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Persistent URL: http://dml.cz/dmlcz/135312

ROBUST QUASI NID AIRCRAFT 3D FLIGHT CONTROL UNDER SENSOR NOISE[1]

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In the paper the design of an aircraft motion controller based on the Dynamic Contraction Method is presented. The control task is formulated as a tracking problem for Euler angles, where the desired decoupled output transients are accomplished under assumption of high-level, high-frequency sensor noise and incomplete information about varying parameters of the system and external disturbances. The resulting controller has a simple form of a combination of a low-order linear dynamical system and a matrix whose entries depend nonlinearly on certain measurable flight variables.

1. INTRODUCTION

The progress in control theory is a result of its inner research activity as well as stimulation coming from other fields. One of them is aerospace industry which generates new interesting control problems. Many of today's evolving aircraft configurations present a considerable design challenge due to the use of innovative control effectors to increase maneuver capabilities and expand operating envelopes. The design of flight control systems for these configuration is complicated by both the number of control effectors, highly nonlinear response characteristic and uncertainty in disturbances as well as in a model of the aircraft (damage assessment). The design of aircraft control system for large, simultaneous longitudinal and lateral maneuvers as well as control of an aircraft at high angles of attack are interesting but – due to the complexity caused by the presence of nonlinearities, cross-coupling effects, disturbances and the system parameter variations – complicated control problems.

In fly-by-wire aircraft system the vehicle management system processes the pilot's control signals and commands independent actuators thus making it possible to realize highly sophisticated control algorithms.

In general, the goal of the design of an aircraft control system is to provide decoupling [11], i.e. each output should be independently controlled by a single input, and to provide desired output transients under assumption of incomplete information about varying parameters of the aircraft model and unknown external disturbances.

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[1] A version of this paper was presented at the 5th Mediterranean Conference on Control and Systems held in Paphos (Cyprus) on June 21–23, 1997.

Although various problem statements are possible, in the present paper we state the control task as a tracking problem of the Euler angles, which determine orientation of the airplane with respect to the inertial coordinate system. In addition we require that transient processes have desired dynamic properties, are mutually independent and are independent of airplane parameters and disturbances. Feedback data for the regulator are three Euler angles $\theta(t), \phi(t), \psi(t)$, or, more accurately, their noisy measurements. Control signals are: an aileron displacement δ_l, an elevator command δ_h and a rudder deflection δ_v.

In a survey paper by Wise [32] a classification and overview of particular control problems are given. Additionally for each problem the methods of solution are presented and evaluated. Some of them have been already applied in the aerospace industry and some seem to be potentially useful. The cited methods are: Nonlinear Inverse Dynamics (NID) and its modifications in the case when the number of controls is greater than dynamic quantities to be controlled, feedback linearization, nonlinear H_∞, linear matrix inequalities, l_1 optimal control and LQR. Moreover, an on-line identification method is considered for adaptive control in the case of failure of aircraft characteristics.

The control of non-linear systems through the use of their inverse dymamics [6, 15, 19] is a topic that has received a great deal of attention. Control laws that are based on the Non-linear Inverse Dynamics (NID) method theoretically offer the potential for providing improved levels of performance over conventional flight control designs in the extreme flight conditions [1, 11, 16, 20]. The shortcoming of this approach is the assumption that the dynamics of the system are exactly known.

Recently, some attempt has been made to design control systems for uncertain nonlinear systems using Variable Structure Systems (VSS) theory [19, 24]. As a result, a discontinuous control law was obtained which switches as the trajectory crosses switching surface in the state space. A crucial feature of the VSS control is that in the sliding phase the motion of the system is insensitive to parameter variations and disturbances in the system.

Another possibility to reach the goal is the algorithmical approach to inverse dynamics problem solution which allows to deal with unceratain nonlinear dynamical systems. For example, to form desired output transients for nonlinear time-varying systems a control method, called Localisation Method (LM), was proposed [25, 26, 27, 28, 29, 30]. The peculiarity of LM is the application of higher order derivatives [9, 18] jointly with a high gain [12, 13] in the control law. Methods of singularly perturbed equations [8, 17, 22] are used in LM to analyse closed loop system properties.

An algorithmic approach to solution of the inverse dynamics problem based on the Gradient Descent Method (GDM) has been discussed in references [2, 7, 10, 14].

In this paper Dynamic Contraction Method (DCM) [33, 34, 36], which is a further development of LM, is applied. At the same time the proposed approach can also be seen as a generalization of GDM [14]. In particular, the structure of the control law discussed here follows from a higher order optimization procedure [23].

The main purpose of this paper is to present a controller design procedure which reduces the control input variability caused by the high frequency sensor noise.

The paper is a continuation of [3, 4, 5] and is organized as follows: first, a model of the aircraft motion is defined, next a background of the discussed method and the method itself are summarized, and finally the design of an aircraft control system followed by an example are presented.

