BOOK REVIEWS

Misha Gromov: METRIC STRUCTURES FOR RIEMANNIAN AND NON-RIEMANNIAN SPACES. Edited by J. LaFontaine and P. Pansu, English translation by Sean Michael Bates, Progress in Mathematics, vol. 152. Birkhäuser, Boston, 1998, ISBN 0-8176-3898-9, DM 198,–.

The French version of this monograph appeared already in 1979. The monograph under review can be only partially described as a translation of its French predecessor because, as the author mentions in the preface, during the years the monograph has approximately quadrupled in size. There are three appendices: P. Pansu: "Quasiconvex" domains in $\mathbb{R}^n$, S. Semmes: Metric spaces and mappings seen at many scales, and M. Katz: Systolically free manifolds. The parts added in the English version can be easily recognized being denoted by the subscript $_+$. The monograph represents a fundamental treatise on metric structures in mathematics. The main parts concerned are analysis, differential geometry and topology, and we find also applications in physics. The book starts from the very beginning and leads the reader to the level of the contemporary research which, especially in the last two decades, has been very intensive. For mathematicians investigating metric structures the book is quite indispensable. They will find there many conjectures, suggestions for further research, and many invaluable remarks. To a great extent the book is accessible even to undergraduate students provided they are patient enough. The prerequisites required are relatively mild. Of course, they depend on the part of the book the reader intends to read, because the book has a very wide scope. Generally, it is an excellent monograph and can be strongly recommended to all mathematicians and to all libraries.

Jiří Vanžura, Brno

I. Gohberg, S. Goldberg, N. Krupnik: TRACES AND DETERMINANTS OF LINEAR OPERATORS. Operator Theory, Advances and Applications, vol. 116, Birkhäuser, Basel, 2000, DM 178,–.

The book presents a systematic survey of various generalizations of traces and determinants for operators on infinite dimensional Banach spaces.

The properties of the trace of finite rank operators and the determinant of operators of the form $I + F$ where F is finite rank are well-known. A natural idea is to extend the trace and determinant to a wider class by continuity. However, the trace and determinant are discontinuous with respect to the operator norm. Therefore it is necessary to consider a stronger norm on an algebra of operators containing a dense subset of finite rank operators such that the trace (determinant) is continuous with respect to the new norm. This concept is called an embedded algebra.

Using this method, the authors obtain in a unified way many classical extensions of the trace and determinant (due to Hill, van Koch, Fredholm, Poincaré, Ruston, Grothendieck and others).

One of the main conclusions of the book is that the trace and determinant of an operator depend on the algebra containing the operator. In fact, the trace and determinant of a non-nuclear operator can be almost any complex number. However, each of these determinants still has good properties; for example, an operator is invertible if and only if each of the determinants is different from zero.

The material is self-contained and can be used for advanced courses and seminars. The monograph will be interesting both for postgraduate students and for researchers in its own and related fields.

Vladimír Müller, Praha

N. Bellomo, M. Pulvirenti (eds.): MODELING IN APPLIED SCIENCES. A Kinetic Theory Approach. Birkhäuser, Boston, 2000, xiv + 419 pages, ISBN 0-8176-4102-5, DM 168,–.

In contrast to the more usual approach to modeling real complex systems based on solutions of partial differential equations equipped with suitable initial and/or boundary conditions, the present book reviews models based on the nonlinear kinetic theory generalizing the Boltzmann statistical method of treating large systems of non distinguishable objects.

The book opens by an introductory chapter written by the editors, continues by eight review papers covering various areas of applications and closes by a chapter summarizing computational schemes recurring throughout the book.

Flow of granular media consisting of monodisperse particles undergoing dissipative collisions is treated in the second and third chapters. The results lead to the understanding how a small inelasticity on the system changes its macroscopic behaviour and find applications in a variety of industries (handling and transport of coal, food transport, agriculture), where serious undesired effects such as clogging, erratic behaviour, undesired segregation and structural failures can occur.

