Mathematica Bohemica

Book reviews

Mathematica Bohemica, Vol. 132 (2007), No. 1, 109–112

Persistent URL: http://dml.cz/dmlcz/133988

BOOK REVIEWS

Onésimo Hernández-Lerma, Jean Bernard Lasserre: MARKOV CHAINS AND INVARIANT PROBABILITIES. Progress in Mathematics, Vol. 211, Birkhäuser, Basel, 2003, xvi+205 pages, ISBN 3-7643-7000-9, EUR 66.34.

Let $\xi = (\xi_i;\ i \geqslant 0)$ be a time-discrete homogeneous Markov chain on a measurable space $(X, \mathscr{B})$ with a transition probability P. One of the basic ways how to understand the long-time behaviour of the chain ξ is to study the iterates P^k of the kernel P by means of ergodic theory: one aims at describing all invariant probability measures for P (i.e., distributions of the initial state ξ_0 such that ξ is a strictly stationary process) and at investigating their attractivity properties (that is, the convergence of $P^k(x,\cdot)$ to an invariant measure in a suitable sense), which are closely related to recurrence properties of the chain.

O. Hernández-Lerma and J. B. Lasserre addressed this complex of problems in a long series of papers in the nineties of the last century, and now, in the monograph under review, they present their results in a systematic manner, including also some hitherto unpublished theorems. The book is reasonably self-contained and a bit heterogeneous material is organized very thoughtfully, although it is focused on the authors' results and, as a rule, only those are given a proof.

Let us describe the contents of the book in some detail: The first chapter contains auxiliary results from measure theory and functional analysis. The remaining eleven chapters are divided into three parts. In Part I (Markov chains and ergodicity) the reader may find, besides preliminaries on ergodic theorems for Markov operators and on limit theorems for chains on countable state spaces (Chapters 2 and 3, respectively), results based on a thorough study of the behaviour of expected occupation measures $P^{(n)}(x,\cdot)$, which are defined as the Cesàro means $P^{(n)} = n^{-1}\sum_{k=0}^{n-1} P^k$. In Chapter 4, they are employed to characterize positive Harris recurrent chains. The next chapter is devoted to chains with a locally compact metric state space X and contains theorems concerning the explicit form of limits in ergodic theorems or generalizing Yosida's ergodic decomposition. A complete classification of Markov chains (again on locally compact spaces) in terms of occupation measures is developed in Chapter 6. Part II of the book is entitled Further ergodicity properties and contains Chapters 7 to 9. In the first of them, the main results concern quasi-Feller chains, a class of chains that violate the Feller condition, nevertheless, share with Feller chains their limiting behaviour. Conditions for existence of a solution (g,h) to a probabilistic Poisson equations $g = Pg$, $g + h - Ph = f$ are treated in Chapter 8, without any hypotheses on the topology of the state space or upon uniqueness of the invariant measure. Chapter 9 contains several theorems about strong and uniform ergodicity, improving on results due to N. V. Kartashov. Part III (Existence and approximation of invariant probability measures) consists also of three chapters. Existence of invariant measures, of strictly positive invariant measures, and existence and uniqueness of strictly positive invariant measures are studied in Chapter 10, while the following chapter deals with analogous problems for Markov operators on L^1-spaces, that is, invariant densities are looked for. In both cases, the authors' original approach relies on a generalized Farkas theorem and leads to necessary and sufficient conditions. Finally, numerical approximation schemes for invariant measures based on a sequence of linear or semidefinite programs are proposed in the last chapter.

The monograph is a welcome addition to the existing literature on ergodic theory of Markov processes, since it makes the results of its prolific authors easily available, being written in a careful and lucid manner. However, it should be noted that the bibliografic notes at the end of each chapter cannot be always taken literally. (For example, I. V. Girsanov might be rather surprised, had he the opportunity to read on page 102 that "the definition of [...] strong-Feller MCs is due to W. Feller (whence the name)".)

Jan Seidler, Praha

V. Müller: SPECTRAL THEORY OF LINEAR OPERATORS AND SPECTRAL SYSTEMS IN BANACH ALGEBRAS. Birkhäuser, Basel, 2003, 381 pages, hardcover, ISBN 3-7643-6912-4, EUR 108.–.