2. AIRCRAFT CONTROL PROBLEM

2.1. Aircraft model

Assuming that an airplane is a rigid body with six degrees of freedom, the state of aircraft motion is defined by twelve coordinates. The following state vector is adopted: $[v', \omega', \varphi', p']'$ where $v = [u, v, w]'$, $\omega = [p, q, r]'$, $\varphi = [\theta, \phi, \psi]'$, $p = [x, y, z]'$ are, respectively, the velocity vector, the angular velocity vector, the vector of Euler angles and the inertial position vector. Here u, v, w and p, q, r are projections of velocity and angular velocity onto the x_s, y_s and z_s axes of the body axis system, respectively; see [21] and [31] for the meaning of Euler angles, velocities and coordinate systems.

The system of state equations is as follows:

$$\dot{v} = \Omega(\omega)\,v + D_{sg}(\theta, \phi, \psi)\,g + \gamma\delta_c + \frac{1}{2m}\rho v_\alpha^2 D_{sa}(\alpha, \beta)\,Sc(\alpha, \beta, \omega), \quad (1)$$

$$\dot{\omega} = -J^{-1}\Omega(\omega)\,J\omega + \frac{1}{2m}\rho v_\alpha^2 J^{-1}D_{sa}(\alpha, \beta)\,SLm(\alpha, \beta, \omega, \delta_u), \quad (2)$$

$$\dot{\varphi} = T_\omega(\theta, \phi)\,\omega, \quad (3)$$

$$\dot{p} = D'_{sg}(\theta, \phi, \psi)\,v, \quad (4)$$

with

$$\Omega(\omega) = \begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix}, \quad J = \begin{bmatrix} J_x & 0 & J_{xz} \\ 0 & J_y & 0 \\ J_{xz} & 0 & J_z \end{bmatrix},$$

$S = \mathrm{diag}(S_x, S_y, S_z)$, $L = \mathrm{diag}(L_x, L_y, L_z)$, $c = [c_x, c_y, c_z]'$ and $m = [m_x, m_y, m_z]'$. Here $\delta_u = [\delta_h, \delta_v, \delta_l]'$, is a vector of control signals consisting of an aileron displacement δ_l, an elevator command δ_h and a rudder deflection δ_v; α denotes an angle of attack and β denotes a sideslip angle.

Denoting the velocities

$$v = [u, v, w]', \quad v_\alpha = [v_{\alpha x}, v_{\alpha y}, v_{\alpha z}]'$$

and the wind velocity

$$v_w = [v_{wx}, v_{wy}, v_{wz}]'$$

then the relationships between the angles α, β, and the velocities v, v_w and v_α are given as follows:

$$v_\alpha = v - D_{sg}v_w,$$

$$\alpha = \arctan(v_{\alpha z}/v_{\alpha x}),$$

$$\beta = \arcsin(v_{\alpha y}/v_\alpha),$$

$$v_\alpha = (v_{\alpha x}^2 + v_{\alpha y}^2 + v_{\alpha z}^2)^{\frac{1}{2}}.$$

The transformation matrices $D_{sg}(\theta, \phi, \psi)$, $D_{sa}(\alpha, \beta)$ and $T_\omega(\theta, \phi)$ are defined e. g. [3] as follows:

$$D_{sg}(\theta, \phi, \psi) = \begin{bmatrix} c\theta c\psi & c\theta s\psi & -s\theta \\ s\phi s\theta c\psi - c\phi s\psi & s\phi s\theta s\psi + c\phi c\psi & s\phi c\theta \\ c\phi s\theta c\psi + s\phi s\psi & c\phi s\theta s\psi - s\phi c\psi & c\phi c\theta \end{bmatrix}, \tag{5}$$

$$T_\omega(\theta, \phi) = \begin{bmatrix} 0 & c\phi & -s\phi \\ 1 & s\phi s\theta / c\theta & c\phi s\theta / c\theta \\ 0 & s\phi / c\theta & c\phi / c\theta \end{bmatrix}, \tag{6}$$

$$D_{sa}(\alpha, \beta) = \begin{bmatrix} c\alpha c\beta & -c\alpha s\beta & -s\alpha \\ s\beta & c\beta & 0 \\ s\alpha c\beta & -s\alpha s\beta & c\alpha \end{bmatrix} \tag{7}$$

where s and c are abbreviations of 'sin' and 'cos'.