The fourth chapter is concerned with problems of the dynamics of dispersed particles interacting through a fluid and two simple cases are examined. The perfect incompressible and irrotational flow relates to bubbles in the fluid whereas the Stokes flow finds applications in the treatment of suspension and sedimentation processes.

Condensing and evaporating gases are modeled by means of the modified Becker-Döring equations in the fifth chapter; important analogies between Becker-Döring and Boltzmann equations are emphasized and the ability of the theory to describe phase transitions, behaviour of nonuniform mixtures, clustering and other cooperative phenomena is underlined. Nonlinear kinetic models with chemical reactions of the next chapter can be used in the description of combustion phenomena, in physical chemistry of the upper atmosphere, spacecraft flight modeling, plasma and nuclear physics etc.

The last three review chapters cover three very different areas, namely kinetic Boltzmann type models in mathematical biology (e.g. population dynamics, epidemiology and immunology), traffic flow models and large communication networks modeled with use of Markov processes.

The book with its more than 300 references can serve as a review of ideas, recent solutions and unsolved problems and will certainly be appealing to students, engineers and mathematicians interested in modeling large complex systems. However, many readers will in vain look for a missing subject index.

Ivan Saxl, Praha

P. G. L. Dirichlet: LECTURES ON NUMBER THEORY WITH SUPPLEMENTS BY R. DEDEKIND. Amer. Math. Soc., London Math. Soc., Providence, Rhode Island, 1999, xx+275 pages, ISBN 0-82-18-2017-6, $ 49,–.

This book is an English translation from the German original "Vorlesungen über Zahlentheorie" by P. G. Lejeune-Dirichlet, F. Vieweg und Sohn, Braunschweig, 1863. These Vorlesungen (lectures) belong to the most important number-theoretical books of the 19th century. This book contains the fundamentals of number theory: divisibility theory, congruences, quadratic residues, quadratic forms, primitive roots, and Euler's totient function.

In fact, this text was written by Dedekind according to Dirichlet's lectures; Dedekind added supplements to the second and later editions. The translator included Supplements I–IX because they fill some gaps in the main text and they showcase Dirichlet's theorem on primes in arithmetic progressions and his "pigeonhole" solution of Pell's equation. On the other hand, Supplements X and XI concerning ideal theory were omitted.

Reading this book requires only the most elementary knowledge from mathematics; the reader can learn from it the foundations of number theory.

Contents: Translator's introduction, 1. On the divisibility of numbers, 2. On the congruence of numbers, 3. On quadratic residues, 4. On quadratic forms, 5. Determination of the class number of binary quadratic forms, Supplements: I. Some theorems from Gauss's theory of circle division, II. On the limiting value of an infinite series, III. A geometric theorem, IV. Genera of quadratic forms, V. Power residues for composite moduli, VI. Primes in arithmetic progressions, VII. Some theorems from the theory of circle division, VIII. On the Pell equation, IX. Convergence and continuity of some infinite series, Index.

Ladislav Skula, Brno

H. Koch: NUMBER THEORY. ALGEBRAIC NUMBERS AND FUNCTIONS. American Mathematical Society, Providence, Rhode Island, 2000, xviii+368 pages, ISBN 0-8218-2054-0, $ 59,–.

This book is a translation of the German original "Zahlentheorie. Algebraische Zahlen und Funktionen", 1st edition by F. Vieweg und Sohn, Braunschweig/Wiesbaden, 1997, into English.

This volume is devoted to the foundations of algebraic numbers, including the geometry of numbers, Dedekind's theory of ideals for integral domains, and the theory of valuations. Algebraic functions are investigated by the means developed for the algebraic numbers. Analytic methods are used for the Riemann zeta-function, the Hecke L-series, and the Dedekind zeta-function. These methods are also applied to the study of the distribution of prime ideals in algebraic number fields.

