Marek Fila; Ján Filo

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A BLOW-UP RESULT FOR NONLINEAR DIFFUSION EQUATIONS

MAREK FILA and JÁN FILO

1. Introduction

This paper deals with the initial-boundary value problems of the form

$$\text{(I)}\qquad \begin{aligned} &u_t = \Delta(u^m) + u^p - g(u) && x \in D,\ t > 0,\\ &u(x, t) = 0 && x \in \partial D,\ t > 0,\\ &u(x, 0) = u_0(x)\,(\geqslant 0) && x \in D,\end{aligned}$$

where D is a smoothly bounded domain in $\mathbf{R}^N$ and m, p are positive constants $1 < p$, $0 < m \leqslant p$. Precise conditions concerning the data g and u_0 will be given later, but till then, we shall consider as a model term $g(u) = \alpha u^q$ for $u \geqslant 0$ and $g(u) \equiv 0$ for $u < 0$, where $\alpha \geqslant 0$, $1 \leqslant q < p$.

In connection with the question of the nonexistence of global solutions to problems related to (I), a number of authors (e.g. Ball [2], Fujita [8], Galaktionov [9], Levine and Sacks [11], Nakao [12], Sacks [14], Sattinger and Payne [15], Tsutsumi [16], Filo [5]) have investigated conditions under which weak solutions will blow up in a finite time. In the paper presented we extend, in a certain sense, the blow-up result given by Sattinger and Payne [15] concerning the semilinear parabolic equations to nonlinear diffusion problems including the absorptive term g.

In order to describe our results, let us take $m < p$ and define

$$\boldsymbol{d} = k \inf_{\substack{w \in H_0^1(D)\\ w \geqslant 0,\ w \not\equiv 0}} \left(\frac{\left(\int_D (|\nabla w|^2 + g(w^{1/m})\, w)\, dx \right)^{1/2}}{\left(\int_D w^{1 + p/m} dx \right)^{m/(p+m)}} \right)^{\frac{2(p+m)}{p-m}},$$

where $k = \min\left(\frac{1}{2}, \frac{m}{m + q\,\operatorname{sign}(\alpha)}\right) - \frac{m}{m+p}$ and in case of $N \geqslant 3$ we add the assumption $pm^{-1} \leqslant (N+2)/(N-2)$. From the Sobolev embedding theorem one can then see that $\boldsymbol{d}$ is positive and we can introduce the "unstable" set $\boldsymbol{B}$ given by

$$\boldsymbol{B} = \left\{ w \in H_0^1(D),\ w \geqslant 0\colon J(w) < \boldsymbol{d} \text{ and } \int_D (|\nabla w|^2 + g(w^{1/m})\, w - w^{1+p/m})\, dx < 0 \right\},$$

where J is the functional of the potential energy associated with (I), i.e.

$$(1.1)\qquad J(w) = \int_D \left(\frac{1}{2} |\nabla w|^2 + \int_0^w g(r^{1/m})\, dr - \frac{m}{m+p}\, w^{1+p/m} \right) dx.$$

We prove that if $u_0^m \in \boldsymbol{B} \cap L^\infty(D)$, then the mth power of the solution u to Problem (I) does not leave the set $\boldsymbol{B}$ and tends to infinity in finite time in the L^∞ norm. For $g \equiv 0$ the number $\boldsymbol{d}$ is just the depth of the potential well introduced by Sattinger (see [15] and references there). If $m = 1$, then the set $\boldsymbol{B}$ concides with the set of those initial data in $H_0^1(D)$ for which Sattinger and Payne ([15]) proved the blowing up of solutions to a problem with a reaction term like u^p. Though our treatment is based on the study of an analogous "unstable" set $\boldsymbol{B}$, the nonlinearity in diffusion as well as the absorptive term g cause that our method is completely different. Some of our arguments are of a similar nature as those used by Nakao [12], [13] in order to prove the global existence of solutions to Problem (I) with $g \equiv 0$ and $m > 1$.

The more delicate case $m = p > 1$ is considered separately and only for a more special choice of g.

First, however, we shall need some preliminaries.