It is assumed that the aerodynamic moment and force coefficients depend on the angle of attack and the sideslip angle, angular velocities ω and on control surfaces deflections δ_u and can be expressed with sufficient accuracy as follows:

$$m(\alpha, \beta, \omega) = \frac{1}{v_\alpha} m^0(\omega) + m^1(\alpha, \beta) + M(\alpha, \beta)\delta_u, \tag{8}$$

$$c(\alpha, \beta, \omega) = \frac{1}{v_\alpha} c^0(\omega) + c^1(\alpha, \beta) + C(\alpha, \beta)\delta_u. \tag{9}$$

2.2. Aircraft control problem

In the present paper control task is stated as a tracking problem for the Euler angles:

$$\lim_{t\to\infty}[\theta_0(t) - \theta(t)] = 0, \quad \lim_{t\to\infty}[\phi_0(t) - \phi(t)] = 0$$

$$\lim_{t\to\infty}[\psi_0(t) - \psi(t)] = 0.$$

where $\theta_0(t), \phi_0(t), \psi_0(t)$ are the desired values of the considered angles. In addition, we require that transient processes have desired dynamic properties, are mutually independent and are independent of airplane parameters and disturbances. Feedback data for the regulator are three Euler angles $\theta(t), \phi(t), \psi(t)$. Control signals are: an aileron displacement δ_l, an elevator command δ_h and a rudder deflection δ_v.

3. SOLUTION OF A NON–LINEAR CONTROL PROBLEM BY DCM

3.1. Control Problem Statement

Let us consider a nonlinear time-varying system in the following form

$$\dot{x} = h(t, x, u), \quad x(0) = x_0, \tag{10}$$

$$y = g(t, x) \tag{11}$$

where $x(t)$ is an n-dimensional state vector, $y(t)$ is a p-dimensional output vector and $u(t)$ is a p-dimensional control vector.

Here, the dependence of $h(t, x, u)$ and $g(t, x)$ of time t expresses the influence of external disturbances.

Let us assume that the first $m - 1$ time derivatives of the output $y(t)$ do not depend explicitly on the control vector $u(t)$ and

$$y^{(m)} = f(t, x) + B(t, x)u \tag{12}$$

where $\det B(t, x) \neq 0$. Assume also that a reference model for transients of $y(t)$ is given in the form of the following vector differential equation:

$$y^{(m)} = F(\bar{y}, \bar{r}) \tag{13}$$

where

$$\bar{y} = [y', y^{(1)\prime}, \ldots, y^{(m-1)\prime}], \quad \bar{r} = [r', r^{(1)\prime}, \ldots, r^{(m-1)\prime}],$$

$y = r$ at the equilibrium and $r(t)$ is the reference. For example, (13) may have the form of a linear vector equation:

$$y^{(m)} = -\sum_{i=0}^{m-1} \alpha_i y^{(i)} + \sum_{i=0}^{m-1} \beta_i r^{(i)}. \tag{14}$$

If $\alpha_{m-1} \cdots, \alpha_0$ are diagonal matrices then transients of the components of $y(t)$ are assumed to be mutually independent. If, additionally, $\beta_{m-1} \cdots, \beta_0$ are diagonal then we require the decoupling of control channels.

Denote $\Delta(t) = r(t) - y(t)$ the tracking error. The task of a control system is stated so as to provide that

$$\lim_{t \to \infty} \Delta(t) = 0.$$

Moreover, the transients $y(t)$ should have a desired behaviour defined in (13) which does not depend either on the external disturbances or on the possibly varying parameters of equations (10) and (11). Let us denote

$$\Delta^F = F(\bar{y}, \bar{r}) - y^{(m)}. \tag{15}$$

Then equation (13) defining the desired dynamics is fulfilled if and only if

$$\Delta^F = 0 \tag{16}$$

holds. So the control action $u(t)$ which provides the control problem solution is the root of equation (16). Expression (16) is the insensitivity condition of the output transient performance indices with respect to external disturbances and varying parameters of the system in $(10)-(11)$.

3.2. DCM solution to the control problem

In what follows a historical way leading to DCM as well as the basic properties of the control law are presented for the noisless case.

Non-linear inverse dynamics method. The well known NID control algorithm $u(t) = u^a(t)$ [19] bases on the analytic solution of eq. (16)

$$u^a = B(t, x)^{-1}[F(\bar{y}, \bar{r}) - f(t, x)].$$

Obviously, the above control law may only be used if complete information is available about the disturbances, model parameters and the state of the system in eqn. (10)–(11).

Localization method. Another way to solve equation (16) is the implicit function method applied jointly with a high gain. In particular, from the equation

$$u = kK_0\Delta^F \qquad (17)$$

it results that $u(t)$ approaches $u^a(t)$ for large k in the closed loop system. The control law in eqn. (17) forms a basis of the Localization Method of [25, 27].

Gradient descent method. Equation (16) can be solved by the gradient descent method as a stable equilibrium of the following differential equation [2, 7, 10, 14]:

$$u^{(1)} = kK_0\Delta^F \qquad (18)$$

where $k \to \infty$ and K_0 is a nonsingular matching matrix such that $K_0B(t, x)$ is positive definite. For example, $K_0 \approx \{B\}'$ or $K_0 \approx \{B\}^{-1}$.