The tools of algebraic number theory are demonstrated on the example of quadratic number fields, where many results are clearer than in the general case. In the last chapter a glimpse of class field theory is presented. At the end some notions and results from divisibility theory, field extensions and topological groups are explained, in the Appendices.

Contents: Preface, Notation, 1. Introduction, 2. The Geometry of Numbers, 3. Dedekind's Theory of Ideals, 4. Valuations, 5. Algebraic Functions of One Variable, 6. Normal Extensions, 7. L-Series, 8. Applications of Hecke L-Series, 9. Quadratic Number Fields, 10. What Next?, Appendix A. Divisibility Theory, Appendix B. Trace, Norm, Different, and Discriminant, Appendix C. Harmonic Analysis on Locally Compact Abelian Groups, References, Index.

The translator made only small changes in the correction of a few small errors and the enlargement of the index. For basics of abstract algebra he cites the English book "Algebra" by S. Lang (1965) instead of the German book "Algebra" by E. Kunz (1991).

This book is very suitable for researchers and graduate students who are interested in basic algebraic number theory up to the beginning of class field theory.

Ladislav Skula, Brno

J. Albree, D. C. Arney, V. F. Rickey: A STATION FAVORABLE TO THE PURSUITS OF SCIENCE. Primary materials in the history of mathematics at the United States military academy. History of Mathematics, vol. 18. American Mathematical Society. London Mathematical Society, 2000, 272 pages, ISBN 0-8218-2059-1, \$ 59,–.

This book reveals a rich collection of mathematical works located at the U.S. Military Academy at West Point. This school was founded on March 16, 1802, when the United States was mathematically still a weak, lost English colony. The development of mathematics in this country started after the War of 1812.

The first chapter (about 40 pages) is devoted to the development of Military Academy and its library. The main part of the first chapter describes the history of mathematical and mechanical education. We can find there a list of professors of mathematics and mechanics from 1802 to 1931. The next part deals with the contents of mathematical education. Then the history of the library and its mathematical collection is described.

The second chapter (about 200 pages) is a catalog of the West Point Collection. There are 798 authors and 1195 books in this catalog, which represents 1340 different works, where all volumes and editions of the books are included. The entries in this bibliography are alphabetized by the author's name. The oldest book is Regiomontanus's *Epytoma Joánis de móte regio in almagestii ptolomei*, Venice 1496, and the newest one is Darboux's *Principes de géométrie analytique*, Paris 1917.

The book contains lots of interesting photographs and valuable details about the West Point collection.

Pavel Šišma, Brno

John B. Conway: A COURSE IN OPERATOR THEORY. Graduate Studies in Mathematics, vol. 21, American Mathematical Society, Providence, Rhode Island, 2000, \$ 49,–.

The book is an introduction to the theory of Hilbert space operators and operator algebras. It assumes only the basic knowledge of functional analysis and provides a foundation for the study of operator theory.

The book studies in detail the most important classes of operators on Hilbert spaces, especially the normal and compact operators. Further, it examines the Hilbert-Schmidt and trace class operators, subnormal and essentially normal operators, and weighted shifts and their connections with Hardy and Bergman spaces.

Together with the theory of single operators the book studies also operator algebras. It explores C^*-algebras and gives an extended introduction to von Neumann algebras. The last chapter is devoted to reflexivity and invariant subspaces.

John B. Conway belongs to the best authors of basic textbooks (let me mention his Functions of One Complex Variable and A Course in Functional Analysis that were both published by Springer-Verlag). The present book continues this tradition of clear and elegant way of presentation. Although most of the material can be found in other sources, this book can be highly recommended for students of operator theory as well as to experts in the field who will find many interesting ideas there.

Vladimír Müller, Praha

A. Candel, L. Conlon: FOLIATIONS I. Graduate Studies in Mathematics, vol. 23, American Mathematical Society, 2000, Providence, R. I., ISBN 0-8218-0809-5, \$ 54,–.

The book is the first of two volumes of a monograph on foliations. It is divided into three parts.

