) for our convenience and ready reference.

$$U(t) = -(K_{af} + \Delta K_{as})\, X_a(t) + V_p^* + V_f^* \tag{42}$$

where the feedback gain K_{af} is kept constant, but the proper choice of fuzzy switching gain ΔK_{as} can accelerate the state trajectories to reach the sliding hyper plane, and thus the dynamic performances will may be improved. The function of each part of the control (20) is already discussed in detail in Section 2. Now, we consider the design procedure of the fuzzy switching gain matrix $\overline{\Delta K}^*_{as}$ as a part of the control signal that will drive the state trajectories from any initial state condition to the sliding surface.

4.1. Design of switch gain matrix elements $\overline{\Delta K}^*_{as,ij}$ based on fuzzy logic approach

We have considered $\bar{\sigma}_i \bar{x}_{a,j}$ (see equation (26)) as the linguistic input fuzzy variable and the corresponding transformed switch gain elements $\overline{\Delta K}^*_{as,ij}$ as the output variable. Non-fuzzy variable $\bar{\sigma}_i \bar{x}_{a,j}$ is quantized into five/six linguistic variables and similarly the qualitative linguistic variable $\overline{\Delta K}^*_{as,ij}$ is quantized into five/six linguistic output variables. The universe of discourse for each membership function is selected based on some trials and these are shown in Figure 1.

Table 2.

<table>
<tr><td>Input variables ⟶</td><td colspan="6">Linguistic variables</td></tr>
<tr><td>$\bar{\sigma}_i \bar{x}_{a,i}$, $i = 1,2,\ldots,m$</td><td>Positive Large (PL)</td><td>Positive Big (PB)</td><td>Positive Medium (PM)</td><td>Positive Small (PS)</td><td colspan="2">Zero (ZE)</td></tr>
<tr><td>$\bar{\sigma}_i \bar{x}_{a,j}$ $i = 1,2,\ldots,m$, $j \neq i$</td><td>Positive Large (PL)</td><td>Positive Medium (PM)</td><td>Positive Zero (PZ)</td><td>Negative Zero (NZ)</td><td>Negative Medium (NM)</td><td>Negative Large (NL)</td></tr>
<tr><td>Output variables ⟶</td><td colspan="6">Linguistic variables</td></tr>
<tr><td>$\overline{\Delta K}^*_{as,ii}$, $i = 1,2,\ldots,m$</td><td>Positive Large (PL)</td><td>Positive Big (PB)</td><td>Positive Medium (PM)</td><td>Positive Small (PS)</td><td colspan="2">Zero (ZE)</td></tr>
<tr><td>$\overline{\Delta K}^*_{as,ij}$ $i = 1,2,\ldots,m$, $j = 1,2,\ldots,m$, $j \neq i$</td><td>Positive Large (PL)</td><td>Positive Medium (PM)</td><td>Positive Zero (PZ)</td><td>Negative Zero (NZ)</td><td>Negative Medium (NM)</td><td>Negative Large (NL)</td></tr>
</table>

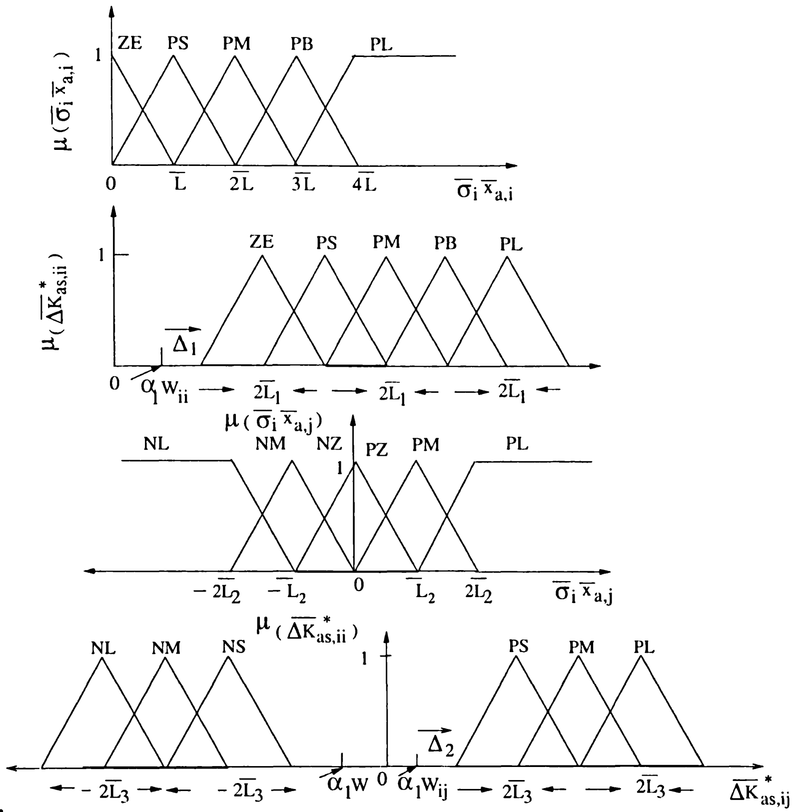

Fig. 1. Membership functions for each input and output.