Dynamic contraction method. [33, 36] Let us denote:

$$u = K_0K_1v \qquad (19)$$

where K_1 is a diagonal matrix. Then, following the above ideas, a useful generalization of (18) and (17) is:

$$\mu^q v^{(q)} + \sum_{i=0}^{q-1} \mu^i d_i v^{(i)} = k\Delta^F, \quad \bar{v}(0) = \bar{v}_0 \qquad (20)$$

where $\bar{v} = [v', v^{(1)'}, \cdots, v^{(q-1)'}]'$; μ is a small positive parameter, $\mu > 0$; $d_{q-1}, \cdots, d_0$ are diagonal matrices and k is a gain. As a result, when taken together, equations (13), (14) and (20) define the dynamic control law:

$$\sum_{i=0}^{q} \mu^i d_i v^{(i)} = \left[\sum_{i=0}^{m-1} \beta_i r^{(i)} - \sum_{i=0}^{m} \alpha_i y^{(i)}\right]. \qquad (21)$$

Equation (20) is called differential contraction equation. Assuming that $q \geq m$, then the control law (21) is proper and it may be realized without any differention.

Let us assume that there is a sufficient time-scale separation, represented by a small parameter μ, between the fast and slow modes in the closed loop system. Methods of singularly perturbed equations can then be used to analyse the closed loop system [17, 22] and, as a result, slow and fast motion subsystems can be analysed separatcly.

Fast motions. From (15), (12) and (19), (20) it follows that the closed loop system equations may be rewritten in the form

$$y^{(m)} = f(t, x) + B(t, x)K_0K_1v, \tag{22}$$

$$\sum_{i=1}^{q} \mu^i d_i v^{(i)} + \delta_0 v = k[F(\bar{y}, \bar{r}) - f(t, x)] \tag{23}$$

where

$$\delta_0 = d_0 + kB(t, x)K_0K_1. \tag{24}$$

Let us introduce the new fast time scale $\tau = t/\mu$ into the closed loop system equations of (22), (23) then finding the limit $\mu \to 0$ and returning to the primary time scale $t = \mu\tau$ we obtain the following fast-motion subsystem:

$$\sum_{i=1}^{q} \mu^i d_i v^{(i)} + \delta_0 v = k[F - f] \tag{25}$$

where $\bar{v}(0) = \bar{v}_0$ and we assume that the state vector of the system of (10) is constant during the transients in the system of (25).

The asymptotic stability of the fast-motion subsystem of (25). and desired transients of $v(t)$ can be achieved by a proper choice of the parameters k, μ, d_0, d_1, ..., d_{q-1}. At the same time, if we have a steady state (more precisely, quasi-steady state) in the fast-motion subsystem, then $v(t) = v^s(t)$ where

$$v^s = k\delta_0^{-1}[F - f]. \tag{26}$$

Let us denote $u^a = K_0K_1v^a$ where $u^a(t)$ is the non-linear inverse dynamic control. Then

$$v^s = v^a + \delta_0^{-1}d_0[BK_0K_1]^{-1}[f - F]. \tag{27}$$

Slow motions. If the fast-motion subsystem (25) is in the steady-state (quasi steady-state) then the closed loop system equations of (22), (23) imply that

$$y^{(m)} = F(\bar{y}, \bar{r}) + k^{-1}d_0[k^{-1}d_0 + BK_0K_1]^{-1}[f(t, x) - F(\bar{y}, \bar{r})] \tag{28}$$

is an equation of the slow-motion subsystem. If $d_0 = 0$ or $k \gg 1$, then the slow-motion subsystem (28) approaches the form of (13). If $d_0 = 0$ then $y = r$ at the equilibrium. The transients of $y(t)$ in the closed loop system (12) and (20) are close to the transients of $y(t)$ in the slow-motion subsystem (28) if the fast-motion subsystem (25) is asymptotically stable and the transients in the fast-motion subsystem are much faster than the transients of the slow-motion subsystem and faster than the external disturbance.

4. THE CASE OF NOISY MEASUREMENTS

4.1. Problem statement

Let us assume that sensor outputs are corrupted by a zero-mean, high frequency noise $n(t)$. Then instead of $y(t)$, only $z(t)$,

$$z = y + n, \tag{29}$$

is available for control. If we replace $y(t)$ for $z(t)$ in equation (21) then the control algorithm becomes:

$$\sum_{i=0}^{q} \mu^i d_i v^{(i)} = k \left[\sum_{i=0}^{m-1} \beta_i r^{(i)} - \sum_{i=0}^{m} \alpha_i z^{(i)} \right]. \tag{30}$$

Let us replace Δ^F in (20) by $\Delta_z^F = F(\bar{z}, \bar{r}) - z^{(m)}$ where $\bar{z} = [z', z^{(1)\prime}, \ldots, z^{(m-1)\prime}]$. Then instead of (23) we have:

$$\sum_{i=0}^{q} \mu^i d_i v^{(i)} + \delta_0 v = k[F(\bar{z}, \bar{r}) - f(t, x) - n^{(m)}]. \tag{31}$$

Observe that if $\mu \to 0$ then from (31), (24) it follows that

$$v^s = [k^{-1} d_0 + B K_0 K_1]^{-1} [F(\bar{z}, \bar{r}) - f(t, x) - n^{(m)}].$$

Inserting this into (12) yields:

$$
\begin{aligned}
y^{(m)} = {} & F(\bar{z}, \bar{r}) - n^{(m)} \\
& + k^{-1} d_0 [k^{-1} d_0 + B K_0 K_1]^{-1} [f(t, x) - F(\bar{z}, \bar{r}) + n^{(m)}]. \tag{32}
\end{aligned}
$$

As a result, if $\mu = 0$ then, e.g. in the steady-state case, $r(t) = r_{ss} = $ const, $z(t) = z_{ss} = $ const, the output $y(t) = z_{ss} - n(t)$ reproduces $n(t)$. It is not what we expect not only from the output but also from the control variable whose magnitude is very large in this case.

