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GENERALIZED IMMERSION AND NONLINEAR ROBUST OUTPUT REGULATION PROBLEM[1]

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The problem of output regulation of the system affected by unknown constant parameters is considered here. Under certain assumptions, such a problem is known to be solvable using error feedback via the so-called immersion to an observable linear system with outputs. Nevertheless, for many interesting cases this kind of finite dimensional immersion is difficult or even impossible to find. In order to achieve constructive procedures for wider classes, this paper investigates a more general type of immersion. Such a *generalized* immersion enables to solve robust output regulation problem via dynamic feedback compensator using error and exosystem state measurement. When the exosystem states are not completely measurable, a modified observed-based generalized immersion is then presented. The main result obtained here is that under reasonable assumptions both the full and partial exosystem measurement problems are equivalently solvable. Examples together with computer simulation are included to clarify the suggested approach.

Keywords: output regulation, robust, nonlinear, immersion

AMS Subject Classification: 93C10, 93D20

1. INTRODUCTION

A central problem in control theory and applications is to design a control law to achieve asymptotic tracking with disturbance rejection in nonlinear systems. When the class of reference inputs and disturbances are generated by an autonomous differential equations, this problem is called nonlinear output regulation problem, or, alternatively, nonlinear servomechanism problem, see e. g. [8] and [9]. The problem can precisely be formulated as follows:

Consider a nonlinear plant described by

$$\begin{aligned} \dot{x} &= f(x, w, u, \mu) \\ e &= h(x, w, \mu) \end{aligned} \tag{1}$$

where the first equation of (1) describes the dynamics of a *plant*, whose *state* x is defined in a neighborhood U of the origin in $\mathbb{R}^n$, with *control input* $u \in \mathbb{R}^m$ and

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subject to a set of *exogenous* input variables $w \in \mathbb{R}^r$, which includes *disturbances* (to be rejected) and/or *references* (to be tracked) and $\mu \in \mathbb{R}^p$ is a vector of unknown parameters. The second equation defines an *error* variable $e \in \mathbb{R}^s$, which is expressed as a function of the state x, the exogenous input w and the vector of unknown parameters μ. Suppose $\mu = 0$ to be a nominal value of the parameter μ and assume $f(x, w, u, \mu)$ and $h(x, w, \mu)$ to be smooth functions of their arguments with $f(0, 0, 0, \mu) = 0$ and $h(0, 0, \mu) = 0$ for each value of μ.

The family of the exogenous inputs $w(\cdot)$ affecting the plant will be taken as the family of all functions of time which are the solution of the autonomous differential equation

$$\dot{w} = s(w) \tag{2}$$

with initial condition $w(0)$ ranging on some neighborhood W of the origin of R^r. This system, which is viewed as a mathematical model of a *"generator"* of all possible exogenous input functions, is called exosystem. Through the paper, (2) is assumed to be neutrally stable, which is a standard assumption for exogenous systems.

Beginning with the pioneering works [8] and [6], the nonlinear output regulation problem has been studied intensively during the last decade. Basic results on full information feedback case, error feedback case and the so-called robust output regulation are collected in [9], some results on full information nonsmooth feedback were obtained in [2], and [3]. For further robust aspects of the output regulation see [7] and [1] and references within there. In particular, it has been shown that the inclusion of an internal model in the controller structure was necessary and sufficient for having robust regulation [5]. Following these ideas in [9], an error feedback controller which relies on the existence of an internal model, which represent an immersion of the exosystem dynamics into an observable one, was presented. This immersion allows to generate, as in the linear case, all the possible steady state inputs for the admissible values of the system parameters.

Nevertheless, in general, the corresponding necessary and sufficient conditions are quite abstract and nonconstructive in the general case, even if only sufficient conditions are taken by requiring to find an immersion of the exosystem into a linear observable system. This later approach has been shown to have an explicit solution for the class of systems for which the steady state input is given in a polynomial form with respect of the exosystem states. This paper explores the possibility of using a generalized immersion, in the sense that the immersion is allowed to depend explicitly on the exosystem states. By some characteristic examples, it is shown that in some cases in which the linear immersion is difficult, or even impossible to find, the suggested approach gives an alternative solution to the Robust Regulation Problem (RORP).

More precisely, it was shown in [4] that the existence of the generalized immersion provides solvability of RORP, provided the measurement of the exosystem state is allowed. Here, we aim to further enhance this result by showing that (under certain reasonable condition), one may use a suitable exosystem output measurement only. Typical candidate for such an output might be reference to be tracked in case when RORP represents tracking problem. In other words, it seems to be quite realistic in many applications to assume that tracking error is not measured directly but is computed as difference between the measured output to be controlled and measured reference to be tracked.