Based on the expressions (27) – (28) (derived in the previous section), we can compute the switch gain matrix elements $\overline{\Delta K}^*_{as,\,ij}$ using the following decision rules.

Fuzzy rules for $i = j$ and $i = 1, 2, \ldots, m$ *are given below:*

R1: If $\bar{\sigma}_i \bar{x}_{a,\,i}$ is PL then $\overline{\Delta K}^*_{as,\,ii}$ is PL for $i = j$ and $i = 1, 2, \ldots, m$.

This rule indicates that when the transformed state is leaving the sliding surface quickly, then a positive large $\overline{\Delta K}^*_{as,\,ii}$ is required to decrease $\dot{\bar{\sigma}}$ quickly to make $\bar{\sigma}_i$ near the sliding hyper plane.

R2: If $\bar{\sigma}_i \bar{x}_{a,i}$ is PB then $\overline{\Delta K}^*_{as,\,ii}$ is PB.

R3: If $\bar{\sigma}_i \bar{x}_{a,i}$ is PM then $\overline{\Delta K}^*_{as,\,ii}$ is PM.

R4: If $\bar{\sigma}_i \bar{x}_{a,i}$ is PS then $\overline{\Delta K}^*_{as,\,ii}$ is PS.

R5: If $\bar{\sigma}_i \bar{x}_{a,i}$ is ZE then $\overline{\Delta K}^*_{as,\,ii}$ is ZE.

For $i \neq j$ and $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, m$, the following fuzzy rules are described as

R$_6$: If $\bar{\sigma}_i \bar{x}_{a,j}$ is PL then $\overline{\Delta K}^*_{as,ij}$ is PL.

R$_7$: If $\bar{\sigma}_i \bar{x}_{a,j}$ is PM then $\overline{\Delta K}^*_{as,ij}$ is PM.

R$_8$: If $\bar{\sigma}_i \bar{x}_{a,j}$ is PZ then $\overline{\Delta K}^*_{as,ij}$ is PS.

R$_9$: If $\bar{\sigma}_i \bar{x}_{a,j}$ is NZ then $\overline{\Delta K}^*_{as,ij}$ is NS.

R$_{10}$: If $\bar{\sigma}_i \bar{x}_{a,j}$ is NM then $\overline{\Delta K}^*_{as,ij}$ is NM.

R$_{11}$: If $\bar{\sigma}_i \bar{x}_{a,j}$ is NL then $\overline{\Delta K}^*_{as,ij}$ is NL.

Defuzzification. The crisp output $\overline{\Delta K}^*_{as,ij}$ is obtained by choosing the center-of-area (centroid) defuzzification method and it is given by

$$\overline{\Delta K}^*_{as,ij} = \frac{\int \mu_{\left(\overline{\Delta K}^*_{as,ij}\right)} \overline{\Delta K}^*_{as,ij} \, \mathrm{d}\left(\overline{\Delta K}^*_{as,ij}\right)}{\int \mu_{\left(\overline{\Delta K}^*_{as,ij}\right)} \mathrm{d}\left(\overline{\Delta K}^*_{as,ij}\right)}. \tag{43}$$

It has been observed that a large switching gain with proper sign of $\overline{\Delta K}^*_{as,ij}$ will drive the state trajectories to approach the sliding surface rapidly and vice versa. Transformed switch gain matrix $\overline{\Delta K}^*_{as,ij}$ is then changed to switch gain matrix ΔK_{as} by using the inverse of the matrix $W^T B$. Furthermore, when the state trajectories hitting the sliding surface an equivalent control law (35) is then applied to maintain the motion of the states along sliding hyper plane and ensures the trajectory remains on the surface once it gets there.