It is important to notice here that the influence of the noise on the behaviour of the control variable and its effect on the output variable are identical in all systems where the output is required to meet eqn. (13) [27]. In particular, NID systems with ideal output differentiating, VSS controllers with ideal observations and LM based systems with ideal differentiation share the same problem with DCM when $\mu = 0$.

4.2. Controller design

Introducing appropriately chosen q and $\mu > 0$ can reduce large and fast changes of $u(t)$ which, in turn, contributes to the reduction of the variability of $y(t)$. This is not only important for the realization of the control task defined in (13) but also for the endurance of the actuators.

The method [35] is used here for designing the discussed control law. Let us now write eqn. (31) in the form

$$\sum_{i=1}^{q} \mu^i d_i v^{(i)} + \delta_0 v = k[F(\bar{y}, \bar{r}) - f(t, x) + F(\bar{n}, 0) - n^{(m)}], \qquad (33)$$

with $\bar{v}(0) = \bar{v}_0$ where $\bar{n} = [n', n^{(1)'}, \ldots, n^{(m-1)'}]$. The fast-motion subsystem (33) is a linear system in which $F(\bar{y}, \bar{r})$ and $f(t, x)$ can be considered to be time invariant when compared to the transients in (33). Therefore the frequency domain methods may be used for designing the control law. In particular, if we assume that $K_0 \approx B^{-1}$ then the fast-motion subsystem (31) is decomposed on p fast-motion subsystems for each control channel:

$$\sum_{j=1}^{q_i} \mu_i^j d_{i,j} v_i^{(j)} + \delta_{i,0} v_i = k[F_i(\bar{y}_i, \bar{r}_i) - f_i(t, x) + F_i(\bar{n}_i, 0) - n_i^{(m)}] \qquad (34)$$

where $\delta_{i,0} \approx d_{i,0} + k k_i$ and $i = 1, 2, \ldots, p$.

Let us apply the Laplace transform to (34). Then, assuming zero initial conditions, we get

$$v_i(s) = W_{i,f}(s) \frac{F_i - f_i}{s} + W_{i,n}(s) n_i(s) \qquad (35)$$

where

$$W_{i,f}(s) = k_{i,f} D_i^{-1}(s), \qquad (36)$$

$$W_{i,n}(s) = k_{i,n} D_i^{-1}(s) A_i(s), \qquad (37)$$

$$A_i(s) = \left[\sum_{j=0}^{m-1} \alpha_{i,j} s^j + s^m \right] / \alpha_{i,0}, \qquad (38)$$

$$D_i(s) = \left[\delta_{i,0} + \sum_{j=1}^{q_i} \mu_i^j s^j \right] / \delta_{i,0}. \qquad (39)$$

From the closed-loop model specifications and information on disturbance characteristics a frequency band $[0, \omega_{i,0}]$ can be defined in which $|W_{i,f}(j\omega)|$ should not depend on the frequency ω, i.e:

$$|W_{i,f}(j\omega)| \approx k_{i,f} \approx k_i^{-1}, \quad \omega \leq \omega_{i,0}. \qquad (40)$$

Assume that the main part of the power spectrum of n_i is placed in the interval $[\omega_{i,n}, \infty)$ and $\omega_{i,0} < \omega_{i,n}$. In order to suppress the influence of the high frequency sensor noise n_i on the variability of the control input v_i let us introduce the requirement:

$$|W_{i,n}(j\omega)| \leq \varepsilon_{i,n}(\omega), \quad \omega \geq \omega_{i,n}. \qquad (41)$$

Here $\varepsilon_{i,n}(\omega)$ is the function which expresses the desired degree of noise suppression. For example, we can choose $\varepsilon_{i,n}(\omega) = \varepsilon_{i,n}/\omega^{\vartheta_i}$, where $\vartheta_i \geq 0$. From (41) it follows that

$$|W_{i,f}(j\omega)| \leq \varepsilon_{i,n}(\omega) \omega^m, \quad \omega \geq \omega_{i,n}. \qquad (42)$$