2. Preliminaries

We start by introducing some notation: $Q_T = D \times (0, T)$, $S_T = \partial D \times (0, T)$, $|D|$ — Lebesgue measure of the set D, $|u|_q = \|u\|_{L^q(D)}$, $1 \leqslant q \leqslant \infty$,

$$\overset{+}{H}{}_0^1 = \{ w \in H_0^1(D)\colon w \not\equiv 0,\ w \geqslant 0 \ a.e.\ on\ D \},$$

$$\|w\| = \left(\int_D |\nabla w|^2 dx \right)^{1/2}, \quad \int_D h(t) = \int_D h(x, t)\, dx, \quad \iint_{Q_T} h = \iint_{Q_T} h(x, t)\, dxdt,$$

$$(u, v) = \int_D u(x)\, v(x)\, dx.$$

Now, before specifying the meaning of the solution of Problem (I) we note that except of the case $m = 1$, Problem (I) does not necessarily have a classical solution even if the data are smooth and so it is necessary to consider some well-defined generalized solution.

Definition 1. *By a solution of Problem* (I) *on* $[0, T]$ *we mean a nonnegative function u such that*

$$u \in C([0, T]; L^2(D)) \cap L^\infty(Q_T), \quad u^m \in L^\infty(0, T; H_0^1(D))$$

and u satisfies

$$(2.1) \qquad (u(t), \varphi(t)) - \int_0^t ((u, \varphi_t) - (\nabla u^m, \nabla \varphi) + (u^p - g(u), \varphi)) = (u_0, \varphi(0))$$

for a.e. $t \in [0, T]$ *and all* φ *such that* $\varphi \in L^2(0, T; H_0^1(D))$, $\varphi_t \in L^2(Q_T)$.

A subsolution (*supersolution*) *is defined as above with equality in* (2.1) *replaced by* $\leqslant$ ($\geqslant$) *whenever* $\varphi \geqslant 0$.

Further, in this paper we shall always use the following assumptions about the domain D and the initial function u_0:

(H1) *D is a bounded domain in* $\mathbf{R}^N$ *whose boundary,* ∂D, *is of class* C^3.

(H2) $u_0(x)$ *is a nonnegative function defined on D such that* $u_0^m \in H_0^1(D) \cap L^\infty(D)$.

We shall also refer to these assumptions collectively as (H).

Next we shall need the following comparison and local existence results

Proposition 1. *Suppose that D satisfies* (H1) *and that* u_0 *and* v_0 *both satisfy* (H2), $g(0) = 0$ *and g is locally Lipschitz continuous. Let u be a subsolution and v be a supersolution of Problem* (I) *on* $[0, T]$ *with initial functions* u_0 *and* v_0, *respectively.*

Then $u_0 \leqslant v_0$ *a.e. in D implies* $u \leqslant v$ *a.e. in* Q_T.

Proposition 2. *If* (H) *holds and g is locally Lipschitz continuous,* $g(0) = 0$, *then there exists a time* T_{max}, $0 < T_{max} \leqslant \infty$ (*which depends only on the data m, N, p, g,* u_0 *and D*) *such that Problem* (I) *possesses a unique solution u on* $[0, T]$ *for any* $0 < T < T_{max}$. *If* $T_{max} < \infty$, *then*

$$(2.2) \qquad \lim_{t \to T_{max}^-} |u(t)|_\infty = +\infty.$$

Moreover, for $0 \leqslant s < t < T_{max}$ *u satisfies*

$$(2.3) \qquad \frac{4m}{(m+1)^2} \int_s^t |(u^{(m+1)/2})_t|_2^2 + J(u^m(t)) \leqslant J(u^m(s)),$$

where J is given by (1.1).

For the proof of Proposition 1 in the case of $m \geqslant 1$ we refer to [1], in case of $0 < m < 1$, for example, to [5] (where the method of [1] is adapted). Proposition 2 for $m \geqslant 1$ is proved in [11] and for $0 < m < 1$, see e.g. [5].

3. Main results

We begin by formulating the precise conditions concerning the function g, which will be called hypotheses (A):
$g \in C^1([0, \infty))$, $g(0) = 0$, $g(u) \geqslant 0$ for $u \geqslant 0$ and if we define $G(u) = g(u^{1/m})$,

$$G'(u)\,u \leqslant \vartheta G(u) \quad \textit{for some} \quad 0 < \vartheta < p/m \quad \textit{and all} \quad u \geqslant 0. \tag{3.1}$$

If G' has a singularity at 0, which might occur in the case of slow diffusion, i.e. for $m > 1$, we shall need the additional assumption, namely, that (3.1) holds for some $0 < \vartheta' \leqslant 1$ in a neighbourhood of the origin.