5. SIMULATION RESULTS

We consider load-frequency control problem of two area interconnected power system to demonstrate the effectiveness of the proposed controllers in presence of parameter perturbation and external disturbances. The nominal system is represented in the state space form by the equation

$$\dot{X}(t) = AX(t) + BU(t) + \Gamma d(t) \tag{44a}$$

$$Y(t) = CX(t) \tag{44b}$$

where, $X(t) = \begin{bmatrix} \Delta f_1 & \Delta P_{g1} & \Delta X_{g1} & \Delta P_{tie} & \Delta f_2 & \Delta P_{g2} & \Delta X_{g2} \end{bmatrix}^T$, Δf_1 and Δf_2 are the deviation in frequencies, ΔP_{tie} is the change in tie-line power, ΔP_{g1} and ΔP_{g2} are the change in turbine-generator outputs, ΔX_{g1} and ΔX_{g2} are the change in outputs of the governors. Furthermore,

$$U = \begin{bmatrix} \Delta P_{c1} & \Delta P_{c2} \end{bmatrix}^T, \quad d = \begin{bmatrix} d_1 & d_2 \end{bmatrix}^T.$$

Area-control error in Area-1 $\text{ACE}_1 = \Delta f_1 + \Delta P_{tie}$ and in Area-2 $\text{ACE}_2 = \Delta f_2 - \Delta P_{tie}$ are the outputs of the composite system. The following are the nominal system matrices:

$$A = \begin{bmatrix} -\frac{1}{T_{P1}} & \frac{K_{P1}}{T_{P1}} & 0 & -\frac{K_{P1}}{T_{P1}} & 0 & 0 & 0 \\ 0 & -\frac{1}{T_{T1}} & \frac{1}{T_{T1}} & 0 & 0 & 0 & 0 \\ -\frac{1}{R_1 T_{G1}} & 0 & -\frac{1}{T_{G1}} & 0 & 0 & 0 & 0 \\ T_{12}^* & 0 & 0 & 0 & -T_{12}^* & 0 & 0 \\ 0 & 0 & 0 & \frac{K_{P2}}{T_{P2}} & -\frac{1}{T_{P2}} & \frac{K_{P2}}{T_{P2}} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{T_{T2}} & \frac{1}{T_{T2}} \\ 0 & 0 & 0 & 0 & -\frac{1}{R_2 T_{G2}} & 0 & -\frac{1}{T_{G2}} \end{bmatrix}$$

$$B^T = \begin{bmatrix} 0 & 0 & \frac{1}{T_{G1}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{T_{G2}} \end{bmatrix}, \quad \Gamma^T = \begin{bmatrix} -\frac{K_{P1}}{T_{P1}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{K_{P1}}{T_{P1}} & 0 & 0 \end{bmatrix}.$$

The following nominal parameters are used [5]: $T_P = T_{P1} = T_{P2} = 20.0\,\text{s}$; $T_T = T_{T1} = T_{T2} = 0.3\,\text{s}$; $T_G = T_{G1} = T_{G2} = 0.08$ s; $K_P = K_{P1} = K_{P2} = 120\,\text{Hz / p.u.MW}$; $R = R_1 = R_2 = 2.4$ Hz / p.u.MW.

There are always errors present in such models due to linearization, unmodelled dynamics, etc. Moreover, the power system operating conditions change with time leading to changes in system linearized parameters and the following range of system parameter variations are considered:

$$\frac{1}{T_P} \in [\ 0.025 \quad 0.075\ ], \quad \frac{K_P}{T_P} \in [\ 3.0 \quad 9.0\ ], \quad \frac{1}{T_T} \in [\ 2.333 \quad 4.333\ ]$$

$$\frac{1}{RT_G} \in [\ 2.6041 \quad 7.8124\ ], \quad \frac{1}{T_G} \in [\ 8.75 \quad 16.25\ ].$$

The nominal system matrices are as follows:

$$A = \begin{bmatrix} -0.05 & 6.0 & 0.0 & -6.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & -3.33 & 3.33 & 0.0 & 0.0 & 0.0 & 0.0 \\ -5.2083 & 0.0 & -12.5 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.545 & 0.0 & 0.0 & 0.0 & -0.545 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 6.0 & -0.05 & 6.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -3.33 & 3.33 \\ 0.0 & 0.0 & 0.0 & 0.0 & -5.2083 & 0.0 & -12.5 \end{bmatrix}$$

$$B^T = \begin{bmatrix} 0.0 & 0.0 & 12.5 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 12.5 \end{bmatrix}$$

$$\Gamma^T = \begin{bmatrix} -6.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & -6.0 & 0.0 & 0.0 \end{bmatrix}$$

and $$C = \begin{bmatrix} 1.0 & 0.0 & 0.0 & 1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & -1.0 & 1.0 & 0.0 & 0.0 \end{bmatrix}.$$