In a broader perspective, our approach may be used to distinguish between exosystem components describing the reference and those describing unknown disturbances. Main idea here would be to find immersions that would eliminate only the disturbance related components, thereby soften restrictions for its existence. The present paper, nevertheless, will consider the more simple version when the whole exosystem state is observable from a suitable additional exosystem output and will show how to combine this property with existing (if any) generalized immersion.

The paper is organized as follows. In next section we summarize the basic definitions and known results on the robust output regulation problem. The contribution of the paper is presented in Section 3, while Section 4 presents several illustrative examples and simulations. Conclusions and some ideas for the future research are drawn in the final section.

2. ELEMENTS OF ROBUST OUTPUT REGULATION

2.1. Error feedback solution to RORP and linear immersion problem

Definition 1. (Robust Output Regulation Problem (RORP).) Given a nonlinear system of the form (1) and a neutrally stable exosystem (2), find , if possible, an integer ν, two mappings $\theta(\xi)$ and $\eta(\xi, e)$ (with $\xi \in \Xi \subset \mathbb{R}^\nu, \theta : \mathbb{R}^\nu \to \mathbb{R}^m, \eta : \mathbb{R}^\nu \times \mathbb{R}^s \to \mathbb{R}^\nu$) and a neighborhood $\mathcal{P}$ of $\mu = 0$ in $\mathbb{R}^p$ such that, for each $\mu \in \mathcal{P}$:

(S) the equilibrium $(x, \xi) = (0, 0)$ of

$$\dot{x} = f(x, 0, \theta(\xi), \mu)$$
$$\dot{\xi} = \eta(\xi, h(x, 0, \mu))$$

is asymptotically stable in the first approximation,

(R) there exists a neighborhood $V \subset U \times \Xi \times W$ of $(0, 0, 0)$ such that, for each initial condition $(x(0), \xi(0), w(0)) \in V$, the solution of

$$\dot{x} = f(x, w, \theta(\xi), \mu)$$
$$\dot{\xi} = \eta(\xi, h(x, w, \mu))$$
$$\dot{w} = s(w)$$

is such that $\lim_{t\to\infty} e(t) = 0$.

The following result, which can be found in [9], gives conditions for the existence of a solution to the RORP.

Theorem 1. (N&S condition for RORP.) The Robust Output Regulation Problem is solvable if and only if there exist mappings $x_{ss} = \pi^a(w, \mu)$ and $u_{ss} = c^a(w, \mu)$, with $\pi^a(0, \mu) = 0$ and $c^a(0, \mu) = 0$, both defined in a neighborhood $W^o \times \mathcal{P} \subset W \times \mathbb{R}^p$ of the origin, satisfying the conditions

$$\begin{aligned} \frac{\partial \pi^a(w,\mu)}{\partial w} s(w) &= f(\pi^a(w,\mu), w, c^a(w,\mu), \mu) \\ 0 &= h(\pi^a(w,\mu), w, \mu) \end{aligned} \tag{3}$$

for all $(w, \mu) \in W^o \times \mathcal{P}$, and such that the autonomous system with output denoted as $\{W^o \times \mathcal{P}, s^a, c^a\}$ and given by

$$\frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} w \\ \mu \end{bmatrix} = \begin{bmatrix} s(w) \\ 0 \end{bmatrix} := s^a(w),\ u = c^a(w,\mu),\ (w,\mu) \in W^o \times \mathcal{P}$$

is immersed into a system

$$\begin{aligned} \dot{\xi} &= \varphi(\xi) \\ u &= \gamma(\xi) \end{aligned}$$

defined on a neighborhood Ξ^o of the origin in $\mathbb{R}^\nu$, in which $\varphi(0) = 0$ and $\gamma(0) = 0$, and the two matrices

$$\Phi = \left[\frac{\partial \varphi}{\partial \xi}\right]_{\xi=0}, \quad \Gamma = \left[\frac{\partial \gamma}{\partial \xi}\right]_{\xi=0}$$

are such that the pair

$$\begin{pmatrix} A(0) & 0 \\ NC(0) & \Phi \end{pmatrix}, \begin{pmatrix} B(0) \\ 0 \end{pmatrix}$$

is stabilizable for some choice of the matrix N, and the pair

$$\begin{pmatrix} C(0) & 0 \end{pmatrix}, \begin{pmatrix} A(0) & B(0)\Gamma \\ 0 & \Phi \end{pmatrix}$$

is detectable, where $A(\mu) = \left[\frac{\partial f}{\partial x}\right]_{(0,0,0,\mu)}; B(\mu) = \left[\frac{\partial f}{\partial u}\right]_{(0,0,0,\mu)}; C(\mu) = \left[\frac{\partial h}{\partial x}\right]_{(0,0,\mu)}$.