Since we restrict ourselves to $u_0 \geqslant 0$, the solution $u(x, t)$ is nonnegative too, thus the behaviour of g for $u < 0$ is irrelevant and we can put $g \equiv 0$ for $u < 0$.

Now let us first consider the case $m < p$. Put

$$\boldsymbol{d} = k \inf_{w \in \overset{+}{H}{}_0^1} \left(\frac{(\|w\|^2 + (G(w), w))^{1/2}}{|w|_{1+p/m}} \right)^{2(p+m)/(p-m)}, \tag{3.2}$$

where $k = i(\vartheta) - m(m+p)^{-1}$, $i(\vartheta) = \min(2^{-1}, (1 + \vartheta_g)^{-1})$ and $\vartheta_g = 0$ if $g \equiv 0$ and $\vartheta_g = \vartheta$ otherwise. By the Sobolev embedding theorem it is not difficult to see that $\boldsymbol{d}$ is positive if

$$\begin{cases} p \textit{ is arbitrary } (m < p) \textit{ for } N = 1, 2 \textit{ and} \\ pm^{-1} \leqslant (N+2)/(N-2) \textit{ for } N \geqslant 3, \end{cases} \tag{3.3}$$

and using the notation

$$K(w) = \|w\|^2 + (G(w), w) - |w|_{1+p/m}^{1+p/m}$$

we put

$$\boldsymbol{B} = \begin{cases} \{w \in \overset{+}{H}{}_0^1 \colon J(w) < \boldsymbol{d} \textit{ and } K(w) < 0\} \textit{ if } (3.3) \textit{ holds and} \\ \{w \in \overset{+}{H}{}_0^1 \cap L^\infty(D) \colon J(w) \leqslant 0\} \textit{ otherwise.} \end{cases} \tag{3.4}$$

We note that the assumption (3.1) yields

$$\int_0^u G(r)\,dr \geqslant (1 + \vartheta)^{-1} G(u)\,u \qquad \text{for } u \geqslant 0, \tag{3.5}$$

and that, using (3.5), it is not difficult to find that $J(w) \leqslant 0$ for $w \in \overset{+}{H}{}_0^1$ implies $K(w) < 0$. The main results read then as follows.

Theorem 1. *Assume that D and u_0 satisfy* (H), $0 < m < p$, $1 < p$, g *satisfies* (A) *and let $u(t, u_0)$ denote the solution of Problem* (I) *with initial value u_0.*

If $u_0^m \in \boldsymbol{B}$ then $u^m(t, u_0) \in \boldsymbol{B}$ for $0 \leqslant t < T_{max}$ and

$$T_{max} \leqslant \left(\frac{1}{|D|} \int_D u_0^{m+1} \right)^{(1-p)/(1+m)} \Big/ (p-1)(1-\kappa) < \infty, \tag{3.6}$$

where the constant $\kappa \in (0, 1)$ *depends only on* u_0, m, p, ϑ (*c.f.* (4.5)), *i.e. the solution* $u(t, u_0)$ *blows up in a finite time in* L^∞ *norm for* $u_0^m \in \boldsymbol{B}$.

Remark 1. Assume that D satisfies (H1) and let (3.3) hold with $pm^{-1} < (N+2)/(N-2)$ if $N \geqslant 3$. Moreover, suppose either (i) $g \equiv 0$ or (ii) $m = 1$ and $g(u) = au$ for some $a > 0$. Then it is not difficult to verify that

$$\boldsymbol{d} = \inf_{w \in \overset{+}{H}{}_0^1} \left(\sup_{0 \leqslant \lambda < \infty} J(\lambda w) \right) \tag{3.7}$$

and that the unstable set $\boldsymbol{B}$ given by (3.4) is equal to the set

$$\{ w \in \overset{+}{H}{}_0^1 : J(\lambda w) < \boldsymbol{d} \text{ for } \lambda \in [1, \infty) \}$$

(see, e.g., [16], where a similar result for a potential well is demonstrated).