Note that our nominal system is not in a regular form of equation (17) and (18). One can use a suitable sate transformation to get the desired form. In the present example, the states are rearranged to obtain the system description in regular form and it is given by

$$\overline{X}(t) = \begin{bmatrix} \Delta f_1 & \Delta P_{g1} & \Delta P_{g2} & \Delta P_{tie} & \Delta f_2 & \Delta X_{g1} & \Delta X_{g2} \end{bmatrix}^T$$

$$\overline{A} = \begin{bmatrix} -0.05 & 6.0 & 0.0 & -6.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & -3.33 & 0.0 & 0.0 & 0.0 & 3.33 & 0.0 \\ 0.0 & 0.0 & -3.33 & 0.0 & 0.0 & 0.0 & 3.33 \\ 0.545 & 0.0 & 0.0 & 0.0 & -0.545 & 0.0 & 0.0 \\ 0.0 & 0.0 & 6.0 & 6.0 & -0.05 & 0.0 & 0.0 \\ -5.2083 & 0.0 & 0.0 & 0.0 & 0.0 & -12.5 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & -5.2083 & 0.0 & -12.5 \end{bmatrix}$$

$$\overline{B}^T = \begin{bmatrix} 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 12.5 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 12.5 \end{bmatrix}.$$

Note that the physical interpretation of the states is remaining same after transformation and in general, it is not true. Corresponding transformed nominal system matrices are

$$\overline{\Gamma}^T = \begin{bmatrix} -6.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & -6.0 & 0.0 & 0.0 \end{bmatrix}$$

and

$$\overline{C} = \begin{bmatrix} 1.0 & 0.0 & 0.0 & 1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & -1.0 & 1.0 & 0.0 & 0.0 \end{bmatrix}.$$

Define $\overline{X}_1^T = \begin{bmatrix} \Delta f_1 & \Delta P_{g1} & \Delta P_{g2} & \Delta P_{tie} & \Delta f_2 \end{bmatrix}$ and $\overline{X}_2^T = \begin{bmatrix} \Delta X_{g1} & \Delta X_{g2} \end{bmatrix}$.

Case A. It should be pointed out that if the original system parameters are free from perturbation and not excited by external disturbance then we need to design a P-type sliding surface. When the system state trajectory comes on the sliding surface the closed loop dynamics are described by reduced order model (7). The state feedback control gain S_1 of the reduced order model (7) (switching function) can be found out by minimizing the performance index

$$J = \int_0^t (\overline{X}_1^T Q \overline{X}_1 + \overline{X}_2^T R \overline{X}_2)\,\mathrm{d}t \tag{45}$$

where $Q = 15\, I_{5\times5}$ and $R = 10\, I_{2\times2}$. The resulting value of switching surface gain matrix

$$S = \begin{bmatrix} 1.2053 & 1.6153 & -0.0016 & -0.8141 & -0.0023 & 1 & 0 \\ -0.0023 & -0.0016 & 1.6153 & 0.8141 & 1.2053 & 0 & 1 \end{bmatrix}. \tag{46}$$

The equivalent control law (9) is given by

$$U_{\text{eq}}(t) =$$

$$-\begin{bmatrix} -0.4568 & 0.1479 & -0.0007 & -0.5797 & 0.0355 & -0.5693 & -0.0004 \\ 0.0355 & -0.0007 & 0.1479 & 0.5797 & -0.4568 & -0.0004 & -0.5693 \end{bmatrix} \overline{X}(t). \tag{47}$$

The range space eigenvalues are located at 0.4 and 0.3 that are unstable and the corresponding fixed gain matrix is given by

$$K_f = \begin{bmatrix} -0.4954 & 0.0962 & -0.0006 & -0.5536 & 0.0356 & -0.6013 & -0.0004 \\ 0.0356 & -0.0006 & 0.1091 & 0.5601 & -0.4857 & -0.0004 & -0.5933 \end{bmatrix}.$$

To satisfy the reaching conditions (28) based on SVD method the value of switch gain matrix is chosen as

$$\overline{\Delta K_s^*} = \begin{bmatrix} 5 & \pm 2 & 0 & 0 & 0 & 0 & 0 \\ \pm 2 & 5 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Computation of $\overline{\Delta K}^*_{as,ij}$ based on fuzzy logic approach (see Figure 1) yields:

$$\alpha_1 \overline{W}_{11} = 1.1275, \quad \alpha_1 \overline{W}_{22} = 1.1275, \quad \alpha_1 \overline{W}_{12} = \alpha_1 \overline{W}_{21} = 0.1611.$$

For $i = j$, $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, m$.
Width of input $(\bar{\sigma}_i \bar{x}_{a,i})$ membership function, $2\overline{L} = 1.0$.
Width of output $(\overline{\Delta K}^*_{as,ij})$ membership function $2\overline{L}_1 = 2$.
For $i \neq j$, $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, m$.
Width of input $(\bar{\sigma}_i \bar{x}_{a,j})$ membership function, $2\overline{L}_2 = 2$.
Width of output $(\overline{\Delta K}^*_{as,ij})$ membership function $2\overline{L}_3 = 2$.

