Kybernetika

Michel Fliess; Hugues Mounier

On a class of linear delay systems often arising in practice

Kybernetika, Vol. 37 (2001), No. 3, [295]--308

Persistent URL: http://dml.cz/dmlcz/135410

KYBERNETIKA — VOLUME 37 (2001), NUMBER 3, PAGES 295–308

ON A CLASS OF LINEAR DELAY SYSTEMS OFTEN ARISING IN PRACTICE

MICHEL FLIESS AND HUGUES MOUNIER

We study the tracking control of linear delay systems. It is based on an algebraic property named π-freeness, which extends Kalman's finite dimensional linear controllability and bears some similarity with finite dimensional nonlinear flat systems. Several examples illustrate the practical relevance of the notion.

INTRODUCTION

We will here describe recent works [14, 25, 26, 27, 29] on the control tracking of linear delay systems. Our philosophy is guided by two major concerns: the first one (*practical concern*) is to discover structural properties that occur most frequently in practical applications; the second one (*simplicity concern*), related to the previous one, is to obtain the simplest properties for each class of applications. The practical concern has led us to a new property, called *π-freeness*, which allows the tracking of a reference trajectory in a way which bears some analogy with flat finite dimensional nonlinear systems (see[12, 13] and the references therein). Through the simplicity concern, we discovered a novel class named quasi-finite delay systems, the controllability and stabilization of which is very simple, and quite analogous to the one of systems without delays. π-freeness is an extension of the classic Kalman linear controllability, when viewed in the module-theoretic language of [7, 8, 14, 25]. Then a linear system is a finitely generated module over the principal ideal ring of linear differential operators. Kalman's controllability is equivalent to the freeness of this module, or what amounts to the same, to its torsion freeness. Infinite dimensional systems, like delay ones, yield modules over more general rings, where freeness and torsion freeness are no longer equivalent (see [10] for a more general discussion on controllability issues). Concrete case-studies (see [25, 27]) show that the corresponding modules are mostly not free but only torsion free. Freeness may nevertheless be recovered via a suitable localization, i.e., by taking the inverse of an element π of the ring. Any basis of this free module is called a *flat*, or *basic*, *output*; it plays the same role as a *flat*, or *linearizing*, *output* of a flat finite dimensional system (see also [23]). Quasi-finite systems are, roughly speaking, systems where the only variable that is delayed is the input. This class seems to encompass nearly all technological examples of linear delay systems. To our knowledge, the only important practical class that does not belong to the quasi-finite one comprises systems modeled by the wave equation without damping [27]. The paper is organized as follows. We first introduce abstract linear systems over arbitrary commutative rings and π-freeness in this context. Finite dimensional linear systems are then presented within this framework. We proceed to delay systems, their controllability properties, and Quasi-finite systems. Some technological examples are then analyzed, and seen to be quasi-finite. Finally, an example of a flexible rod with an end mass relates to the boundary control of the wave equation. This example is not quasi-finite, but π-free.

1. ABSTRACT LINEAR SYSTEM THEORY

1.1. Basic definitions

Any ring R is commutative, with 1 and without zero divisors.

Notation. The submodule spanned by a subset S of an R-module M is written $[P]$.

An *R-system* Λ, or a *system over R*, is an R-module. Two R-systems Λ_1 and Λ_2 are said to be *R-equivalent*, or *equivalent over R*, if the R-modules Λ_1 and Λ_2 are isomorphic. An *R-dynamics*, or a *dynamics over R*, is an R-system Λ equipped with an *input*, i. e., a subset $\boldsymbol{u}$ of Λ which may be empty, such that the quotient R-module $\Lambda/[\boldsymbol{u}]$ is torsion. The input $\boldsymbol{u}$ is *independent* if the R-module $[\boldsymbol{u}]$ is free, with basis $\boldsymbol{u}$. An *output* $\boldsymbol{y}$ is a subset, which may be empty, of Λ. An *input-output R-system*, or an *input-output system over R*, is an R-dynamics equipped with an output.

Remark 1.1.1. Kalman's module-theoretic setting [18] is related to the state variable description, whereas our module description encompasses all system variables without any distinction.

Let A be an R-algebra et and Λ be an R-system. The A-module $A \otimes_R \Lambda$ is an A-system, which *extends* Λ.

1.2. Relations

Let Λ be an R-system. There exists an exact sequence of R-modules [34]

$$0 \to N \to F \to \Lambda \to 0 \tag{1}$$

where F is free. The R-module N, which is sometimes called the *module of relations*, should be viewed as a *system of equations* defining Λ. Associate to Λ a *free presentation* [34], i. e., the short exact sequence of R-modules

$$F_1 \to F_0 \to \Lambda \to 0$$

where F_0 and F_1 are free. The R-module Λ is said to be *finitely generated*, or of *finite type*, if there exists a free presentation where any basis of F_0 is finite. It is said to be *finitely presented* if there exists a free presentation where any basis of F_0

and F_1 is finite. The matrix corresponding, for some given bases, to the mapping $F_1 \to F_0$ is called a *presentation matrix* of Λ. If the ring R is Noetherian, it is known [34] that the conditions of being of finite type and of being finitely presented coincide. Then (1) may be chosen such that both F and N are of finite type. This latter case will always be verified in the sequel.