The first part treats the foundations of the theory. Foliations are defined and basic methods of constructing and studying these objects are introduced. The second part is dedicated to codimension one foliations. Among other things, various stability theorems are proved and Poincaré-Bendixson theory for limit sets of flow lines is presented. The third part contains selected topics on higher codimensions and abstract foliated spaces. The main notions of this part are foliation cycles and entropy of foliations. The authors also explain how some methods presented there generalize to 'abstract laminations.'

The book is addressed to students and researchers interested in the theory of foliations as one of aspects of differential geometry. Numerous examples, exercises, applications and illustrations are presented. Reading the book requires only a basic familiarity with differential geometry and topology.

Martin Markl, Praha

Jean Fresnel: ESPACES QUADRATIQUES, EUCLIDIENS, HERMITIENS. Hermann, Actualités scientifiques et industrielles 1445, Paris 1999, 135 pages.

The book under review forms one whole with the textbook Algèbre des matrices by the same author, which consists of a single chapter A. Consequently it consists of three chapters: B. Espaces vectoriels quadratiques, C. Espaces vectoriels euclidiens, D. Espaces vectoriels hermitiens.

Chapter B introduces and investigates (finite dimensional) vector spaces over an arbitrary field (mostly of characteristic $\neq 2$) endowed with a symmetric bilinear form. To this form the notion of orthogonality and of isotropic vectors is associated in a natural way. The linear bijections conserving the bilinear structure form the orthogonal group of the space. The isomorphism of two quadratic spaces (the isometry) is defined and the essential result, the theorem of Witt on the extension of any isometry between two subspaces of a given non degenerate quadratic space E over a field of characteristic $\neq 2$ to an orthogonal automorphism of E is proved. The theorem of Witt is used to prove the existence of a unique (up to an isometry) decomposition of any non degenerate quadratic space as a direct orthogonal sum of a hyperbolic space and a space without isotropic vectors. In the end the orthogonal group of a quadratic space is studied from a geometric point of view.

Chapter C is devoted to real quadratic spaces. They are completely classified by Sylvester's „lex inertiae“. The signature (p, q) and the dimension are orthogonal invariants characterizing (up to an isometry) the real quadratic space. The euclidian vector space E is defined by a positive definite symmetric bilinear form (of signature $(n, 0)$, $n = \dim E$), which induces a norm (Schwarz inequality) on E. The orthogonal group of E and, more generally, of a real quadratic space, is studied in detail.

In Chapter D the complex vector spaces endowed with a hermitian bilinear form, which is an appropriate generalization of symmetric bilinear forms to the complex space, are studied.

The important part of the book are about 150 exercises. They contain, e.g., the following fine results: the classification of quadratic forms over finite fields, the dimension of the vector space of nilpotent matrices, the decompositions of Iwasawa and of Cartan, the theorem of Fischer-Cochran, the theorems of Bieberbach on crystallographic groups, the Cholesky factorization and finally the well known theorems of Burnside, Schur, Jordan on torsion subgroups of $GL_n(\mathbb{C})$ and many others.

Jaroslav Fuka, Praha

Jacques Hadamard: NON-EUCLIDEAN GEOMETRY IN THE THEORY OF AUTOMORPHIC FUNCTIONS. J. J. Gray and Abe Shenitzer (eds.), AMS, LMS, History of Mathematics, vol. 17, AMS, LMS, Providence, R. I., 1999, 95 pages, $ 19,–.