The computer simulation of the composite system has been performed taking a initial state disturbance of $\overline{X}(0) = \begin{bmatrix} 0.5 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$. Design of switch gain matrix based on fuzzy logic approach (soft computing) is compared with that of hard switching gain matrix and comparison of system responses using the proposed control strategies are shown in Figures 2 – 7. It is observed that the switched gain components designed based on fuzzy logic approach are much smooth than the hard switched gain components (see Figures 5 – 6). As a result, the system responses based on soft switching seem to be better than the hard switching control actions. Furthermore, Figure 3 shows that the reaching time to a final sliding surface based on soft switching is decreased in comparison with the hard switching. Figures 7 shows the robustness of the proposed controller in presence of parameter perturbation and external disturbances.

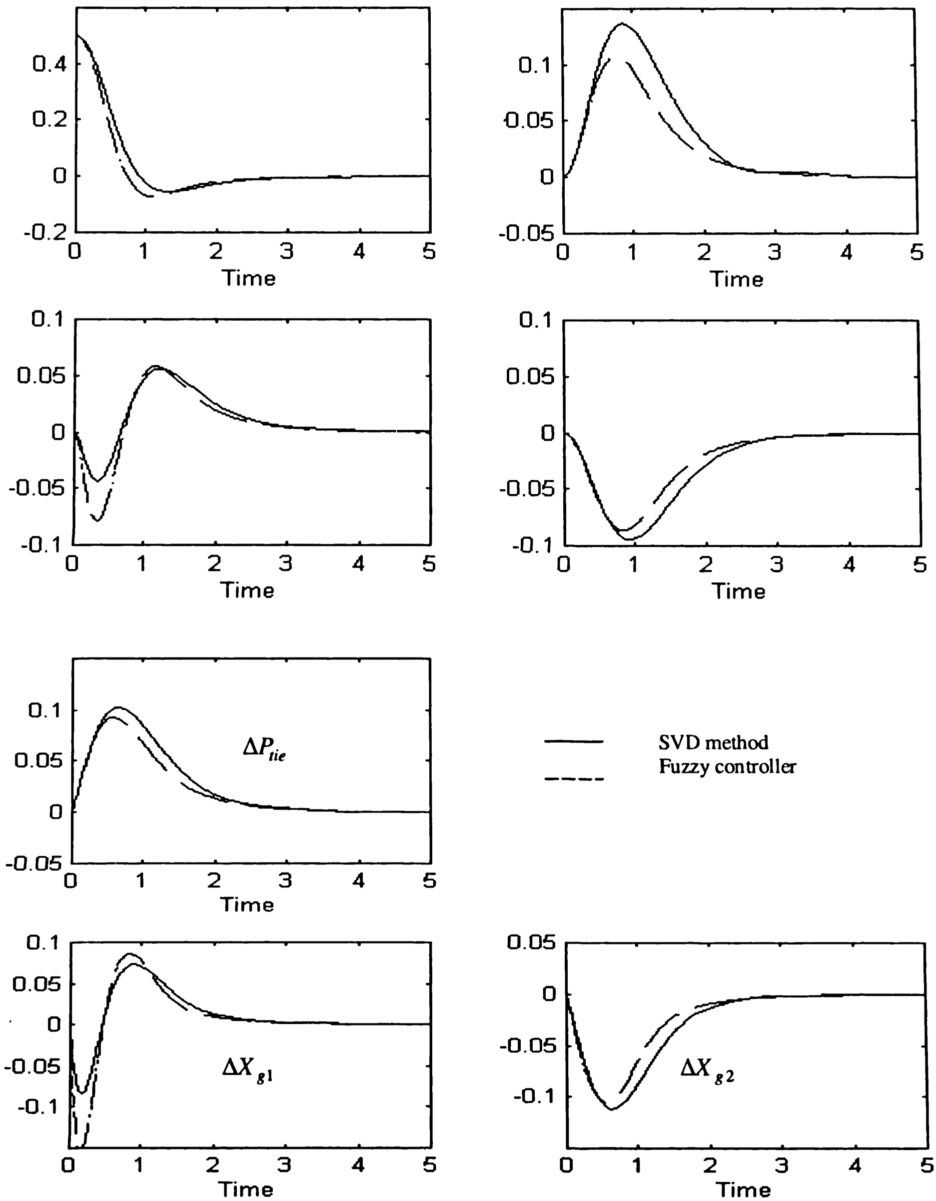

Fig. 2. Comparison of nominal system responses with state disturbance only.