Example 1.2.1. Let us determine the R-module Λ corresponding to a system of R-linear equations

$$\sum_{\kappa=1}^{\mu} a_{\iota\kappa}\xi_\kappa = 0, \quad a_{\iota\kappa} \in A, \iota = 1, \dots, \nu$$

where $\xi_1, \dots, \xi_\mu$ are the unknowns. Let F be the free R-module spanned by $f_1, \dots, f_\mu$. Let $N \subseteq F$ be the module of relations, i. e., the submodule spanned by $\sum_{\kappa=1}^{\mu} a_{\iota\kappa} f_\kappa$, $\iota = 1, \dots, \nu$. Then, $\Lambda = F/N$. The ξ_κ's are the *residues* of the f_κ's, i. e., the canonical images of the f_κ's.

1.3. Different notions of controllability

An R-system Λ is said to be *R-torsion free controllable* (resp. *R-projective controllable*, *R-free controllable*) if the R-module Λ is torsion free (resp. projective, free). Elementary homological algebra (see, e. g., [34]) yields the

Proposition 1.3.1. R-free (resp. R-projective) controllability implies R-projective (resp. R-torsion free) controllability.

Take an R-free controllable system Λ with a finite output y. This output is said to be *flat*, or *basic*, if y is a basis of Λ.

1.4. π–freeness

The next result [14] follows at once from [35, Proposition 2.12.17, p. 233]:

Theorem and Definition 1.4.1. Let Λ be an R-system, A an R–algebra, and $\mathcal{S}$ a multiplicative part of A such that Λ is $\mathcal{S}^{-1}R$-free controllable. Then, there exists an element π in $\mathcal{S}$ such that Λ is $R[\pi^{-1}]$-free controllable. The preceding system will then be called *π-free*. An output being a basis of $R[\pi^{-1}] \otimes_R \Lambda$ is called *π-flat* or *π-basic*.

2. FINITE DIMENSIONAL LINEAR SYSTEMS

2.1. Modules over principal ideal rings

In this section R is the principal ideal ring $k[\frac{d}{dt}]$, whose elements are of the form $\sum_{\text{finite}} a_\alpha \frac{d^\alpha}{dt^\alpha}$, $a_\alpha \in k$, where k is a field. All $k[\frac{d}{dt}]$-modules are finitely generated and, therefore, finitely presented.

2.2. Controllability

The three notions of free, projective and torsion free controllability over $k[\frac{d}{dt}]$ coincide. A $k[\frac{d}{dt}]$-system Λ is therefore said to be *controllable* [7], if the $k[\frac{d}{dt}]$-module Λ is free.

3. DELAY SYSTEMS AND CONTROLLABILITY

3.1. Linear delay systems

Let R be the ring $k[\frac{d}{dt}, \delta_1, \dots, \delta_r] = k[\frac{d}{dt}, \delta]$ of polynomials in $r+1$ indeterminates over a commutative field k, where the δ_i's are (*localized*) *delay* operators of non commensurate amplitudes. A (*linear*) *delay system* (resp. *dynamics*) is a $k[\frac{d}{dt}, \delta]$-system (resp. $k[\frac{d}{dt}, \delta]$-dynamics).

3.2. Controllability

The resolution of Serre's conjecture [36] due to Quillen [32] and Suslin [39] (*see also* [19, 42] for a detailed exposition) states that, on a polynomial ring, a projective module is free. Thus, in the present context, Quillen–Suslin's theorem may be stated as [14]:

Proposition 3.2.1. A delay system is $k[\frac{d}{dt}, \delta]$-free controllable if, and only if, it is $k[\frac{d}{dt}, \delta]$-projective controllable.

Very many notions can then be considered (through torsion freeness and freeness on the one hand, and through the variation of the ground ring on the other hand). Among these, the $k[\frac{d}{dt}, \delta]$-free controllability is certainly the most appealing from an algebraic viewpoint. The existence of a basis is an extremely useful feature; but this notion seems quite rare in practice (see, e. g., [25, 27]). The π-freeness retains the main advantage of freeness (existence of a basis) while being almost always satisfied in applications. Indeed, we have [14]

Proposition 3.2.2. A $k[\frac{d}{dt}, \delta]$–torsion free controllable $k[\frac{d}{dt}, \delta]$–system Λ is π–free, where π may be chosen in $k[\delta]$.

3.2.1. Criteria for $k[\frac{d}{dt}, \delta]$–free and $k[\frac{d}{dt}, \delta]$–torsion free controllability

We establish [14] two criteria for $k[\frac{d}{dt}, \delta]$–free and $k[\frac{d}{dt}, \delta]$–torsion free controllability. The first one uses the resolution of Serre's conjecture [32, 39], and the second one[1] uses [41].

[1] *See* [43] for related results.