Finally the Bode plot may be used to form $W_{i,f}(j\omega)$ such that the requirements (40) and (42) are met. Then from inspection of $W_{i,f}(j\omega)$ both the degree q_i and the parameters of the control law (20) follow. In particular, (40) yields:

$$\mu_i \le \omega_{i,0}^{-1}(d_{i,0} + kk_i)^{\frac{1}{q_i}},\qquad(43)$$

whereas from (42) it results:

$$\mu_i \ge \left(k\omega_{i,n}^{m+\vartheta_i-q_i}\varepsilon_{i,n}^{-1}\right)^{\frac{1}{q_i}},\qquad(44)$$

where $q_i \ge m + \vartheta_i$. Finally, from (43), (44) the required order q_i is

$$q_i \ge m + \vartheta_i + \log\left(\frac{k\omega_{i,0}}{\varepsilon_{i,n}[d_{i,0} + kk_i]}\right)\{\log(\omega_{i,n})\}^{-1}.$$

5. AIRCRAFT CONTROLLER DESIGN

5.1. Design principles

From equations (1) and (2) it follows that the second time derivatives of Euler angles depend algebraically on the control vector $\boldsymbol{\delta_u} = [\delta_h, \delta_v, \delta_l]'$:

$$\boldsymbol{\varphi}^{(2)} = \boldsymbol{f}(\theta, \phi, \omega, v, v_w) + \boldsymbol{B}(\cdot)\,\boldsymbol{\delta_u}\qquad(45)$$

where:

$$\boldsymbol{B}(\theta, \phi, \alpha, \beta) = \frac{1}{2}\rho v_\alpha^2 \boldsymbol{T_\omega}\boldsymbol{J}^{-1}\boldsymbol{D_{sa}}\boldsymbol{SLM}(\alpha, \beta),$$

$$\boldsymbol{M}(\alpha, \beta) = \begin{bmatrix} m_x^h & m_x^v & m_x^l \\ m_y^h & m_y^v & m_y^l \\ m_z^h & m_z^v & m_z^l \end{bmatrix}$$

and $\boldsymbol{T_\omega}(\theta, \phi)$, $\boldsymbol{D_{sa}}(\alpha, \beta)$ are defined as in (5)-(7). Obviously, $\det\boldsymbol{B}(\cdot) \ne 0$ and the values of functions $|f_i(\cdot)|$ $i = \theta, \phi, \psi$ are bounded. Let us assume that the desired dynamics are determined by a set of mutually independent differential equations:

$$\begin{aligned}
\theta^{(2)} &= \tau_\theta^{-2}(-2\alpha_\theta\tau_\theta\theta^{(1)} - \theta + \theta_0), \\
\phi^{(2)} &= \tau_\phi^{-2}(-2\alpha_\phi\tau_\phi\phi^{(1)} - \phi + \phi_0), \\
\psi^{(2)} &= \tau_\psi^{-2}(-2\alpha_\psi\tau_\psi\psi^{(1)} - \psi + \psi_0).
\end{aligned}\qquad(46)$$

Parameters $\tau_\theta, \tau_\phi, \tau_\psi$ and $\alpha_\theta, \alpha_\phi, \alpha_\psi$ have a very well known meaning and have to be specified by the designer.

From equations (24) and (25) we see that it would be convenient to have the matrix $\boldsymbol{B}(\theta, \phi, \alpha, \beta)\,\boldsymbol{K}_0$ diagonal. This would be possible by taking $\boldsymbol{K}_0 = \boldsymbol{B}(\theta, \phi, \alpha, \beta)^{-1}$. When the values of α and β are small, $\boldsymbol{D_{sa}}(\alpha, \beta)$ is close to the unity matrix. Therefore a convenient choice in this case is

$$\boldsymbol{K}_0 = \left[\frac{1}{2}\rho v_\alpha^2 \boldsymbol{T_\omega}(\theta, \phi)\,\boldsymbol{J}^{-1}\boldsymbol{SLM}(\alpha, \beta)\right]^{-1}.\qquad(47)$$

The controller equations are as follows:

$$
\begin{bmatrix} \delta_h \\ \delta_v \\ \delta_l \end{bmatrix} = K_0 K_1 \begin{bmatrix} v_\theta \\ v_\phi \\ v_\psi \end{bmatrix},
\tag{48}
$$

where $K_1 = \mathrm{diag}(k_\theta, k_\phi, k_\psi)$ and the control law has the following form

$$
\mu_i^{q_i} v_i^{(q_i)} + \cdots + \mu_i d_{i,1} v_i^{(1)} + d_{i,0} v_i = -k[y_i^{(2)} + \tau_i^{-2}(2\alpha_i \tau_i y_i^{(1)} + y_i - r_i)],
\tag{49}
$$

where $i = \theta, \phi, \psi$.

5.2. Design example

For an airplane whose parameters are given in Table 1 the following parameters of the multivariable regulator have been calculated using discussed approach: $\alpha_\theta = \alpha_\phi = \alpha_\psi = 0.7$, $\tau_\theta = \tau_\phi = \tau_\psi = 1.4$ $k_\theta = 1$, $k_\phi = 4$, $k_\psi = 4$, $k = 100$, $d_{i,0} = 0$ or $d_{i,0} = 1$, $\log_{10}(\omega_{i,0}) = 2.4$, $\log_{10}(\omega_{i,n}) = 4$ where $i = \theta, \phi, \psi$. The exemplary Bode plots of $W_{i,f}(j\omega)$ for $i = \theta$ is presented in Figure 1.

