Book Reviews

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BOOK REVIEWS

S. Helgason: THE RADON TRANSFORM. Second Edition. Birkhäuser-Verlag, Boston-Basel-Berlin, 1999, xii+193 pages, 22 figures. ISBN 3-7643-4109-2, price DM 88.–.

The first edition of this book published by Birkhäuser in 1980 as Vol. 5 of the Series Progress in Mathematics was primarily based on the author's papers on the Radon transform and its generalizations published during 1959–1965 and on his lectures given in 1966 at MIT. At that time, it represented the first systematic exposition of this integral geometric (in the sense introduced by Gelfand, which differs from the more common one closely connected with the geometric probability) topic by one of its chief contributors. The present second edition is revised and extended in several directions, namely as summarized in the preface: "Many examples with explicit inversion formulas and range theorems have been added, and the group-theoretic viewpoint emphasized." At the same time, the author tried to preserve the introductory character of the first edition. However, as already the reviewer of the first edition remarked (Math. Rev. 1983, 83f: 43012) "even though the background knowledge required to read most of the book is on a rather elementary level, ... it takes some mathematical maturity to gain a real understanding of the proofs and to appreciate the many remarks and notes ... " This reviewer's feeling remains valid even for the present edition, in which the flow of deep remarks, notes and examples is much stronger.

The first chapter develops the properties of the Radon transform and its dual on $\mathbb{R}^n$. The inversion formulas are proved first for the hyperplanes, then also on the d-planes, $0 < d < n$, with particular attention to the $d = 1$ case called the X-ray transform for its application to the picture reconstruction in X-ray tomography. The theory is extended also to the distributions, and the applications in partial differential equations (the method of plane waves etc.) are outlined.

The general problem of determining a function on a manifold by means of its integrals over certain submanifolds is treated in Chapter II in terms of *homogeneous spaces in duality* (double fibration). Several examples of the introduced abstract formalism cover e.g. the original Funk transform on the two-dimensional sphere $\mathbf{S}^2$, transformations in the hyperbolic plane $\mathbf{H}^2$ (including X-ray transform and horocycles), the relation between the Radon transform and the Poisson integral, inversions on Grasmannian manifolds etc. The extreme variety of results achieved leads the author to the curious suspicion that the approach accepted might be a framework for examples rather than a basis for some general theory.

In the following two chapters, the solution of the general problem is solved for Riemannian manifolds (by considering the totally geodesic submanifolds) of constant curvature and for isotropic Lorentz spaces (including orbital integrals and their relation to Huyghens' principle, integration over Lorentzian spheres, use of Riesz potentials).

The last chapter is an outgrowth of §8, Chap. I of the first edition and may be considered a self-contained Appendix or a short course on Schwartz' distributions and Fourier transforms with an excursion to selected results concerning the Riesz potential used in the book.

Each chapter is again concluded by excellent bibliographical notes reviewing the relevant references in the historical and topical context. The number of references increased more than three times to approximately 230, which testifies the attention attracted by this area of mathematics with extremely useful applications (besides those mentioned above, at least radioastronomy, image analysis, data compression and seismology should be mentioned, and thousands of references gush out when searching Internet for Radon transform).

The book is suitable for graduate students. Mathematicians interested in integral transforms and their applications will appreciate the extension and updating of the wellknown book. However, for the copy of the "somewhat inaccessible" Radon pioneering paper from 1917, they should go back to the first edition, in which it was reproduced.

Ivan Saxl

C. Truesdell, K. R. Rajagopal: AN INTRODUCTION TO THE MECHANICS OF FLUIDS. Birkhäuser-Verlag, Boston, 2000, x+277 pages. ISBN 3-7643-4014-2, price DM 148,–.

The publication may in a prevailing way be characterized as an introduction to fluid mechanics with emphasis on non-linear models. It is articulated as a textbook with lessons and excercises.

At the beginning the notion of the abstract body motion and its transformations are introduced and basic quantities with fundamental material relations are established. Then boundary conditions are discussed. In the second chapter the kinematics of the fluid (body) motion and the basic laws of the motion are derived. Then in Chapter 3 it is shown how the diversity of materials can be expressed through the so-called constitutive equations—a complement of basic principles necessary to close the particular models of concrete flows and to express their characteristic features. Further, internal constraints as incompressibility, rigidity, etc. are discussed from quite a general point of view. In the next, fourth chapter the class of simple fluids and the monotonous motion are singled out to show how the models can be simplified by restricting considerations to special fluids and/or special motions. In the following Chapter 5, incompressible fluid flows in a general setting are treated. Here many examples of particular flows are given. Chapter 6 concerns nonlinear fluids characterized by a nonlinear stress-strain relation as fluids of higher-order grade and discusses the necessary modifications of boundary conditions in these cases. Then a variety of special flows is investigated in more detail. In Chapter 7 this issue is continued by analyzing some flows of second-grade fluids. In Chapters 8 and 9 the classical Navier-Stokes theory and the theory of incompressible and compressible Euler fluids are presented and many illustrating examples are given. This part also includes more mathematical treatment and references concerning such questions as existence, uniqueness and stability for particular fluid flow models. Finally, in Chapter 11, singular phenomena as motion of singular surfaces are studied while admitting that some smoothness assumptions on the hydrodynamical quantities could be relaxed.

The book is completed by a survey of selected elementary results from classical mathematical analysis, solutions to excercises and subject index.

The book can be read with a standard knowledge of undergraduate mathematics and will be useful to graduate students, applied mathematicians and engineers interested in the theory and practice of fluid flows modelling.

Ivan Straškraba