In addition, the infimum in (3.7) is attained at a stationary solution to Problem (I) (see, e.g., [3, Theorem 6.3.9] or also the proof of Theorem 2 in this paper), hence

$$\boldsymbol{d} = \min_{v \in E^+} J(v^m),$$

where E^+ denotes the set of all positive stationary solutions to Problem (I), thus E^+ is nonempty. By a stationary solution to Problem (I) we mean a nonnegative function v such that $v^m \in C^2(D) \cap C^1(\bar{D})$, $v = 0$ on ∂D and

$$\Delta(v^m) + v^p - g(v) = 0 \qquad \text{on} \quad D.$$

Remark 2. It follows from the comparison principle stated in Proposition 1 that we can obtain nonexistence results for Problem (I) with a more general growth term $f(u)$ assuming $f(u) \geqslant u^p - g(u)$ for p, g as above, and for any initial data $v_0 \in L^\infty(D)$ such that $v_0 \geqslant u_0$, where u_0 satisfies (H2), $u_0^m \in \boldsymbol{B}$ (with $\boldsymbol{B}$ defined by the growth term $u^p - g(u)$). For the solvability of Problem (I) for $u_0 \in L^\infty(D)$ only, see, e.g. [1], [11], [14], [5].

In order to describe our result concerning the case $m = p > 1$, we need the following

Lemma 1. *Suppose that the domain D is sufficiently "large", i.e. that the first eigenvalue* λ_1 *of the Dirichlet problem* $\Delta\varphi + \lambda_1 \varphi = 0$ *in* D, $\varphi = 0$ *on* ∂D *is less than* 1, *and that* $m = p > 1$. *Let*

$$g(u) = au^q \quad \text{for } u \geqslant 0, \; g(u) \equiv 0 \quad \text{for } u < 0, \tag{3.8}$$

where $1 < q < m$, $0 < a$ *and*

$$\boldsymbol{d} = \inf_{w \in \overset{+}{H}{}_0^1} \left(\sup_{0 \leqslant \lambda < \infty} J(\lambda w) \right). \tag{3.9}$$

Then we have $0 < \boldsymbol{d} < \infty$.

Now we can introduce the "unstable" set $\boldsymbol{B}$:

(3.10) $$\boldsymbol{B} = \{w \in \overset{+}{H}{}_0^1 \colon J(w) < \boldsymbol{d} \text{ and } K(w) < 0\},$$

and formulate

Theorem 2. *Assume that D and u_0 satisfy* (H), $m = p > 1$ *and $g(u)$ is given by* (3.8). *Suppose further that the domain D satisfies the hypothesis of Lemma* 1. *Then $u_0^m \in \boldsymbol{B}$ implies that*

(3.11) $$T_{max} \leqslant \left(\frac{1}{|D|}\int_D u_0^{m+1}\right)^{(1-q)/(m+1)} (m+q)/\alpha(q-1)(m-q)(1-\nu) < \infty,$$

where the constant $\nu \in [0, 1)$ is such that $J(u_0^m) \leqslant \nu\boldsymbol{d}$, i.e. the solution $u(t, u_0)$ of Problem (I) *blows up in finite time for $u_0^m \in \boldsymbol{B}$. Moreover,*

(3.12) $$\boldsymbol{d} = \min_{v \in E} J(v^m),$$

where E is the set of all nontrivial nonnegative stationary solutions to Problem (I), *hence E is nonempty.*

We show that the following known result (see [9]) is a simple consequence of Theorem 2.

Corollary 1. *Let us consider Problem* (I) *with $g \equiv 0$ and let the hypotheses of Theorem 2 hold. Then $u(t, u_0)$ blows up for every $u_0 \not\equiv 0$, $u_0^m \in \overset{+}{H}{}_0^1$.*

Remark 3. It is not difficult to verify that if $\xi \in \overset{+}{H}{}_0^1$, $J(\xi) \leqslant 0$, then $\xi \in \boldsymbol{B}$, $\boldsymbol{B}$ given by (3.10).

Now we can proceed to the proofs of the above assertions.