Table 1. Aircraft parameters.

i	m_i^0	m_i^α	m_i^β	m_i^h	m_i^v	m_i^l
x	0.0	0.0	0.0	0.0	-0.004	-0.04
y	0.0	0.057	0.0	-0.01	0.0	0.0
z	0.0	0.0	-0.011	0.0	0.0008	-00002
i	S_i	c_i^0	c_i^α	$c_i^{\alpha 2}$	c_i^β	$c_i^{\beta 2}$
x	0.5	-0.2	0.0	-0.002	0.0	0.0
y	2.0	0.0	0.0	0.0	-0.005	0.0
z	10.0	-0.15	-8.6	0.0	0.0	0.0057
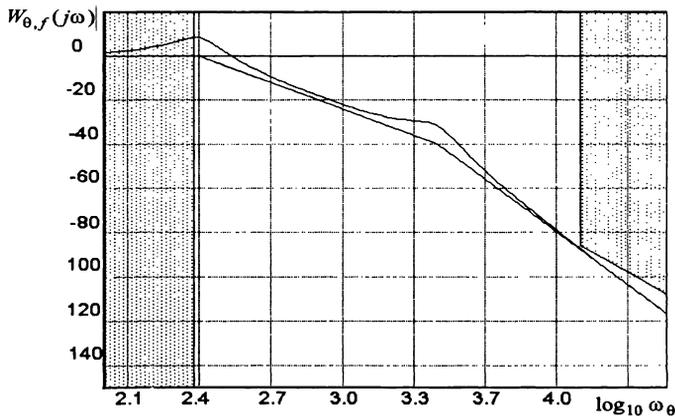
Fig. 1. The Bode plot for the angle θ.

Table 2. Controller parameters.

i	$\log(\varepsilon_{i,n})$	ϑ_i	q_i	μ_i	$d_{i,1}$	$d_{i,2}$	$d_{i,3}$
θ	35	1	4	0.00398	44	102.6	44
ϕ	35	1	3	0.0159	56.12	26.78	–
ψ	15	0	2	0.024	76.6	–	–

The simulated flight for examination of the designed controller was organized as follows. First, a flight along a straight line with the velocity 200 m/s took place and then a flight along a circle with the radius 10.000 m was required. Simulation of large maneuvers performed for a noisless case show a good agreement with the theory presented in the paper.

6. CONCLUSION

Based on the Dynamic Contraction Method, control laws have been derived which accomplish Euler angles tracking with prescribed second order dynamics by an aircraft in spite of noisy measurements and the presence of uncertainty in the system.

The presented procedure allows a controller to be designed with a reduced control signal variability caused by a high frequency sensor noise.

The resulting controller has a simple form of a combination of a low-order linear dynamical system and a matrix whose entries depend nonlinearly on certain measurable flight variables.

The flight controller obtained by DCM holds the properties of linearization and invariance to parameter variations of the plant model in the entire state space whereas in VSS this properties only hold on a hyperplane. Moreover, an integral action can be incorporated in the control loop if DCM is used, as opposed to LM, without increasing the controller's order.

REFERENCES

[1] G. Balas, W. Garrard and J. Reiner: Robust dynamic inversion control laws for aircraft control. In: Proc. of the AIAA Guidance, Navigation and Control Conference, 1992, pp. 192–205.

[2] A. P. Batenko: Final State Control of Driven Plants. Moscow, 1977. In Russian.

[3] M. J. Błachuta, V. D. Yurkevich and K. Wojciechowski: Aircraft control based on localization method. In: Preprints of the 2nd IFAC Workshop "Motion Control", Perugia 1992, vol. 2, pp. 19–24.

[4] M. J. Błachuta, V. D. Yurkevich, and K. Wojciechowski: Aircraft motion control by Dynamic Contraction Method. In: Preprints of the International Conference on Recent Advances in Mechatronics – "ICRAM'95", Istanbul 1995, pp. 404–411.

[5] M. J. Błachuta, V. D. Yurkevich and K. Wojciechowski: Design of aircraft 3D motion control using dynamic contraction method. In: Proceedings of the IFAC Workshop "Motion Control", Munich 1995, pp. 323–330.

[6] L. M. Boychuk: An inverse method of the structural synthesis of automatic control nonlinear systems. Automation 6 (1966), 7–10.

[7] A. L. Fradkov: Adaptive Control in Large–Scale Systems. Nauka, Moscow 1990. In Russian.

[8] E. I. Gerashchenko and S. M. Gerashchenko: A Method of Motion Separation and Optimization of Non–Linear Systems. Nauka, Moscow 1975. In Russian.