Theorem 3.2.1. A delay system Λ with presentation matrix P_Λ of full generic rank β is $k[\frac{\mathrm{d}}{\mathrm{d}t}, \delta]$–free controllable if, and only if,

$$\forall (s, z_1, \ldots, z_r) \in \bar{k}^{r+1}, \quad \mathrm{rk}_{\bar{k}}\, P_\Lambda(s, z_1, \ldots, z_r) = \beta$$

where $\bar{k}$ is the algebraic closure of k. This rank criterion is equivalent to the common minors of P_Λ of order β having no common zero in $\bar{k}^{r+1}$.

Theorem 3.2.2. A delay system Λ is $k[\frac{\mathrm{d}}{\mathrm{d}t}, \delta]$–torsion free controllable if, and only if, the gcd of the $\beta \times \beta$ minors of P_Λ belongs to k.

1. The system $\dot{y} + \delta y = u$ is $k[\frac{\mathrm{d}}{\mathrm{d}t}, \delta]$-free controllable, with basis y.
2. The system $\dot{y} = \delta u$ is $k[\frac{\mathrm{d}}{\mathrm{d}t}, \delta]$-torsion free controllable, but not $k[\frac{\mathrm{d}}{\mathrm{d}t}, \delta]$-free controllable.

3.2.2. Reachability, weak controllability

Consider [14] the $k[\frac{\mathrm{d}}{\mathrm{d}t}, \delta]$–dynamics $\Gamma = [\boldsymbol{x}, \boldsymbol{u}]$ with equations

$$\dot{\boldsymbol{x}} = F(\delta)\, \boldsymbol{x} + G(\delta)\, \boldsymbol{u}$$

where $\boldsymbol{x} = (x_1, \ldots, x_n)$, $\boldsymbol{u} = (u_1, \ldots, u_m)$, and the matrices $F(\delta) \in k[\delta]^{n \times n}$ and $G(\delta) \in k[\delta]^{n \times m}$. The classic notions of *reachability* and *weak controllability* may be found in [24, 37].

Proposition 3.2.3. The dynamics Γ is reachable if, and only if, Γ is free controllable over $k[\frac{\mathrm{d}}{\mathrm{d}t}, \delta]$.

Proposition 3.2.4. The dynamics Γ is weakly controllable if, and only if, the $k(\delta)[\frac{\mathrm{d}}{\mathrm{d}t}]$-module $k(\delta)[\frac{\mathrm{d}}{\mathrm{d}t}] \otimes_{k[\frac{\mathrm{d}}{\mathrm{d}t}, \delta]} \Gamma$ is free, where $k(\delta)$ denotes the quotient field of $k[\delta]$.

3.2.3. Spectral controllability

We use the ring $k[s, e^{-\boldsymbol{h}s}]$, viewed as a subring of the convergent power series ring $k\{\{s\}\}$ (where s plays the role of $\frac{\mathrm{d}}{\mathrm{d}t}$, and $e^{-\boldsymbol{h}s} = (e^{-h_1 s}, \ldots, e^{-h_r s})$, the h_i's ($h_i \in \mathbb{R}, h_i > 0$) being the *amplitudes* of the corresponding delays). The mapping $\frac{\mathrm{d}}{\mathrm{d}t} \mapsto s$, $\delta_i \mapsto e^{-h_i s}$ yields an isomorphism between the rings $k[\frac{\mathrm{d}}{\mathrm{d}t}, \delta]$ and $k[s, e^{-\boldsymbol{h}s}]$. Thus, by a slight abuse of language, a finitely generated $k[s, e^{-\boldsymbol{h}s}]$-module will still be called a delay system [14, 26]. The following definition of *spectral controllability* extends previous ones (see, e. g., [4, 33]) in our context.

Definition 3.2.1. Let Λ be a delay system defined over the ring $k[s, e^{-hs}]$, with presentation matrix P_Λ of full generic rank β. It is called *spectrally controllable* if

$$\forall\, s \in \mathbb{C}, \quad \mathrm{rk}_{\mathbb{C}}\, P_\Lambda(s, e^{-hs}) = \beta.$$

Set $\mathfrak{S}_r = k(s)[e^{-hs}, e^{hs}] \cap \mathfrak{E}$, where $\mathfrak{E}$ denotes the ring of entire functions. We have the following interpretation of spectral controllability [26]:

Proposition 3.2.5. Let Λ be a delay system over $k[s, e^{-hs}]$, such that Λ is $k[s, e^{-hs}, e^{hs}]$-torsion free controllable. Then Λ is spectrally controllable if, and only if, it is $\mathfrak{S}_r$-torsion free controllable.

The following result [14] gives implication relationships between the notion of $\delta_1^{\alpha_1} \ldots \delta_r^{\alpha_r}$-freeness, $\alpha_1 + \ldots + \alpha_r > 0$, and the above quoted ones.