Corollary 1. The RORP is solvable by means of a linear controller if the pair $(A(0), B(0))$ is stabilizable, the pair $(C(0), A(0))$ is detectable, there exist mappings $x_{ss} = \pi^a(w,\mu)$ and $u_{ss} = c^a(w,\mu)$, with $\pi^a(0,\mu) = 0$ and $c^a(0,\mu) = 0$, both defined in a neighborhood $W^o \times \mathcal{P} \subset W \times \mathbb{R}^p$ of the origin, satisfying the conditions (3) and such that, for some set of q real numbers $a_0, a_1, ..., a_{q-1}$,

$$\begin{aligned} L_s^q c^a(w,\mu) &= a_0 c^a(w,\mu) + a_1 L_s c^a(w,\mu) + \dots \\ &\quad \dots + a_{q-1} L_s^{q-1} c^a(w,\mu) \end{aligned} \tag{4}$$

for all $(w, \mu) \in W^o \times \mathcal{P}$, and, moreover, the matrix

$$\begin{pmatrix} A(0) - \lambda I & B(0) \\ C(0) & 0 \end{pmatrix}$$

is nonsingular for every λ which is a root of the polynomial

$$p(\lambda) = a_0 + a_1\lambda + \dots + a_{q-1}\lambda^{q-1} - \lambda^q$$

having non-negative real part.

Remark 1. The mapping $x_{ss} = \pi^a(w,\mu)$ represents the steady state zero output submanifold and $u_{ss} = c^a(w,\mu)$ is the steady state input which makes invariant the steady state zero output submanifold. Conditions (4) expresses the fact that this steady state input can be generated, independently of the values of the parameter vector, by the dynamical system

$$\begin{aligned} \dot{\xi}_2 &= \Phi \xi_2 \\ u_{ss} &= \Gamma \xi_2, \end{aligned} \tag{5}$$

where

$$\begin{aligned} \xi_2 &= \begin{pmatrix} \xi_2^1 & \xi_2^2 & \cdots & \xi_2^m \end{pmatrix}^T; \\ \xi_2^i &= \left(c_i^a(w,\mu),\, L_s c_i^a(w,\mu), \ldots, L_s^{q_i-1} c_i^a(w,\mu)\right)^T \\ \Gamma &= \begin{pmatrix} \Gamma_1 & 0 & \cdots & 0 \\ 0 & \Gamma_2 & \ddots & 0 \\ 0 & 0 & \cdots & \Gamma_m \end{pmatrix}; \\ \Gamma_i &= \begin{pmatrix} 1 & 0 & \cdots & 0 \end{pmatrix}_{1\times q_i} \end{aligned}$$

and

$$\begin{aligned} \Phi &= \operatorname{diag}\begin{pmatrix} \Phi_1, & \Phi_2, & \cdots, & \Phi_m \end{pmatrix}; \\ \Phi_i &= \begin{pmatrix} 0 & 1 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 1 \\ a_0^i & a_1^i & \cdots & a_{q_i-1}^i \end{pmatrix}. \end{aligned}$$

2.2. A generalized immersion

As it has been shown in the previous section, if it is possible to find the immersion Φ or equivalently, the constants $a_0, a_1, \ldots, a_{q-1}$, then the robustness property is achieved. Nevertheless, the classes of systems where it is possible are quite narrow. In fact, it is possible to show that for the case of linear systems, and in the case when the mapping $u_{ss} = c^a(w,\mu)$ is polynomial in the state variables w, a solution can be readily obtained [1], but for many interesting cases, however, as in the case when the mapping $u_{ss} = c^a(w,\mu)$ includes sinusoidal, exponential or rational terms, this solution is hard, or even impossible to find, since the dimension of the immersion would be infinite. A possible way to deal with this situation would be to use an approximate solution [1], or to seek for an alternative solution. In this sense, we will show that if we allow the immersion to depend explicitly on w, then in many cases it is possible to find a finite dimensional immersion. We will call this a *generalized immersion.* More precisely, we say that the extended exogenous system

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} w \\ \mu \end{bmatrix} = \begin{bmatrix} s(w) \\ 0 \end{bmatrix} \tag{6}$$

having the output $c^a(w,\mu)$, allows generalized immersion if it is smoothly immersed into a system of the form

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} w \\ \xi \end{bmatrix} = \begin{bmatrix} s(w) \\ \Phi(w)\xi \end{bmatrix}. \tag{7}$$

Obviously, if we were able to find such an immersion, the robust regulator would have also a solution, provided we are allowed to measure directly the exosystem state. Without going into the detailed definition, we will call such a problem in the sequel as the RORP with the full exosystem measurement, cf. [4] where the following result has been obtained.