The notion of spectrum of a linear operator is central in operator theory, and has far-reaching applications in numerous other areas ranging from PDE to control theory or mathematical physics. Over the time, many types of spectra have emerged and proved of importance, both for single operators and for tuples of commuting operators: for instance, the approximate point spectrum, the Kato spectrum, the Taylor spectrum, various kinds of local spectra and essential spectra, and so on. The aim of the present monograph is to provide a treatment of all these types of spectra (and many more) in an unified axiomatic way. The main ingredient for this is the notion of a *regularity* in a Banach algebra, developed by the author; at the same time, however, the book contains a wealth of information about relevant results of other authors, a lot of which appear here for the first time in a book form, as well as a good deal of prerequisites from Banach algebras and operator theory. The exposition is very well organized and easy to follow. The unique combination of all these ingredients makes the book accessible for anyone with basic command of functional analysis, topology and complex function theory, while making it also a worthy and comprehensive reference source for experts in the field. In my opinion, this is a very nice volume which should not be missing on the shelf of anyone interested in, or wishing to learn more about, spectral theory of Banach space operators and elements of Banach algebras.

Miroslav Engliš, Praha

D. Haroske, T. Runst, H.-J. Schmeisser (eds.): FUNCTION SPACES, DIFFERENTIAL OPERATORS AND NONLINEAR ANALYSIS. The Hans Triebel Anniversary Volume. Birkhäuser, Basel, 2003, xii+474 pages, EUR 98.–.

Papers in this collection are based on lectures given at the international conference FSDONA-01, held in Teistungen (Thuringia). It was the fifth conference in a series of meetings held cyclically in Finland, Germany and Czechia and this one was organized by collaborators and colleagues of Hans Triebel on the occasion of his 65th birthday. His name is connected with fundamental contributions to the systematic general theory of function spaces in the Fourier analysis spirit, interpolation theory, entropy and spectral problems, fractal theory etc. and this collection is a representative tribute to that. The whole volume is divided into three parts: Part I consists of two fine surveys, and two other parts contain 7 longer and 24 shorter contributions. The whole volume reflects recent progress in the theory of function spaces and related fields, in particular, in selected areas from the PDE's theory. The spectrum of the problems considered here is naturally mirroring the immense influence of Hans Triebel on the function spaces theory during the last decades. The attached Contents might be helpful to get the idea of the whole volume.

C o n t e n t s. Part I. Oleg Besov and Gennadiy Kalyabin: Spaces of differentiable functions, David E. Edmunds: Entropy, embeddings and equations.

Part II. Claudianor O. Alves and Djairo G. de Figueiredo: Nonvariational elliptic systems via Galerkin methods, Gérard Bourdaud: Superposition operators in Zygmund and BMO spaces, Vladimir Kozlov and Vladimir Maz'ya: Asymptotics of a singular solution to the Dirichlet problem for an elliptic equation with discontinuous coefficients near the boundary, Akihiko Miyachi: Weighted Hardy spaces on a domain and its application, Stanislav Pohozaev: The general blow-up for nonlinear PDE's, Michael Solomyak: Laplace and Schrödinger operators on regular metric trees: the discrete spectrum case, Gunther Uhlmann: Inverse boundary problems in two dimensions.

Part III. Marina Borovikova and Rüdiger Landes: On the regularity of weak solutions of elliptic systems in Banach spaces, Michele Bricchi: Complements and results on h-sets. Viktor I. Burenkov: Lifting properties of Sobolev spaces, António M. Caetano and Dorothee D. Haroske: Sharp estimates of approximation numbers via growth envelopes, Andrea Cianchi: Sharp summability of functions from Orlicz-Sobolev spaces, Serguei Dachkovski: Regularity problems for some semi-linear problems, Stephan Dahlke: Besov regularity for the Neumann problem, Sophie Dispa: Intrinsic descriptions using means of differences for Besov spaces on Lipschitz domains, Pavel Drábek: Landesman-Lazer type like results for the p-Laplacian, W. D. Evans: On the Sobolev, Hardy and CLR inequalities associated with Schrödinger operators, Ji Gao: Mazur distance and normal structure in Banach spaces, Vagif S. Guliev: Some inequalities for integral operators, associated with the Bessel differential operator, Mats Gyllenberg, Andrei Osipov and Lassi Päivärinta: On determining individual behaviour from population data, Yavdat Il'yasov and Thomas Runst: Nonlocal investigations of inhomogeneous indefinite elliptic equations via variational methods, Jon Johnsen: Regularity results and parametrices of semilinear boundary problems of product type, Denis A. Labutin: Potential estimates for large solutions of semilinear elliptic equations, Jan Malý: Coarea properties of Sobolev functions, Osvaldo Méndez and Marius Mitrea: Banach envelopes of the Besov and Triebel-Lizorkin spaces and applications to PDE's, Luc Molinet, Francis Ribaud and Abdellah Youssfi: On the flow map for a class of parabolic equations, David Opěla: Spaces of functions with bounded and vanishing mean oscillation, Bohumír Opic: On equivalent quasi-norms on Lorentz spaces, Evgeniy Pustylnik: Concave functions of second order elliptic operators, kernel estimates and applications, László Simon: On approximation of solutions of parabolic functional differential equations in unbounded domains, Leszek Skrzypczak: Function spaces in presence of symmetries: compactness of embeddings, regularity and decay of functions.