Proposition 3.2.6. Let Λ be a $k[\frac{\mathrm{d}}{\mathrm{d}t}, \delta]$-system. The following chain of implications is true

$$\Lambda \text{ spectrally controllable} \Longrightarrow \Lambda \;\; \delta_1^{\alpha_1} \ldots \delta_r^{\alpha_r}\text{-free} \;\Longrightarrow \Lambda \;\; k[\tfrac{\mathrm{d}}{\mathrm{d}t}, \delta]\text{-torsion free.}$$

Proof. The proof follows directly from the inclusion chain $k[\frac{\mathrm{d}}{\mathrm{d}t}, \delta] \subset k[\frac{\mathrm{d}}{\mathrm{d}t}, \delta, \delta^{-1}] \subset \mathfrak{S}_r$. □

3.3. Quasi-finite systems

3.3.1. Definition

A seemingly important class for applications is one in which only the input is delayed, which will be called quasi-finite systems. Controllability and stabilization of these systems are quite similar to the one of systems without delays, hence the name.

Let $P \in k[\frac{\mathrm{d}}{\mathrm{d}t}, \delta]^{\beta \times \alpha}$ be a presentation matrix of Λ, i.e., Λ is isomorphic to the $k[\frac{\mathrm{d}}{\mathrm{d}t}, \delta]$-module spanned by $\boldsymbol{w} = (w_1, \ldots, w_\alpha)$ subject to

$$P(\tfrac{\mathrm{d}}{\mathrm{d}t}, \delta)\boldsymbol{w} = 0$$

P is said to be *special* if $P \in k[\frac{\mathrm{d}}{\mathrm{d}t}]^{\beta \times \alpha}$. We will say that a $k[\frac{\mathrm{d}}{\mathrm{d}t}, \delta]$-system Λ is *special* if

$$\Lambda = k[\tfrac{\mathrm{d}}{\mathrm{d}t}, \delta] \otimes_{k[\frac{\mathrm{d}}{\mathrm{d}t}]} \Lambda^{\mathrm{spec}}$$

where Λ^{spec} is a finitely generated $k[\frac{\mathrm{d}}{\mathrm{d}t}]$-module. Alternatively, it can be defined by a special presentation matrix. Then, a $k[\frac{\mathrm{d}}{\mathrm{d}t}, \delta]$-dynamics Λ is said to be *special* if there exists Λ^{spec} such that $\boldsymbol{u}]$ belongs to it. A such Λ^{spec}, being unique, will be called the *fundamental dynamics* and designated by $\Lambda_0^{\mathrm{spec}}$. A special $k[\frac{\mathrm{d}}{\mathrm{d}t}, \delta]$-system with input $\boldsymbol{u}$ and output $\boldsymbol{y}$ is said to be *quasi-finite* if the dynamics is special, and if there exists p non negative real numbers $L_1, \ldots, L_p$ such that $\delta_{L_1} y_1, \;\ldots\; \delta_{L_p} y_p \in \Lambda_0^{\mathrm{spec}}$. See [17] for more details.

3.3.2. State representation

All the technological examples presented below will be of the following form

$$\dot{x} = Ax + B\delta u \tag{2}$$

where $A \in k^{n\times n}$, $B \in k^{n\times m}$. For this restricted class, we have the following equivalence result.

Proposition 3.3.1. For (2), the three following conditions are equivalent:

1) it is torsion free controllable,
2) it is spectrally controllable,
3) it is δ-free,
3) $\mathrm{rk}(B, AB, \dots, A^{n-1}B) = n$.

Proof. The equivalence between 1) and 4) has been proved in [14]. The equivalence between 2) and 3) follows from ([30, 21]) and $e^{-hA} = \sum_{\nu\geq 0} h^\nu A^\nu/\nu!$. The implication 3) $\Longrightarrow$ 1) is obvious. Let us show the inverse implication. Set $m_i(s,z)$ ($i \in \{1, \dots, \gamma\}$) as the $n\times n$ minors of $(sI - A \mid -zB)$ (where s stands for $\mathrm{d}/\mathrm{d}t$ and z for δ), and consider them as functions over $\mathbb{C}^2$. Let also $M(s,z) = \sum_{i=1}^{\gamma} |m_i(s,z)|^2$. Then, one has: ((2) δ-free $\Leftrightarrow \forall\,(s,z) \in \mathbb{C}\times\mathbb{C}^*$, $M(s,z) \neq 0$) and ((2) weakly controllable $\Leftrightarrow \forall\, s \in \mathbb{C}$, $M(s,z) \neq 0$). The minor of $(sI - A)$ depends only on s, so that evidently

$$(\forall\,(s,z) \in \mathbb{C}\times\mathbb{C}^*\, M(s,z) \neq 0) \Leftrightarrow (\forall\, s \in \mathbb{C},\ M(s,z) \neq 0)$$

wherefrom the result. □

4. TECHNOLOGICAL EXAMPLES

All of the following examples, like a great majority of technological examples, are quasi-finite.