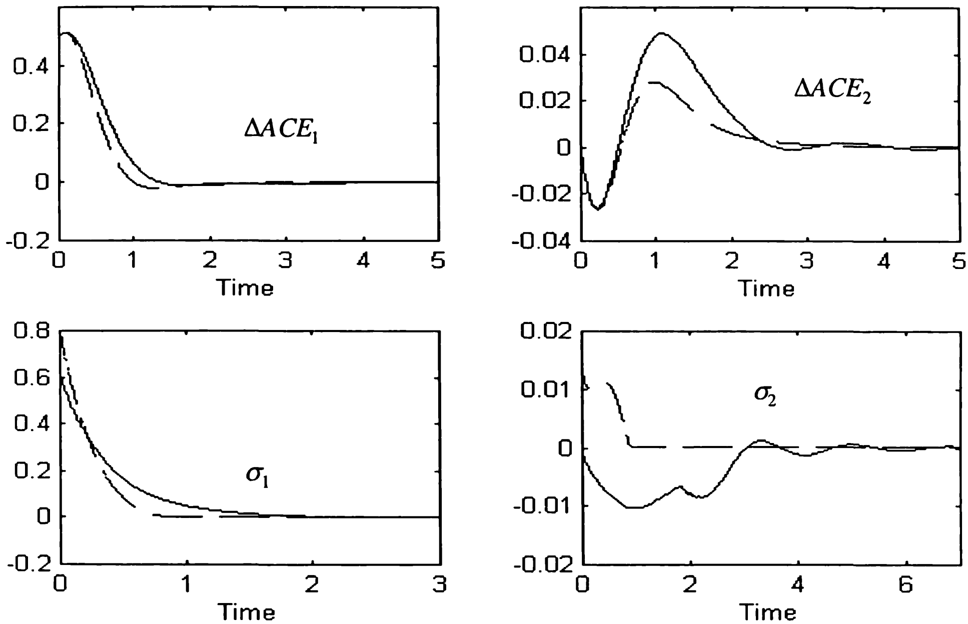

Fig. 3. Area control errors and Sliding surface trajectories
(— with hard switching structure $\overline{\Delta K^*_{as}}$, - - - with soft switching structure $\overline{\Delta K^*_{as}}$).

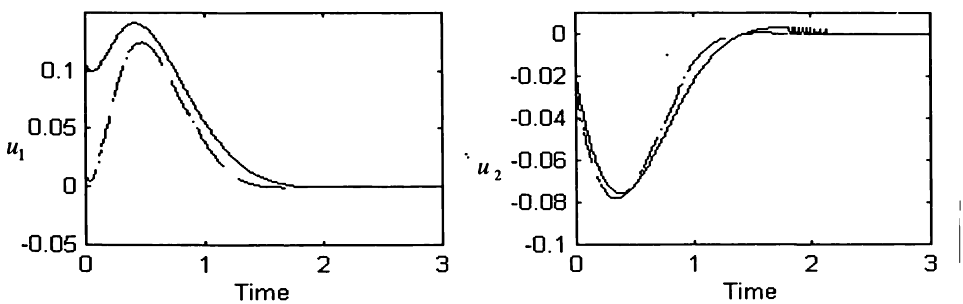

Fig. 4. Comparison of Control input sequences
(— with hard switching structure $\overline{\Delta K^*_{as}}$, - - - with soft switching structure $\overline{\Delta K^*_{as}}$).

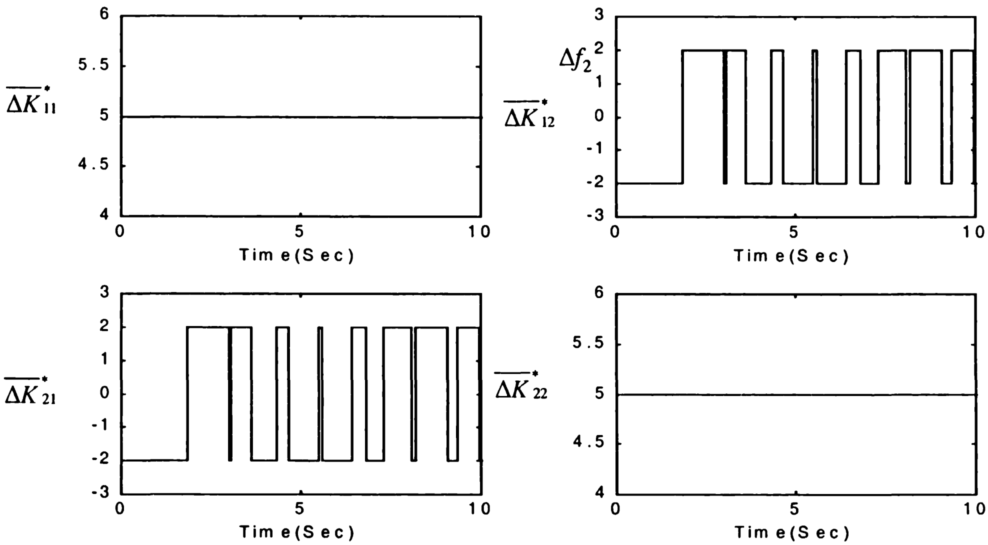

Fig. 5. Hard switching structures (SVD method) $\overline{\Delta K}^*_{as}$.