Theorem 2. Consider system (1) with $s = 1$ and $m = 1$. The RORP with full exosystem measurement is solvable if and only if there exists mappings $\pi^a(w,\mu)$ and $c^a(w,\mu)$, $\pi^a(0,0) = 0$, $c^a(0,0) = 0$, solving the regulator equation (3), such that extended exogenous system (6) with output $c^a(w,\mu)$ is immersed into (7) and the following conditions hold

a) the pair

$$\begin{bmatrix} A & 0 \\ NC & \Phi(0) \end{bmatrix}, \begin{bmatrix} B \\ 0 \end{bmatrix} \tag{8}$$

is stabilizable for some choice of the matrix N;

b) the pair

$$\begin{bmatrix} C & 0 \end{bmatrix}, \begin{bmatrix} A & B\Gamma \\ 0 & \Phi(0) \end{bmatrix}, \Gamma = [\,1\; 0 \cdots 0\,] \tag{9}$$

is detectable.

Remark 2. As a matter of fact, the structure of the controller solving RORP with exosystem measurement is similar to that given in [1], namely,

$$\begin{aligned} \begin{pmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \end{pmatrix} &= \begin{pmatrix} K & 0 \\ 0 & \Phi(w) \end{pmatrix}\begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} + \begin{pmatrix} L \\ N \end{pmatrix} e, \\ u &= M\xi_1 + \Gamma\xi_2, \end{aligned}$$

where the matrices K, L, M, N of the appropriate dimensions are such that

$$\begin{bmatrix} \begin{bmatrix} A & B\Gamma \\ NC & \Phi(0) \end{bmatrix} & \begin{bmatrix} B \\ 0 \end{bmatrix} M \\ L[C\ 0] & K \end{bmatrix} \tag{10}$$

has all eigenvalues with negative real parts. The existence of K, L, M, N is the direct consequence of assumptions **a)** and **b)** of Theorem 2.

3. MAIN RESULT

A natural question arises, whether the measurement of exosystem state does not present additional obstacle for practical implementation. As a reasonable motivation one can consider the case when exosystem produces a reference to be tracked only. In this case it is acceptable to assume that we can measure a desired reference signal. As a matter of fact, it seems to be natural to suppose that in case of tracking the output of the exosystem, say $y_w = h(0, w, 0) =: r(w)$, by the output of the system to be controlled, say $y = h(x)$, not only the error $e = y - y_w$, but also y_w is available for measurement.

Let us note that there still exists uncertainty in the studied problem represented by unknown parameters μ. One can also consider splitting exosystem into two parts, one of them responsible for the reference to be tracked and another one for unknown disturbances to be rejected obtaining thereby further generalization to our problem. This is left to future research, nevertheless, the main message here is that the unknown disturbances should be treated in a different way than the known references to be tracked. When finding the immersion, only unknown part should be eliminated what increases chances for its existence.

For simplicity, we concentrate in this paper to the case when the system $\dot{w} = s(w)$, $y_w = r(w)$ is observable and $y_w = r(w)$ can be independently measured, so that the only uncertainty is due to unknown parameters μ. Anyway, even with such a simplification, this still does not mean we are able to measure the full exosystem state so that the results of [4] are not applicable. For this reason we introduce the RRORP with *partial* exosystem measurement.

Definition 2. (RORP with partial exosystem measurement.) Given a nonlinear system of the form (1) and a neutrally stable exosystem (2) with additional output

$$y_w = r(w), \qquad y_w \in \mathbb{R}^{s'},$$

find , if possible, an integer ν, two mappings $\theta(\xi)$ and $\eta(\xi, e, y_w)$ (with $\xi \in \Xi \subset \mathbb{R}^\nu, \theta : \mathbb{R}^\nu \to \mathbb{R}^m, \eta : \mathbb{R}^\nu \times \mathbb{R}^s \times \mathbb{R}^{s'} \to \mathbb{R}^\nu$) and a neighborhood $\mathcal{P}$ of $\mu = 0$ in $\mathbb{R}^p$ such that, for each $\mu \in \mathcal{P}$:

(S) the equilibrium $(x, \xi) = (0, 0)$ of

$$\begin{aligned} \dot{x} &= f(x, 0, \theta(\xi), \mu) \\ \dot{\xi} &= \eta(\xi, h(x, 0, \mu), r(0)) \end{aligned}$$

is asymptotically stable in the first approximation,

(R) there exists a neighborhood $V \subset U \times \Xi \times W$ of $(0, 0, 0)$ such that, for each initial condition $(x(0), \xi(0), w(0)) \in V$, the solution of

$$\begin{aligned} \dot{x} &= f(x, w, \theta(\xi), \mu) \\ \dot{\xi} &= \eta(\xi, h(x, w, \mu), r(w)) \\ \dot{w} &= s(w) \end{aligned}$$

is such that $\lim_{t \to \infty} e(t) = 0$.