4.1. Crude oil mixing

Consider two tanks of crude oil C_1 and C_2 with respective octane numbers I^1 and I^2. The output of C_1 (resp. C_2) is u (resp. $D - u$) where D is a constant. The two outputs are mixed in a pipe and transported to a tank C_3. The transportation lag corresponds to a delay δ, and the mixture in C_3 has the octane number I^b. Supposing that, at time t, the volume of the mixture is V_t and the octane number in C_3 is I_t^b, one has, from t to $t + \mathrm{d}t$:

$$I_t^b V_t + \delta\big(I^1 u + I^2(D-u)\big) = V_{t+\mathrm{d}t} I_{t+\mathrm{d}t}^b.$$

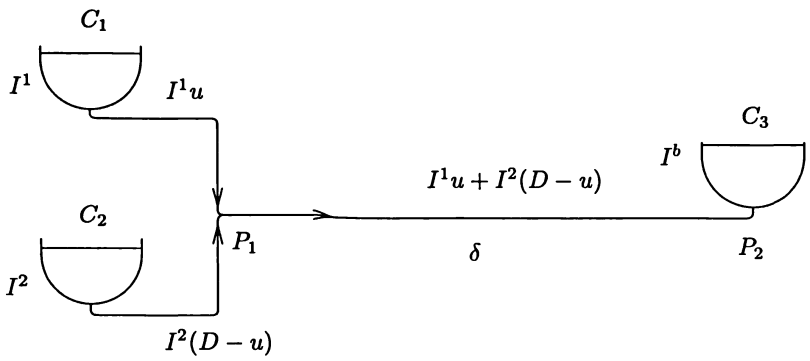

Fig. 1. Crude oil mixing.

Thus: $\frac{d}{dt}(I^b V)(t) = I^2 D + (I^1 - I^2)u(t-h)$ where h designates the amplitude of the delay δ. Set $y = I^b V$ and $v = u + I^2 D/(I^1 - I^2)$:

$$\dot{y} = (I^1 - I^2)v(t-h).$$

The system is quasi-finite, with basis y:

$$v = \frac{1}{(I^1 - I^2)}\delta^{-1}\dot{y}.$$

Suppose we want to obtain a desired output trajectory $y_d = I^b_d V_d$. The following control law allows to achieve the tracking:

$$u_d(t) = \frac{1}{I^1 - I^2}(\dot{y_d}(t+h) - I^2 D).$$

4.2. A teleoperated robot arm

Let us consider a flexible teleoperated robot arm. More precisely, take a simple model of the first mode for a one link flexible robot, actuated by a motor wich recieves its orders from a distant module. Denote:

- by τ the orders transmission time,
- by $q_r(t)$ the rigid displacement,
- by $q_e(t)$ the first mode of the elastic displacement,
- by $C(t)$ the motor's torque actuating the arm.

Newton's law yields:

$$\begin{pmatrix} M_{rr} & M_{re} \\ M_{er} & M_{ee} \end{pmatrix} \begin{pmatrix} \ddot{q}_r \\ \ddot{q}_e \end{pmatrix} + \begin{pmatrix} 0 \\ K_e q_e \end{pmatrix} = \begin{pmatrix} C \\ 0 \end{pmatrix}$$

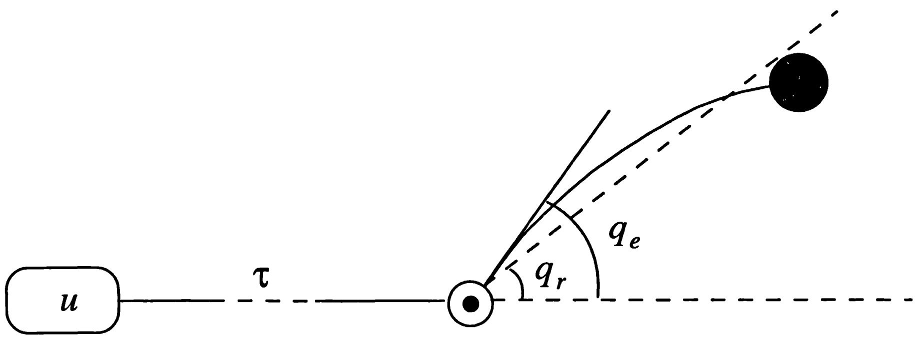

Fig. 2. Teleoperated robot arm.

where M_{xy} denote the equivalent masses and K_e the elastic stiffness. In addition, the motor being teleoperated, the orders $u(t)$ are transmitted from a distant module and arrive with a transmission delay τ:

$$u(t) = \mathcal{C}(t-\tau).$$

The system's equation are then:

$$M_{rr}\ddot{q}_r(t) + M_{re}\ddot{q}_e(t) = u(t-\tau) \tag{3}$$
$$M_{er}\ddot{q}_r(t) + M_{ee}\ddot{q}_e(t) = -K_e q_e(t) \tag{4}$$

This system is quasi-finite, with basis

$$\omega(t) = M_{er}q_r(t) + M_{ee}q_e(t).$$

Indeed:

$$q_r(t) = \frac{1}{M_{er}}\,\omega(t) - \frac{M_{ee}}{K_e M_{er}}\,\omega(t) \tag{5}$$
$$q_e(t) = -\frac{1}{K_e}\,\ddot{\omega}(t) \tag{6}$$
$$u(t) = \frac{M_{rr}}{M_{er}}\,\ddot{\omega}(t+\tau) + \frac{1}{K_e}\left(\frac{M_{rr}M_{ee}}{M_{er}} - M_{re}\right)\omega^{(4)}(t+\tau). \tag{7}$$

For a desired trajectory $\omega_d(t)$ from rest to rest, we obtain an open loop control law yielding an exact tracking of the form:

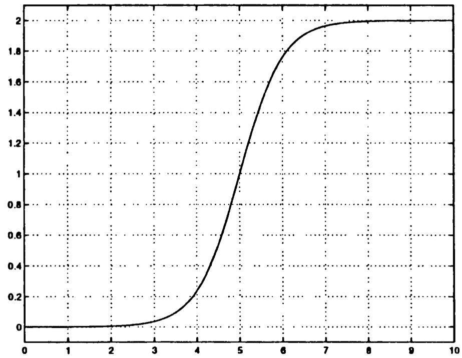

Fig. 3. Desired trajectory $\omega_d(t)$.