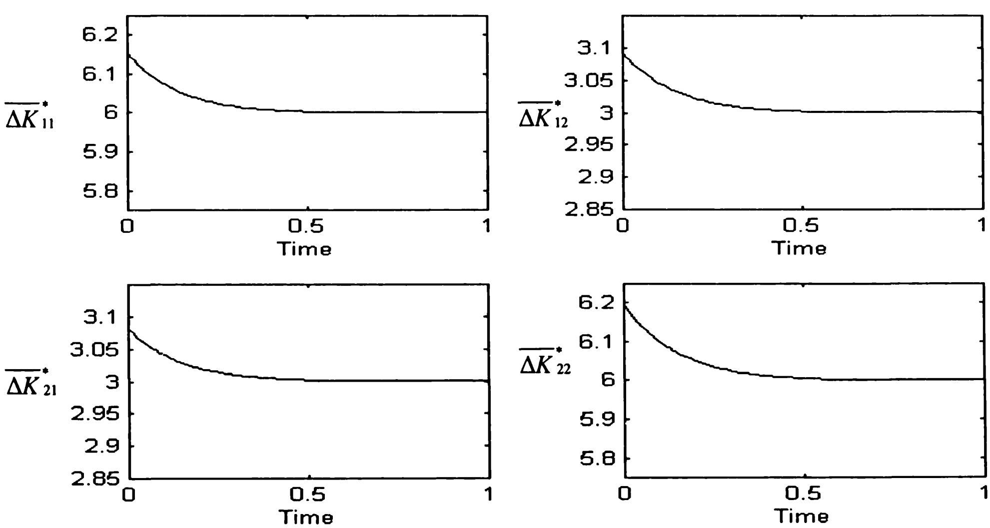

Fig. 6. Soft switching structures (Fuzzy Logic Controller) $\overline{\Delta K}^*_{as}$.

Case B. A PI-type sliding is chosen as in equation (19) when the system is perturbed with parameter perturbations and external disturbances. Switching function is then designed by adopting the procedure as discussed in Section 2.
Selecting the value of $Q_a = 15\, I_{7\times 7}$ and $R_a = 10\, I_{2\times 2}$, the corresponding sliding surface gain matrices are

$$S = \begin{bmatrix} 1.5874 & 1.8662 & 0.0167 & -1.0052 & 0.0266 & 1 & 0 \\ 0.0266 & 0.0167 & 1.8662 & 1.0052 & 1.5874 & 0 & 1 \end{bmatrix}$$

and

$$W = \begin{bmatrix} 1.2247 & 0 \\ 0 & 1.2247 \end{bmatrix}.$$

The range space eigenvalues are placed at 0.4 and 0.3 and the corresponding fixed gain matrix K_{af} of the augmented system is obtained using the expression (23)

$$K_{af} =$$

$$\begin{bmatrix} -0.4195 & 0.2046 & 0.0078 & -0.6190 & 0.0429 & -0.0392 & 0 & -0.5344 & 0.0045 \\ 0.0431 & 0.0079 & 0.2196 & 0.6271 & -0.4068 & 0 & -0.0294 & 0.0045 & -0.5264 \end{bmatrix}.$$

To satisfy the reaching conditions (28) based on SVD method the value of switch gain matrix is chosen as

$$\overline{\Delta K}^{*}_{as} = \begin{bmatrix} 5 & \pm 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \pm 2 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

and simultaneously the reaching conditions must be satisfied.

Simulation results are shown with an initial state disturbance of
$\overline{X}(0) = [\ 0.5 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0\ ]^T$ and 10% step change in load demand in area-1.

Performance of the system based on the proposed variable structure control schemes has been studied qualitatively and system responses are shown in Figure 6, which proves the robustness of the designed techniques. When the system trajectory reaches the sliding surface an equivalent control law (35) where

$$U_{\text{eq}}(t) =$$

$$-\begin{bmatrix} -0.3687 & 0.2644 & 0.0083 & -0.6512 & 0.0437 & -0.5824 & 0.0845 \\ 0.0437 & 0.0083 & 0.2644 & 0.6512 & -0.5770 & 0.0045 & -0.8024 \end{bmatrix} \overline{X}(t)$$

is employed to maintain the state trajectory on the sliding surface.