[9] W. G. Green: Logarithmic navigation for precise guidance of space vehicle. IRE Trans. of Aerospace and Navigational Electronics *2* (1961), 54.

[10] P. D: Krutko: Inverse Problems of Control System Dynamics: Nonlinear Models. Nauka Phys. & Math. Publ., Moscow 1989. In Russian.

[11] S. Lane and R. Stengel: Flight control design using non–linear inverse dynamics. Automatica *24* (1988), 471–483.

[12] M. V: Meerov: Automatic control systems which are stable under any large gain. Avtomat. i Telemekh. (1947), 27–31. In Russian.

[13] M. V. Meerov: Synthesis of Structures of High–Precision Automatic Control Systems. Nauka, Moscow 1959. In Russian.

[14] B. Petrov and P. D. Krutko: Synthesis of flight control algorithms based on solutions of inverse dynamics tasks. Izv. Akad. Nauk USSR. Tekhnicheskaya Kibernetika, *2–3* (1981). In Russian.

[15] W. A. Porter: Diagonalization and inverse for nonlinear systems. Internat. J. Control *11* (1970), 1, 67–76.

[16] J. Reiner, G. Balas and W. Garrard: Robust dynamic inversion for control of highly maneneuverable aircraft. AIAA J. Guidance, Control and Dynamics *18* (1995), 18–24.

[17] V. Saksena, J. O'Reilly and P. Kokotović: Singular perturbations and time-scale methods in control theory: survey 1976-1983. Automatica *20* (1984), 273–293.

[18] G. V. Shchipanov: Theory and methods of the automatic controller design. Avtomat. i Telemekh. (1939), 49–66. In Russian.

[19] J.-J. Slotine and W. Li: Applied Nonlinear Control. Prentice Hall, Englewood Cliffs, N.J. 1991.

[20] S. Snell, D. Enns and W. Garrard: Nonlinear inversion flight control for a supermaneuverable aircraft. In: Proc. of the AIAA Guidance, Navigation and Control Conference, 1990, pp. 808–825.

[21] B. Stevens and F. Lewis: Aircraft Control and Simulation. Wiley, New York 1992.

[22] A. N. Tikhonov: Systems of differential equations with a small parameter in derivatives. Collect. Math. Articles Moscow *31* (1952), 3, 575–586. In Russian.

[23] Ya. Z. Tsypkin: Adaptation and Learning in Automatic Control Systems. Nauka, Moscow 1968. In Russian.

[24] V. Utkin: Sliding Modes and their Application in Variable Structure Systems. Mir Publishers, Moscow 1978. In Russian.

[25] A. S. Vostrikov: On the synthesis of control units of dynamic systems. Systems Sci. *3* (1977), 2, 195–205.

[26] A. S. Vostrikov, V. I. Utkin and G. A. Frantsuzova: Systems with state vector derivative in the control, Automat. Remote Control *43* (1982), 3, Part 1, 283–286.

[27] A. S. Vostrikov: Synthesis of Nonlinear Systems by Means of Localization Method. Novosibirsk University Press, Novosibirsk 1990. In Russian.

[28] A. S. Vostrikov and V. D. Yurkevich: Decoupling of multi–channel non–linear time-varying systems by derivative feedback. Systems Sci. *17* (1991), 21–33.

[29] A. S. Vostrikov and V. D. Yurkevich: Design of control systems by means of localisation method. In: Preprints of 12th IFAC World Congress, Sydney 1993, Vol. 8, pp. 47–50.

[30] A. S. Vostrikov, and V. D. Yurkevich: Design of multi-channel systems with the velocity vector in the control law. Automat. Remote Control *54* (1993), 2, Part 1, 214–226.

[31] M. Vukobratović and R. Stojić: Modern Aircraft Flight Control. (Lecture Notes in Control and Information Sciences 109.) Springer–Verlag, Berlin 1988.

[32] K. A: Wise: Applied controls resarch topics in the aerospace industry. In: Proc. of the 34th Conference on Decision & Control, New Orleans 1995, pp. 751–756.

[33] V. D. Yurkevich: Control of uncertain systems: dynamic compaction method. In: Proc. of the 9th Internat. Conference on Systems Engineering, University of Nevada, Las Vegas 1993, pp. 636–640.

[34] V. D. Yurkevich: Design of aircraft longitudinal motion control using dynamic compaction method. In: Proc. of 3rd Internat. Workshop on Advanced Motion Control, University of California at Berkeley 1994, pp. 1029–1038.

[35] V. D. Yurkevich: On the design of continuous control systems by means of dynamic contraction method: noise influence analysis. In: Proc. of the 2nd Internat. Conference on Actual Problems of Electronic Instrument Engineering (APEIE-94), Novosibirsk 1994, Vol. 4, pp. 75–79.

[36] V. D. Yurkevich: Decoupling of uncertain continuous systems: dynamic contraction method. In: Proc. of the 34th Conference on Decision & Control, New Orleans 1995, pp. 96–201.

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