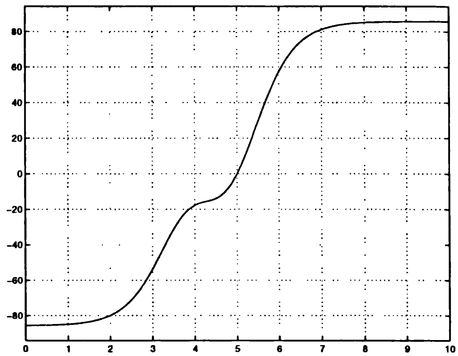

Fig. 4. Open loop control $u_d(t)$.

5. PHYSICAL EXAMPLES: TOWARDS DISTRIBUTED PARAMETER SYSTEMS

5.1. The wave equation: torsional behavior of a flexible rod

Consider [27] the torsional behavior of a flexible rod with a torque applied to one end. A mass is attached to the other end. The system is described by the one dimensional wave equation.

$$\sigma^2 \frac{\partial^2 q}{\partial \tau^2}(\tau, z) = \frac{\partial^2 q}{\partial z^2}(\tau, z) \tag{8}$$

$$\frac{\partial q}{\partial z}(\tau, 0) = -u(\tau), \qquad \frac{\partial q}{\partial z}(\tau, L) = -J\frac{\partial^2 q}{\partial \tau^2}(\tau, L)$$

$$q(0, z) = q_0(z), \qquad \frac{\partial q}{\partial \tau}(0, z) = q_1(z)$$

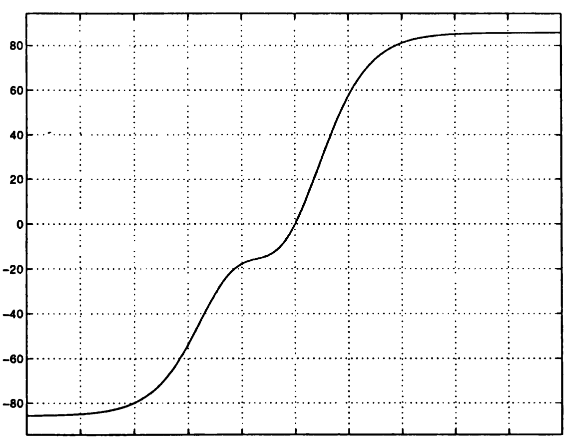

Fig. 5. The flexible rod.

Here $q(\tau, z)$ denotes the angular displacement from the unexcited position at a point $z \in [0, L]$ at time $\tau \geqslant 0$, as shown in Figure 5; L is the length of the rod, σ the inverse of the wave propagation speed, J the inertial momentum of the mass, $u(\tau)$ the control torque and q_0, q_1 describe the initial angular displacement and velocity, respectively.

5.1.1. Delay system model

As well known, the general solution of (8) may be written

$$q(\tau, z) = \phi(\tau + \sigma z) + \psi(\tau - \sigma z)$$

where ϕ and ψ are one variable functions. The control objective will be to assign a trajectory to the angular position of the mass; the output is thus

$$y(\tau) = q(\tau, L).$$

Set $t = (\sigma/J)\tau$, $v(t) = (2J/\sigma^2)u(t)$ and $T = \sigma L$. Easy calculations (see [27] for details) yield the following delay system (compare with [6]):

$$\ddot{y}(t) + \ddot{y}(t - 2T) + \dot{y}(t) - \dot{y}(t - 2T) = v(t - T). \tag{9}$$

One readily has

$$v = (\delta^{-1} + \delta)\ddot{y} + (\delta^{-1} - \delta)\dot{y} \tag{10}$$

which implies

Proposition 5.1.1. System (9) is δ-free, with basis y.

5.1.2. Tracking

Equation (10), yields the open loop control (see Figure 6)

$$v_d(t) = \ddot{y}_d(t + T) + \ddot{y}_d(t - T) + \dot{y}_d(t + T) - \dot{y}_d(t - T)$$

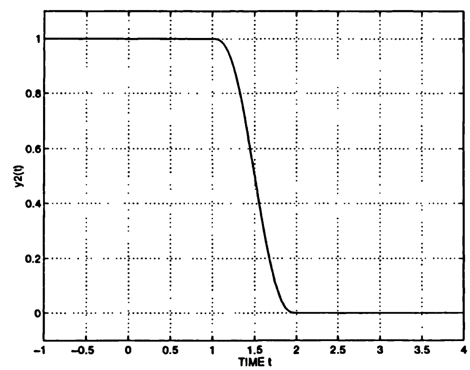

Fig. 6. The desired output y_2.