The main contribution of this paper is the following result showing that under reasonable observability assumption there is no difference between full and partial exosystem measurement. For simplicity and to pick up the key idea of our approach, we limit ourselves to the case when the controlled input, error and the additional exosystem outputs are all scalars.

Theorem 3. Consider system (1) with $s = 1$, $m = 1$ and the exosystem (2) with an additional output $y_w = r(w) \in \mathbb{R}$, i.e. $s' = 1$. Further, let us assume that there exists local asymptotic observer for the exosystem state w given by

$$\dot{\hat{w}} = g(\hat{w}, y_w)$$

with the corresponding error dynamics for $\epsilon = (w - \hat{w})$ as

$$\dot{\epsilon} = \phi(\varepsilon, w)$$

where $\frac{\mathrm{d}\phi}{\mathrm{d}\hat{w}}(0,0)$ is a Hurwitz matrix. Then RORP with partial exosystem measurement is solvable if and only if RORP with full exosystem measurement is solvable. Moreover, the corresponding controller has the following form the controller

$$\begin{pmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \end{pmatrix} = \begin{pmatrix} K & 0 \\ 0 & \Phi(\hat{w}) \end{pmatrix} \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} + \begin{pmatrix} L \\ N \end{pmatrix} e \tag{11}$$

$$\dot{\hat{w}} = g(\hat{w}, y_w) \tag{12}$$

$$u = M\xi_1 + \Gamma\xi_2, \ \Gamma = [\,1\ 0\ \cdots\ 0\,]. \tag{13}$$

where K, L, M, N are as in Remark 1.

Proof. The "only if" part is obvious by formulation and Theorem 2.

To prove "if part", consider the controller (11). Using assumption of the Theorem on exosystem observer, we may represent it as follows

$$\begin{pmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \end{pmatrix} = \begin{pmatrix} K & 0 \\ 0 & \Phi(w-\epsilon) \end{pmatrix} \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} + \begin{pmatrix} L \\ N \end{pmatrix} e \tag{14}$$

$$\dot{\epsilon} = \phi(\epsilon, w) \tag{15}$$

$$u = M\xi_1 + \Gamma\xi_2. \tag{16}$$

Since the matrix $S_\epsilon = \frac{\partial\phi}{\partial\epsilon}(0,0)$ is Hurwitz by assumption and conditions a) and b) of Theorem 2 hold, the following matrix

$$\begin{bmatrix} \begin{bmatrix} \begin{bmatrix} A & B\Gamma \\ NC & \Phi(0) \end{bmatrix} & \begin{bmatrix} B \\ 0 \end{bmatrix} M \\ L[C\ 0] & K \end{bmatrix} & 0 \\ 0 & S_\epsilon \end{bmatrix}, \tag{17}$$

is also Hurwitz. Moreover, (17) is the matrix of linear approximation of system in the condition (S) of Definition 2, so that this condition is proved. To prove condition

(R) of Definition 2, notice that the matrix of linear approximation of the closed loop system considered there is

$$\begin{bmatrix} \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} A & B\Gamma \\ NC & \Phi(0) \end{bmatrix} & \begin{bmatrix} B \\ 0 \end{bmatrix} M \\ L[C\ 0] & K \end{bmatrix} & 0 \\ 0 & S_\epsilon \end{bmatrix} & * \\ 0 & \frac{\partial s}{\partial w}(0) \end{bmatrix}, \tag{18}$$

so that there exists locally attractive center manifold for that closed loop system. At the same time, writting down the partial differential equation for this center manifold graph, one can see that it coincides with PDE part of FIB equation for the extended system. By the theorem assumption on solvability of FIB equation that means, in particular, that also algebraic part of FIB holds. In other words, the closed loop system in condition (R) of the theorem being proved posseses locally attractive center manifold and on this center manifold $e \equiv 0$. That obviously guarantees that locally $e(t) \to 0$ as $t \to \infty$ and the theorem has been proved. □

4. SOME ILLUSTRATIVE EXAMPLES

The main point in the previous discussion is how to find the generalized immersion. The following examples, for which an immersion of the form (5) does not exist, give a possible procedure for some interesting cases. The exosystem for all the cases is a simple linear oscillator given by $\dot{w}_1 = w_2$, $\dot{w}_2 = -w_1$.