4. Proof of Theorem 1

In order to demonstrate that the unstable set $\boldsymbol{B}$ is nonempty put

(4.1) $$j(\lambda) := J(\lambda w) \qquad \text{for } w \in \overset{+}{H}{}_0^1,\ 0 \leqslant \lambda < \infty.$$

The assumption (3.1) implies that there exist nonnegative constants r_0, c such that $G(u) \leqslant cu^{\vartheta}$ for all $u \geqslant r_0$, and as $\vartheta < p/m$, $j(\lambda) \to -\infty$ for $\lambda \to \infty$. Hence $\boldsymbol{B}$ is nonempty.

But as the main aim of this paper is to show the blowing-up from the initial data with positive energy, we should demonstrate that there exists $w_0 \in \overset{+}{H}{}_0^1$ such that $0 < J(w_0) < \boldsymbol{d}$ and $K(w_0) < 0$. To see this, one can easily verify that $j \in C^1([0, \infty))$, $j(0) = 0$ and $j(\lambda) > 0$ in a neighbourhood of the origin, which together with the convergence of j into $-\infty$ for $\lambda \to \infty$ gives the existence of such λ_0 that $0 < j(\lambda_0) < \boldsymbol{d}$ and $j'(\lambda_0) < 0$, hence $w_0 = \lambda_0 w \in \boldsymbol{B}$ as $K(w_0) = = \lambda_0 j'(\lambda_0) < 0$.

Now for a while let us suppose that $\boldsymbol{B}$ is invariant, i.e. that $u^m(t, u_0) \in \boldsymbol{B}$ for $0 \leqslant t < T_{max}$ if $u_0^m \in \boldsymbol{B}$, which is obvious by (2.3) whenever $J(u_0^m) \leqslant 0$. Then according to (2.3) and (3.4) we have

$$J(u^m(t, u_0)) \leqslant v\boldsymbol{d}, \qquad 0 \leqslant t < T_{max}, \tag{4.2}$$

for some constant $v \in (0, 1)$ if p satisfies (3.3) and $0 < J(u_0^m) < \boldsymbol{d}$, or $v = 0$ if $J(u_0^m) \leqslant 0$. Because we have supposed that $u^m(t, u_0) \in \boldsymbol{B}$, (4.2) and (3.2) yield

$$J(u^m(t)) \leqslant vk|u(t)|_{m+p}^{m+p}, \tag{4.3}$$

where for simplicity of notation we write $u(t)$ instead of $u(t, u_0)$. On the other hand, we can estimate $J(u^m(t))$ by (3.5) to obtain from (4.3)

$$\|u^m(t)\|^2 + (g(u(t)), u^m(t)) \leqslant \kappa|u(t)|_{m+p}^{m+p}, \tag{4.4}$$

where

$$\kappa = (vk + m(m+p)^{-1})/i(\vartheta). \tag{4.5}$$

It is not difficult to see that $0 < \kappa < 1$.

Now, using the estimate (4.4), the proof of Theorem 1 can proceed in a standard way. Inserting $u^m(t)$ into (2.1) we obtain

$$|u(t)|_{m+1}^{m+1} - |u_0|_{m+1}^{m+1} = (m+1)\int_0^t (-\|u^m\|^2 - (g(u), u^m) + |u|_{m+p}^{m+p}) \tag{4.6}$$

for a.e. $t \in [0, T_{max})$. We note that it is possible also in the case of $0 < m < 1$, in which $(u^m)_t$ does not always exist. However, in this case by (2.3) and the boundedness of u, u_t exists and (2.1) yields (4.6). Since $y(t) := |u(t)|_{m+1}^{m+1}$ is absolutely continuous on $[0, T]$ for any $T < T_{max}$, we obtain from (4.6), by (4.4) and the Hölder inequality, the differential inequality

$$y'(t) - (m+1)(1-\kappa)|D|^{(1-p)/(1+m)} y^{(m+p)/(m+1)}(t) \geqslant 0 \tag{4.7}$$

for a.e. $t \in [0, T_{max})$. As $(m+p)(m+1)^{-1} > 1$, by the standard comparison theorem for ordinary differential equations we have (3.6), and by (2.2), the assertion of Theorem 1.