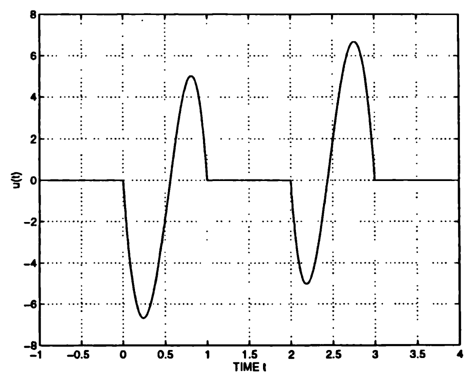

Fig. 7. The control u.

The displacements of the other points of the rod (see Figure 7) can be obtained as (see [27])

$$q_d(z, t) = \frac{1}{2}\Big[y_d(t - z + T) + \dot{y}_d(t - z + T) + y_d(t - T + z) - \dot{y}_d(t - T + z)\Big].$$

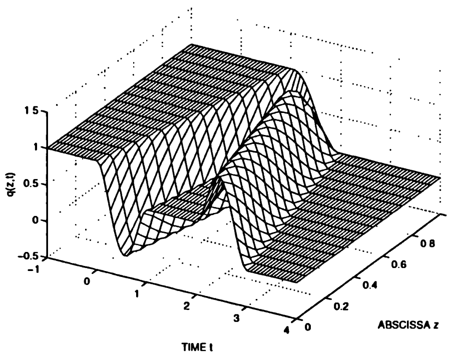

Fig. 8. Angular displacements $q(z,t)$.

Remark 5.1.1. Stabilization around the reference trajectory may be achieved by standard passivity methods or by the following feedback loop [27]

$$v(t) = \lambda_0 \ddot{y}(t-T) - 2\dot{y}(t-T) + \lambda_1 y(t-T)$$

with $\lambda_0 \in]0,2[$, $\lambda_0 \neq 1$ and $\lambda_1 < 1/(\lambda_0 - 2)$ (see [22]). Note that such type of feedback involving past derivatives of the state has already been used for stabilization purposes (see, e. g., [5]).

6. CONCLUSION

Some concrete examples of quasi finite systems as well as robust and simple stabilization schemes through generalized PI controllers and predictors are to be found in [16, 17].

(Received November 22, 2000.)

REFERENCES

[1] Z. Artstein: Linear systems with delayed control: a reduction. IEEE Trans. Automat. Control *27* (1982), 869–879.

[2] A. Bensoussan, G. Da Prato, M. C. Delfour, and S. K. Mitter: Representation and Control of Infinite Dimensional Systems, vol. 1 & 2. Birkhäuser, Boston 1992 & 1993.

[3] D. Brethé and J. J. Loiseau: A result that could bear fruit for the control of delay-differential systems. In: Proc. IEEE MSCA'96. Chania 1996.

[4] K. Bhat and H. Koivo: Modal characterizations of controllability and observability for time-delay systems. IEEE Trans. Automat. Control *21* (1976), 292–293.

[5] C. I. Byrnes, M. Spong, and T. J. Tarn: A several complex variables approach to feedback stabilization of linear neutral delay-differential systems. Math. Systems Theory *17* (1984), 97–133.