Example 1. Consider the term $c^a(w,\mu) = \frac{aw_1}{1+w_2} =: z_1$. Then, differentiating successively this term, we get

$$\begin{aligned} \dot{z}_1 &= z_2 = \frac{aw_1^2}{(1+w_2)^2} + \frac{aw_2}{1+w_2} \\ &= \frac{w_1}{1+w_2} z_1 + \frac{aw_2}{1+w_2} \\ \dot{z}_2 &= \frac{-1+w_2^2+w_1^2}{(1+w_2)^2} z_1 + \frac{w_1}{1+w_2} z_2 \end{aligned}$$

and the immersion is

$$\dot{\xi} = \begin{pmatrix} 0 & 1 \\ \frac{-1+w_2^2+w_1^2}{(1+w_2)^2} & \frac{w_1}{1+w_2} \end{pmatrix} \xi.$$

Example 2. Let $c^a(w,\mu) = w_1 e^{w_2}$. Then

$$\begin{aligned} z_2 &= w_2 e^{w_2} - w_1^2 e^{w_2} = w_2 e^{w_2} - w_1 z_1 \\ \dot{z}_2 &= -w_2 z_1 - w_1 w_2 e^{w_2} - w_1 z_2 \\ &= -(1+2w_2) z_1 - w_1 z_2, \end{aligned}$$

and the immersion is

$$\dot{\xi} = \begin{pmatrix} 0 & 1 \\ -(1+2w_2) & -w_1 \end{pmatrix} \xi.$$

Example 3. For the case $c^a(w,\mu) = aw_1 \cos w_2$, we have:

$$\begin{aligned}
z_2 &= aw_2 \cos w_2 + aw_1^2 \sin w_2 \\
z_3 &= 3aw_1 w_2 \sin w_2 - (1+w_1^2) z_1 \\
z_4 &= 3aw_2^2 \sin w_2 + 3aw_2 \cos w_2 - 5w_1 w_2 z_1 \\
&\quad -(4+w_1^2) z_2 \\
\dot{z}_4 &= -3w_2^2 aw_1 \cos w_2 - 3w_2 aw_1 \sin w_2 \\
&\quad -3aw_1 \cos w_2 - 5w_2^2 z_1 + 5w_1^2 z_1 \\
&\quad -7w_1 w_2 z_2 - (4+w_1^2) z_3 \\
&= \left(-4 + 4w_1^2 - 8w_2^2\right) z_1 - 7w_1 w_2 z_2 \\
&\quad - \left(5 + w_1^2\right) z_3
\end{aligned}$$

and

$$\dot{\xi} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ a_1(w) & a_2(w) & a_3(w) & 0 \end{pmatrix} \xi$$

with

$$\begin{aligned}
a_1(w) &= \left(-4 + 4w_1^2 - 8w_2^2\right) \\
a_2(w) &= -7w_1 w_2 \\
a_3(w) &= -\left(5 + w_1^2\right).
\end{aligned}$$

Example 4. Consider the well-known model of the inverted pendulum together with the exosystem being a simple linear oscillator

$$\dot{x}_1 = x_2,\ \dot{x}_2 = g\sin(x_1) - cu,\ \dot{w}_1 = \alpha w_2,\ \dot{w}_2 = -\alpha w_1,\ \alpha \in \mathbb{R}.$$

The regulation goal is to make zero the following error $y = e = x_1 - w_1$. The parameters g, c are supposed to be known only approximately, so that the algorithm should be robust with respect to them. The solution of the regulator equations (3) is $\pi_1^a = w_1$, $\pi_2^a = \alpha w_2$, and

$$c^a(w,\mu) = \frac{\alpha^2 w_1 + g\sin(w_1)}{c} = aw_1 + b\sin(w_1),$$

where $a = \alpha^2/c$, $b = g/c$ is the reparametrization of the unknown parameters made to simplify further exposition. To find the generalized immersion, let us take

$$\begin{aligned}
z_1 &= c^a(w,\mu) = aw_1 + b\sin(w_1), \\
z_2 &= \dot{z}_1 = aw_2 + w_2 b\cos w_1, \\
z_3 &= -aw_1 - w_1 b\cos w_1 - w_2^2 b\sin w_1 \\
&\quad -aw_1 - w_1 b\cos w_1 - w_2^2 z_1 + aw_1 w_2^2 \\
z_4 &= -aw_2 + aw_2^3 - 2aw_1^2 w_2 + 2w_1 w_2 z_1 - w_2^2 z_2 \\
&\quad -w_2 b\cos w_1 + w_1 w_2 b\sin w_1 \\
&= aw_2^3 - 3aw_1^2 w_2 + 3w_1 w_2 z_1 - (1 + w_2^2) z_2;
\end{aligned}$$

at this step, we note that all the sinusoidal terms have disappeared, and it remain only polynomial terms. Since for polynomial terms, it is possible to find an immersion of the form (5), we may then guarantee that a generalized immersion can be obtained. In fact, in this case, straightforward computations give

$$\begin{aligned}
z_6 &= \dot{z}_5 = 15w_1 w_2 z_1 - \left(9 + 8w_1^2 + w_2^2\right) z_2 \\
&\quad +7w_1 w_2 z_3 - \left(10 + w_2^2\right) z_4,
\end{aligned}$$

i. e. the generalized immersion is

$$\Phi(w) = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ \phi_1 & \phi_2 & \phi_3 & \phi_4 & 0 \end{pmatrix}$$