Hadamard's book consists of 5 chapters: Ch. I. The group of motions of the hyperbolic plane and its properly discontinuous subgroups, Ch. II. Discontinuous groups in three geometries. Fuchsian functions, Ch. III. Fuchsian functions, Ch. IV. Kleinian groups and functions, Ch. V. Algebraic functions and linear algebraic differential equations, Ch. VI. Fuchsian groups and geodesics. Although Hadamard himself did'not work in the theory of automorphic functions, he gives a fascinating and highly instructive brief exposition on Poincaré's creation of the theory of automorphic functions published in 1881–1884. He singles out the hardest part of the Poincaré's new theory (in the 1920s the broad features of the theory remained as they had been established by Poincaré in the 1880s), namely the solution of linear differential equations, and underlines the fundamental rôle of the Lobachevskian metric for this theory (Chapters V and VI). His monograph was written in 1920 in connection with the preparation of the collected works of N. I. Lobachevskiĭ in Russia. It was translated into Russian and published in 1951 as volume 6 in the series The Geometry of Lobachevskiĭ and the Development of Its Ideas. The present translation into English by Abe Schenitzer making this Hadamard's work accessible to all mathematical community is based on the Russian edition, because Hadamard's french original text appears now to be lost.

The historical circumstances, sources and perspectives of Poincaré's discovery are very instructionally and vividly depicted in the attached fine Jeremy Gray's historical essay "Historical Introduction". Generally, this beautiful thin book will please analysts as well as geometers and also all fans of the history of mathematics.

Jaroslav Fuka, Praha

O. Debarre: TORES ET VARIÉTÉS ABÉLIENNES COMPLEXES. Société Mathématique de France, ISBN 2-86883-427-2.

Central objects of the book are n-dimensional complex tori, that is, quotients V/Γ of an n-dimensional complex vector space V modulo a discrete subgroup Γ of maximal rank. The exposition starts with the easiest case $n = 1$ when complex tori are precisely elliptic curves. The corresponding classical theory of Weierstrass functions, theta functions and divisors is also recalled.

The higher-dimensional case is then taken up. For $n \geqslant 2$, the complex torus V/Γ need not necessarily be a complex projective manifold. This happens if and only if there exists an integral Kähler form on Γ. The author explains how the existence of such a form can be characterized in terms of a special basis of Γ.

Final sections of the book are devoted to moduli spaces of polarized abelian varieties (that is, complex tori with a marked Kähler form). The last section is dedicated to subvarieties of these varieties.

Complex tori gave the author an excuse to recall a lot of standard and useful material, as differential forms, sheafs and their cohomology, line bundles and divisors, Riemann-Roch theorem, &c. I warmly recommend the book as an introduction to basic techniques of algebraic geometry. It requires only a preliminary knowledge of algebra and theory of complex functions.

Martin Markl, Praha

R. Hagen, S. Roch, B. Silbermann: C^*-ALGEBRAS AND NUMERICAL ANALYSIS. Pure and Applied Mathematics, A series of Monographs and Textbooks, Marcel Dekker, Inc., New York-Basel, 2001, \$ 165,–.

Many of concrete applications of mathematics in science and technology lead eventually to the problem of solving the equation $Ax = y$ where A is a linear operator between two linear spaces and y a given vector. Related problems are solving the equation $Ax = y$ which is not uniquely solvable, where we are looking for a distinguished solution (for example the least square solution), and computing the eigenvalues and eigenvectors of A.

A direct solution of the equation $Ax = y$ is impossible in most of the practical problems but it is often possible for operators between finite dimensional spaces. Thus it is natural to approximate A by finite matrices A_n.

An approximation method for the operator $A\colon X \to Y$ is a sequence of operators $A_n\colon X \to Y$ such that $A_n \to A$ in the strong operator topology. Typically, A_n have finite rank. The approximation method (A_n) is called applicable if the equation $A_n x = y_n$ is uniquely solvable for all sequences y_n converging to y and all n large enough, and the solutions of $A_n x = y_n$ converge to the solution of the original equation $Ax = y$.

Approximation methods are investigated by algebras techniques. An approximation method (A_n) is considered an element of the algebra of all bounded sequences of operators $X \to Y$ (with the naturally defined algebraic operations) factorized over the ideal of all sequences tending to 0.

The book investigates relations between approximation methods and the properties of this algebra. A typical result is: an approximation method (A_n) is applicable if and only if the class of the sequence (A_n) in the above described quotient algebra is invertible.