So, it remains only to prove that $\boldsymbol{B}$ is invariant in the case of the positive energy of the initial data. It would not be difficult if we knew that the solution $u(t, u_0)$ is sufficiently smooth, say,

$$u^m(t, u_0) \textit{ is a continuous mapping from } [0, T_{max}) \textit{ to } H_0^1, \tag{4.8}$$

(see, e.g. [16], [12]). In fact, let $u^m(t)$ leave the set $\boldsymbol{B}$ at the time t_0. Since $u_0^m \in \boldsymbol{B}$ and $\boldsymbol{B}$ is open with respect to the norm in H_0^1, t_0 is positive. Then in virtue of

(4.8) we obtain that $K(u^m(t_0)) = 0$, as the case $J(u^m(t_0)) = \boldsymbol{d}$ is impossible by (2.3). However, this and (3.5) yield

$$\boldsymbol{d} > J(u^m(t_0)) \geqslant k|u(t_0)|_{m+p}^{m+p} \geqslant \boldsymbol{d},$$

which is a contradiction.

However, since we know of no regularity result like (4.8) if $m \neq 1$, we shall now regularize Problem (I) and present several observations in order to prove the invariance of the set $\boldsymbol{B}$. First, for simplicity of notation, let us denote

$$a(u) = |u|^m \operatorname{sign} u, \quad b(u) = |u|^{1/m} \operatorname{sign} u. \tag{4.9}$$

Now, we shall consider the modified problems

$$(\mathrm{I}_\varepsilon) \qquad \begin{aligned} &u_t = \Delta a_\varepsilon(u) + (a_\varepsilon(u))^{p/m} - F_\varepsilon(a_\varepsilon(u)) \quad \text{in } Q_T, \\ &u(x, 0) = u_{0\varepsilon}(x) \quad \text{in } D,\ u(x, t) = 0 \quad \text{on } S_T, \end{aligned}$$

where $0 < T < T_{max}$, T_{max} has been given for Problem (I) by Proposition 2, $0 < \varepsilon < 1$ and $u_{0\varepsilon}$, a_ε and F_ε are defined as follows.

A. The case $m \geqslant 1$

Let us denote by $\{R_\varepsilon\}$ a set of symmetric mollifiers in one variable with $\operatorname{supp} R_\varepsilon \subset \overline{B(0, \varepsilon)}$ and put

$$a_\varepsilon(u) = (R_\varepsilon * a)(u), \quad b_\varepsilon = a_\varepsilon^{-1}. \tag{4.10}$$

The following properties of a_ε and b_ε are easily verified: $a_\varepsilon, b_\varepsilon \in C^\infty(\mathbf{R})$, $a_\varepsilon(0) = b_\varepsilon(0) = 0$, $0 < \delta(\varepsilon) \leqslant a'_\varepsilon(u) \leqslant K(M) < \infty$ for $|u| \leqslant M$, $M > 0$ and $a_\varepsilon \to a$ in $C^1(\mathbf{R})$, $b_\varepsilon \to b$ in $C^0(\mathbf{R})$, as $\varepsilon \to 0$. From this, according to (H2) it is possible to choose $u_{0\varepsilon} \in C_0^\infty(D)$ such that

$$\begin{aligned} &a_\varepsilon(u_{0\varepsilon}) \to a(u_0) \textit{ strongly in } H_0^1, \textit{ as well as,} \\ &0 \leqslant u_{0\varepsilon} \leqslant |u_0|_\infty + 1 \textit{ and } u_{0\varepsilon} \to u_0 \textit{ strongly in } L^2(D), \\ &\textit{as } \varepsilon \to 0, \end{aligned} \tag{4.11}$$

(see, e.g. [11]). Now, if $G \in C^1([0, \infty))$, we need not to regularize it and put $F_\varepsilon \equiv G$, but if G' has a singularity at 0, let us put, e.g.,

$$G_\eta(u) = \begin{cases} (2G(\eta)\,\eta^{-1} - G'(\eta))\,u + (G'(\eta)\,\eta^{-1} - G(\eta)\,\eta^{-2})\,u^2 \\ \textit{for } 0 \leqslant u \leqslant \eta, \\ G(u) \qquad \textit{for } 0 < \eta \leqslant u < \infty, \end{cases} \tag{4.12}$$

and one can easily verify that $G_\eta \in C^1([0, \infty))$, $G_\eta(0) = 0$. We note, for later reference, that using (3.1), and (4.12), we obtain the analogy of (3.5):

$$\int_0^u G_\eta(r)\,dr \geqslant i(\vartheta)\, G_\eta(u)\, u \qquad \textit{for all } u \geqslant 0. \tag{4.13}$$