Miroslav Krbec, Praha

Erwin Neuenschwander (ed.): WISSENSCHAFT ZWISCHEN QUALITAS UND QUANTITAS. Birkhäuser, Basel, 2003, 456 pages, CHF 118.–/EUR 82.24.

The *Wissenschaftshistorisches Kolloquium* has existed at the University and Technical University (ETH) Zurich for more than 25 years. This book is based on 9 independent lectures given in the framework of this Colloquium during the fall semester 1990/91, and its aim is to describe and analyze the tension and interplay between quality and quantity in science from antiquity up to modern times. The editor provided a unifying introduction and a selected bibiliography to the topic, and E. Brieskorn contributed with an extensive (almost 180 pages) essay entitled *Is there a revival of quality in mathematics?* With one exception, all contributions are in German.

Alois Kufner, Praha

Fuensanta Andreu-Vaillo, Vicent Caselles, José M. Mazón: PARABOLIC QUASILINEAR EQUATIONS MINIMIZING LINEAR GROWTH FUNCTIONALS. Birkhäuser, Basel, 2004, 356 pages, EUR 88.–.

The goal of the book is to present general existence and uniqueness results for quasilinear parabolic equations whose operator is, in divergence form, the subdifferential of a Lagrangian which is convex in $|\nabla u|$ and has linear growth as $|\nabla u| \to \infty$. The monograph pays particular attention to the case of minimizing total variation flow for which the authors study the Neumann, Dirichlet and Cauchy problem in $\mathbb{R}^{\mathbb{N}}$ together with the main qualitative properties of its evolution.

Chapter 1 is devoted to the study of the variational approach to image restoration based on the total variation minimization subject to the constraints given by the image acquisition model. This model was introduced by L. Rudin, S. Osher and E. Fatemi. In Chapter 2 the Neumann problem is studied for the minimizing total variation flow. The Cauchy problem for the total variation flow is studied in Chapter 3. Chapter 4 is devoted to a study of the asymptotic behaviour and qualitative properties of the solutions of the total variation flow in $\mathbb{R}^{\mathbb{N}}$. Chapter 5 deals with the Dirichlet problem for the total variation flow. Chapters 6 and 7 are devoted to a study of the Dirichlet problem for quasilinear parabolic equations whose operator is, in divergence form, the subdifferential of a Lagrangian which is convex and has linear growth in the magnitude of the gradient. These techniques can be used to justify many mathematical models appearing in different contexts like continuum mechanics, faceted crystal growth, and image processing. The book closes with three appendices in which the authors outline some of the main tools used in the above chapters.

The book will be useful to graduate students and researchers in mathematics and image processing.

Šárka Nečasová, Praha

Robert C. Dalang, Marco Dozzi, Francesco Russo (eds.): A SEMINAR ON STOCHASTIC ANALYSIS, RANDOM FIELDS AND APPLICATIONS IV. Progress in Probability, Vol. 58. Birkhäuser, Basel, 2004, xii+328 pages, ISBN 3-7643-7131-5, EUR 108.–.

The first Seminar on Stochastic Analysis, Random Fields and Applications took place at the Centro Stefano Franscini in Ascona (Switzerland) in the year 1993; since then the seminars have been organized each third year. The book under review are proceedings of the fourth seminar, that was held in May 2002. The volume comprises nineteen papers, all of them were refereed and most of them contain new results with complete proofs, the remaining ones providing an overview of their authors' recent research. Both random fields and stochastic analysis are among the crucial concepts of contemporary probability theory and its applications, so it is not surprising that the papers are not focused on any particular topic: the reader may find articles devoted to stochastic partial differential equations, stochastic symplectic geometry, the Sherrington-Kirkpatrick model or to tail estimates for the distribution of the condition number of random matrices. A special section on mathematical finance containing six papers is included, which corresponds to the fact that a Minisymposium on Stochastic Methods in Financial Models was a part of the conference.

Jan Seidler, Praha