[6] S. Drakunov and U. Özgüner: Generalized sliding modes for manifold control of distributed parameter systems. In: Variable Structure and Lyapounov Control (A. S. Zinober, ed., Lecture Notes in Control and Information Sciences 193), Springer, London 1994, pp. 109–129.
[7] M. Fliess: Some basic structural properties of generalized linear systems. Systems Control Lett. *15* (1990), 391–396.
[8] M. Fliess: A remark on Willems' trajectory characterization of linear controllability. Systems Control Lett. *19* (1992), 43–45.
[9] M. Fliess: Une interprétation algébrique de la transformation de Laplace et des matrices de transfert. Linear Algebra Appl. *203–204* (1994), 429–442.
[10] M. Fliess: Variations sur la notion de commandabilité. In: Quelques aspects de la théorie du contrôle. Proc. Journée annuelle Soc. Math. France, Paris 2000, pp. 47–86.
[11] M. Fliess and H. Bourlès: Discussing some examples of linear system interconnections. System Control Lett. *27* (1996), 1–7.
[12] M. Fliess, J. Lévine, P. Martin, and P. Rouchon: Flatness and defect of non-linear systems: introductory theory and applications. Internat. J. Control *61* (1995), 1327–1361.
[13] M. Fliess, J. Lévine, P. Martin, and P. Rouchon: A Lie–Bäcklund approach to equivalence and flatness of nonlinear systems. IEEE Trans. Automat. Control *44* (1999), 922–937.
[14] M. Fliess and H. Mounier: Controllability and observability of linear delay systems: an algebraic approach. ESAIM COCV *3* (1998), 301–314.
URL: `http://www.emath.fr/Maths/Cocv/cocv.html`
[15] M. Fliess and H. Mounier: Quasi-finite linear delay systems: theory and applications. In: Proc. IFAC Workshop Linear Time Delay Systems, Grenoble 1998, pp. 211–215.
[16] M. Fliess, R. Marquez, and H. Mounier: PID like regulators for a class of linear delay systems. In: Proc. European Control Conference, Porto 2001.
[17] M. Fliess, R. Marquez, and H. Mounier: An extension of predictive control, PID regulators and Smith predictors to some linear delay systems. Internat. J. Control. Submitted.
[18] R. E. Kalman, L. Falb, and M. A. Arbib: Topics in Mathematical System Theory. McGraw–Hill, New York 1969.
[19] T. Y. Lam: Serre's Conjecture. Springer, Berlin 1978.
[20] S. Lang: Algebra. Third edition. Addison–Wesley, Reading, MA 1993.
[21] A. J. Manitius and A. W. Olbrot: Finite spectrum assignment problem for systems with delays. IEEE Trans. Automat. Control *24* (1979), 541–553.
[22] J. Marshall, H. Górecki, A. Korytowski, and K. Walton: Time Delay Systems Stability and Performance Criteria with Applications. Ellis Horwood, New York 1992.
[23] P. Martin, R. M. Murray, and P. Rouchon: Flat systems. In: Plenary Lectures and Mini–Courses, ECC 97 (G. Bastin and M. Gevers, eds.), Brussels 1997, pp. 211–264.
[24] A. S. Morse: Ring models for delay-differential systems. Automatica *12* (1976), 529–531.
[25] H. Mounier: Propriétés structurelles des systèmes linéaires à retards: aspects théoriques et pratiques. Thèse, Université Paris-Sud, Orsay 1995.
[26] H. Mounier: Algebraic interpretations of the spectral controllability of a linear delay system. Forum Mathematicum *10* (1998), 39–58.
[27] H. Mounier, P. Rouchon, and J. Rudolph: Some examples of linear systems with delays. J. Europ. Syst. Autom. *31* (1997), 911–925.
[28] H. Mounier and J. Rudolph: Flatness based control of nonlinear delay systems: Example of a class of chemical reactors. Internat. J. Control *71* (1998), 838–871, special issue "Recent Advances in the Control of Non-linear Systems".

[29] H. Mounier, J. Rudolph, M. Fliess, and P. Rouchon: Tracking control of a vibrating string with an interior mass viewed as delay system. ESAIM COCV *3* (1998), 315–321. URL: `http://www.emath.fr/Maths/Cocv/cocv.html`

[30] A. W. Olbrot: Stabilizability, detectability, and spectrum assignment for linear autonomous systems with time delays. IEEE Trans. Automat. Control *23* (1978), 887–890.

[31] N. Petit, Y. Creff, and P. Rouchon: Motion planning for two classes of nonlinear systems with delays depending on the control. In: Proc. 37th IEEE Conference on Decision and Control, 1998.

[32] D. Quillen: Projective modules over polynomial rings. Invent. Math. *36* (1976), 167–171.

[33] P. Rocha and J. C. Willems: Behavioral controllability of D-D systems. SIAM J. Control Optim. *35* (1987), 254–264.

[34] J. Rotman: An Introduction to Homological Algebra. Academic Press, Orlando 1979.

[35] L. H. Rowen: Ring Theory. Academic Press, Boston 1991.

[36] J.-P. Serre: Faisceaux algébriques cohérents. Annals of Math. *61* (1955), 197–278.

[37] E. D. Sontag: Linear systems over commutative rings: a survey. Richerche Automat. *7* (1976), 1–34.

[38] M. W. Spong and T. J. Tarn: On the spectral controllability of delay-differential equations. IEEE Trans. Automat. Control *26* (1981), 527–528.

[39] A. A. Suslin: Projectives modules over a polynomial ring are free (in Russian). Dokl. Akad. Nauk SSSR *229* (1976), 1063–1066. (English translation: Soviet. Math. Dokl. *17* (1976), 1160–1164.)

[40] J. C. Willems: Paradigms and puzzles in the theory of dynamical systems. IEEE Trans. Automat. Control *36* (1991), 259–294.

[41] D. C. Youla and G. Gnavi: Notes on n-dimensional system theory. IEEE Trans. Circuits and Systems *26* (1979), 105–111.

[42] D. C. Youla and P. F. Pickel: The Quillen–Suslin theorem and the structure of n-dimensional elementary polynomial matrices. IEEE Trans. Circuits and Systems *31* (1984), 513–518.

[43] S. Zampieri: Modellizzazione di Sequenze di Dati Mutlidimensionali. Tesi, Università di Padova 1993.

Prof. Dr. Michel Fliess, Centre de Mathématiques et de Leurs Applications, École Normale Supérieure de Cachan, 61 avenue du Président Wilson, 94 235 Cachan. France.
e-mail: fliess@cmla.ens-cachan.fr

Dr. Hugues Mounier, Département AXIS, Institut d'Électronique Fondamentale, Bâtiment 220, Université Paris-Sud, 91 405 Orsay. France.
e-mail: Hugues.Mounier@ief.u-psud.fr