$\phi_1 = 15w_1 w_2$, $\phi_2 = -\left(9 + 8w_1^2 + w_2^2\right)$, $\phi_3 = 7w_1 w_2$, $\phi_4 = -\left(10 + w_2^2\right)$.

Figure 1 shows the behavior of this controller when variations on the parameter d is introduced at time $t = 20$ sec. Figure 2 shows the simulation results of the robust controller when an observer-based generalized immersion is introduced. As we may observe, the performance of the controller is similar to that of the case when the generalized immersion depend directly on the state w.

For the previous examples, even if for a particular case of exosystem, we conjecture that in many interesting cases, like those arising in physical systems, it is possible to obtain a generalized immersion by first performing successive differentiations of the mapping $c^a(w,\mu)$ until an expression containing only polynomial terms is derived and then, depending of the maximum degree of the polynomial terms, a finite additional differentiations will allow to get the desired generalized immersion.

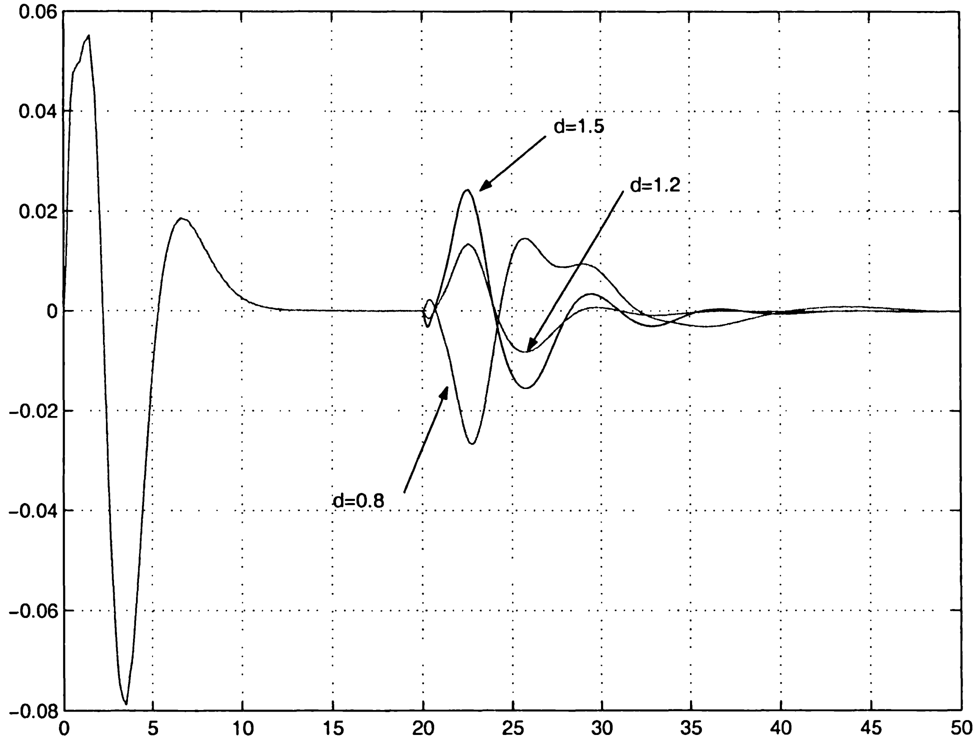

Fig. 1. Output tracking error.

5. CONCLUSIONS

The paper presents an alternative solution to the case when the well-known Isidori and Byrnes solution to the robust output regulation problem is difficult or even impossible to find, namely, the existence of an immersion of the exosystem dynamics into a linear observable one, which generate, as in the linear case, all the possible steady state inputs for the admissible values of the system parameters. Here has been illustrated that by measuring the exosystem state and using it in the design of the compensator, it is possible to find a robust solution for a wider classes of systems, in particular, those for which the steady state input contains non-polynomial terms (i. e. sinuses, cosinuses and exponential terms). Moreover, it has been shown that the introduction of an observer for the exosystem, helps to overcome the necessity of measuring all the exosystem states, thereby further motivating the problem of finding the generalized immersion for a particular system.