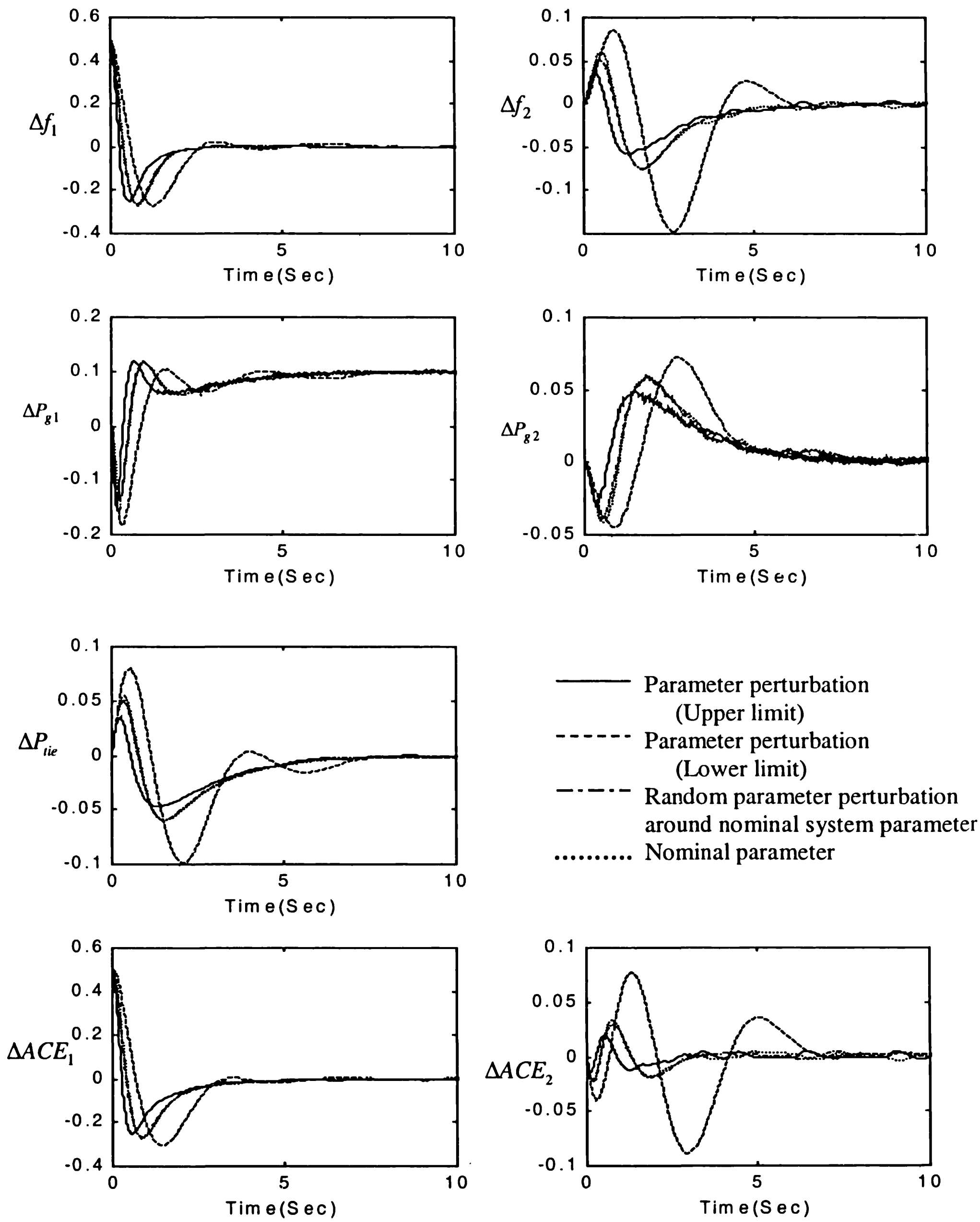

Fig. 7. System responses for state disturbance and 10% step change in the load demand in area-1 (SVD method).

6. CONCLUSIONS

A state feedback VSS with sliding mode controller is designed based on singular value decomposition technique. A proportional plus integral type-sliding surface has been developed while disturbance-matching condition is not satisfied. It has been shown in table that the proposed technique requires less number of switching gain vector components as compared to that of White et al method [11] and moreover, the proposed method does not need to satisfy any additional inequality constraints to reach the sliding surface. This method allows the system to drive from any initial state to the sliding surface in finite interval of time. Design of transformed switch gain matrix elements via fuzzy logic approach provides a smooth control action and also requires less time to hit the sliding surface. Simulation results have confirmed the robustness of the proposed controller in presence of parameter uncertainties and external disturbances.

ACKNOWLEDGEMENT

The authors would like to thank the anonymous reviewers whose critical analysis and suggestions brought insight into the final editing of this paper.

(Received September 18, 2002.)

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