The book is well-written and it is intended both for students who want to see applications of functional analysis and to learn numerical analysis, and for mathematicians and engineers interested in theoretical aspects of numerical analysis.

Vladimír Müller, Praha

V. Rovenski: GEOMETRY OF CURVES AND SURFACES WITH MAPLE. Birkhäuser, Boston, 1999, x + 310 pages, ISBN 3-8176-4074-6, DM 118,–.

The book is an excellent elementary course on visualizing geometrical objects by means of MAPLE software. First two parts are devoted to 2D graphics of functions and planar as well as space curves, whereas 3D graphics of polyhedra and surfaces are treated in the remaining two sections. In order to illustrate the essence of the book, a selection of chapter headings with examples of the covered cases is given in what follows.

Curves in planar coordinates (nets of curvilinear coordinates, level curves, cissoid, strophoid and conchoid of a line, Cassini ovals, working stopwatch), *space curves* (curves on algebraic surfaces, curves with shadows on planar and non-planar surfaces, tangent lines and asymptotes, envelope curves and mathematical embroidery), *length and centre of mass of a curve, curvature and torsion of curves, fractal curves and dimension* (Peano, Koch, Menger and dragon curves), *spline curves, non-Euclidean (hyperbolic) geometry in the half-plane, convex hulls.*

And further, *regular and semi-regular polyhedra* (Platonic solids, star-shaped polyhedra, Archimedean solids), *surfaces in space* (including tangent planes, extrema of functions, singular points), *classes of surfaces* (algebraic, revolutionary, ruled—e.g. conoids, canal surfaces and tubes).

The book also contains definitions of constructed objects, important geometric theorems on which the programs are based, historical notes, Greek etymology of curve names, about

100 exercises and more than 350 figures. Frequently also the curve, surface or body transformations (translations, rotations, inversion, cuts etc.) are described and extensively used to obtain new objects thus establishing the mutual relation between them and promoting a deeper understanding of constructions.

All these features make the book not only a comprehensive and expert guide through MAPLE graphic packages but also a condensed textbook of geometric modelling. It could also be useful for users of other software systems like Mathematica or Mathlab.

Moreover, the detailed contents of the book with extracts from each chapter and illustrative examples are exposed at the author's web page

`http://math2.haifa.ac.il/ROVENSKI/rovenski/Birkhauser.html`

from which also zipped MAPLE code segments (Computer Algebra System Maple V, Release 6) can be downloaded so that the reader will be saved from tedious typing of the programs included in the book. Only the elementary knowledge of analytical geometry is required for the understanding of the book, consequently it is suitable for students, instructors, computer scientists as well as for engineers and mathematicians.

Ivan Saxl, Praha

Titu Andrescu, Razvan Gelca: MATHEMATICAL OLYMPIAD CHALLENGES. Birkhäuser, Basel, 2000, 260 pages, ISBN 0-8176-4155-6, DM 58,–.

The authors of the book under review are two Romanian mathematicians living in the USA since 1991. Both of them had taken part successfully in a number of mathematical competitions and after completing their University studies have engaged themselves in organization of mathematical competitions for high school students, first in Romania and now in USA. They participate intensively in the preparation of American students talented for Mathematics for their participation in International Mathematical Olympiads. In the book they have collected about 400 problems from various fields of Mathematics. They are divided into three Chapters: Geometry and Trigonometry, Algebra and Analysis, Number Theory and Combinatorics. Each chapter is divided into ten sections, e.g. Cyclic Quadrilaterals, Power of a point, Periodicity, The Mean Value Theorem, Pell Equations, etc. Each section presents mathematical assertions used subsequently, several solved problems, and further unsolved problems. Solutions of the latter ones can be found in the other part of the book. The problems come from mathematical journals, mathematical competitions in various countries, as well as from International Mathematical Olympiads. The book can be heartily recommended to students who wish to succeed in mathematical competitions, and of course also to teachers preparing their students for demanding mathematical competitions on both the national and international levels.

Leo Boček, Praha