A procedure for obtaining such generalized immersion has been outlined and shown in several examples. The simulation results on a model of a pendulum demonstrates a potential of the presented approach.

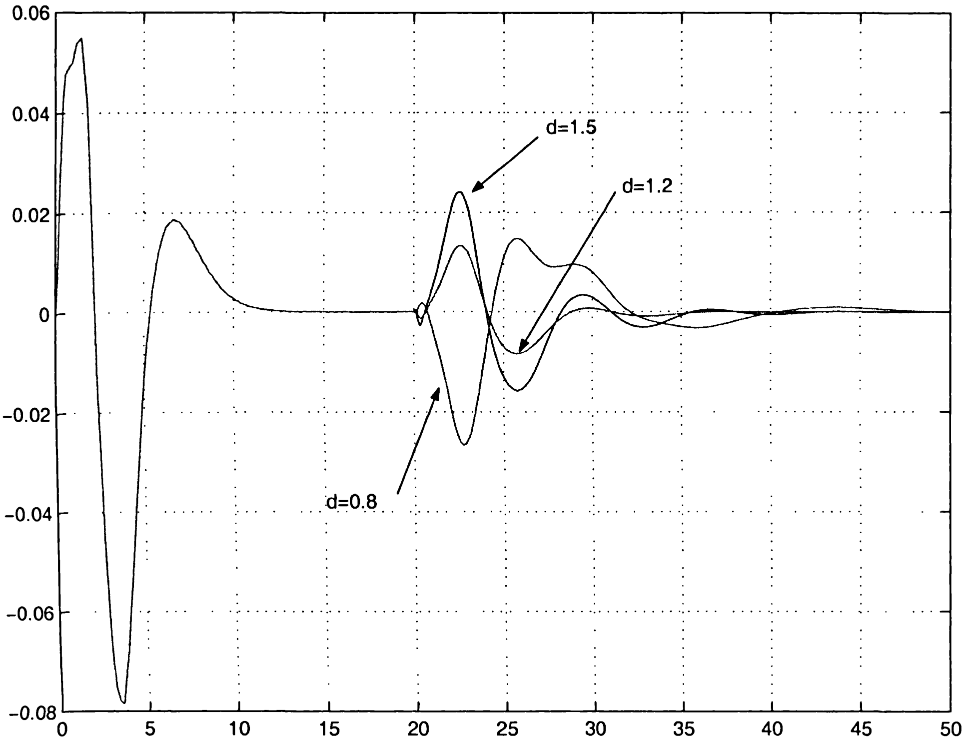

Fig. 2. Output error with observer modification.

Future research will explore the existence of generalized immersion for some more particular classes of systems as well as the possibility of giving a precise characterization of the dimension of the immersion. Another interesting and open problem, is to study an immersion with respect to part of exosystem components only and to find the largest collection of exosystem componets still allowing finite-dimensional immersion.

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REFERENCES

[1] C. I. Byrnes, F. Delli Priscolli, and A. Isidori: Output regulation of uncertain nonlinear systems. Birkhäuser, Boston 1997.

[2] S. Čelikovský and J. Huang: Continuous feedback asymptotic output regulation for a class of nonlinear systems having nonstabilizable linearization. Proc. 37th IEEE Conference on Decision and Control, Tampa, Florida, 1999, pp. 3087–3092.

[3] S. Čelikovský and J. Huang: Continuous feedback practical output regulation for a class of nonlinear systems having nonstabilizable linearization. Proc. 38th IEEE Conference on Decision and Control, Phoenix, Arizona, 2000, pp. 4796–4801.

[4] S. Čelikovský, C. Villanueva-Novelo, and B. Castillo-Toledo: Robust output regulation for nonlinear systems via generalized immersion. Proc. Conference on Systems, Cybernetics and Informatics, *IX*, Orlando, Flo. USA, 2000, pp. 96–101.

[5] J. S. A. Hepburn and W. M. Wonham: Error feedback and internal model on differentiable manifolds. IEEE Trans. Automat. Control *29* (1984), 397–403.

[6] J. Huang and W. J. Rugh: On a nonlinear multivariable servomechanism problem. Automatica *26* (1990), 963–972.

[7] J. Huang: Asymptotic tracking and disturbance rejection in uncertain nonlinear system. IEEE Trans. Automat. Control *40* (1995), 1118–1122.

[8] A. Isidori and C. I. Byrnes: Output regulation of nonlinear systems. IEEE Trans. Automat. Control *35* (1990), 131–140.

[9] A. Isidori: Nonlinear Control Systems. Third edition. Springer–Verlag, London 1995.

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