Now we introduce the dependence of η on ε and put $F_\varepsilon := G_{\eta(\varepsilon)}$. For this purpose, let us define $\boldsymbol{d}_\eta$ like $\boldsymbol{d}$ by (3.2) with G_η instead of G and the set $\boldsymbol{B}_\eta$, and the functionals J_η, K_η in the same way. Then, using (4.12), it can be shown that $\boldsymbol{d}_\eta \to \boldsymbol{d}$ as $\eta \to 0$. Now, by our assumption $a(u_0) \in \boldsymbol{B}$, hence we can choose η_0 such small that $J(a(u_0)) < \boldsymbol{d}' \leqslant \boldsymbol{d}_\eta$ for all $\eta \leqslant \eta_0$ and some $\boldsymbol{d}' < \boldsymbol{d}$. On the other hand, we can choose $\varepsilon_0 > 0$ such small that $J(a_\varepsilon(u_{0\varepsilon})) < \boldsymbol{d}'$ for all $\varepsilon \leqslant \varepsilon_0$ because of (4.11). Now let $0 < \varepsilon \leqslant \varepsilon_0$ be fixed. Then there exists η $(=\eta(\varepsilon))$, $0 < \eta \leqslant \eta_0$ such that

$$J_\eta(a_\varepsilon(u_{0\varepsilon})) < \boldsymbol{d}_\eta, \quad \text{as well as} \quad K_\eta(a_\varepsilon(u_{0\varepsilon})) < 0, \tag{4.14}$$

hence $a_\varepsilon(u_{0\varepsilon}) \in \boldsymbol{B}_\eta$. This results from the construction of G_η (c.f. (4.12)).

So we can return to Problem (I). Put $M = \|u(t, u_0)\|_{L^\infty(Q_T)} + 2$, $f_\varepsilon(u) = (a_\varepsilon(u))^{p/m} - F_\varepsilon(a_\varepsilon(u))$ for $0 \leqslant u \leqslant M$, otherwise smooth and such that $|f_\varepsilon|$, $|f'_\varepsilon| \leqslant K < \infty$ on $\mathbf{R}$, $A_\varepsilon(u) = a'_\varepsilon(u)$ for $|u| \leqslant M$, otherwise smooth and such that $|A_\varepsilon|$, $|A'_\varepsilon| \leqslant K < \infty$ on $\mathbf{R}$, for some positive constant K. With these choices of data we obtain a unique classical solution of the problem

$$(\mathrm{I}'_\varepsilon) \qquad \begin{aligned} &u_t = \operatorname{div}(A_\varepsilon(u)\nabla u) + f_\varepsilon(u) \quad \textit{in } Q_T,\\ &u(x, 0) = u_{0\varepsilon} \quad \textit{in } D,\ u(x, t) = 0 \quad \textit{on } S_T, \end{aligned}$$

which we denote by u_ε, i.e. $u_\varepsilon \in C^{2,1}(\bar{Q}_T)$ (see, e.g. [10, Chapter 5, Theorem 6.1]). For later reference, let us denote $U_\varepsilon = a_\varepsilon(u_\varepsilon)$. The proof of the following lemma will be postponed to the end of this section.

Lemma 2. *There exist a T', $0 < T' \leqslant T$ and $\{\varepsilon\}$, $\varepsilon \to 0$ such that*

$$U_\varepsilon(t) \to a(u(t)) \quad \textit{in } C([0, T'];\ L^{1+p/m}(D)), \tag{4.15}$$

as $\varepsilon \to 0$, where $u(t) = u(t, u_0)$ is the solution of Problem (I), *and $U_\varepsilon \in \boldsymbol{B}_{\eta(\varepsilon)}$ for $0 \leqslant t \leqslant T'$.*

So we are now ready to prove the invariance of the unstable set $\boldsymbol{B}$. Choose constants $\nu \in (0, 1)$, $\delta > 0$ such that $J(a(u_0)) \leqslant \nu \boldsymbol{d}' - \delta$. Then, according to (2.3), the definitions of $\boldsymbol{d}_{\eta(\varepsilon)}$, $\boldsymbol{B}_{\eta(\varepsilon)}$, we have

$$J(a(u(t))) \leqslant \nu k |U_\varepsilon(t)|_{1+p/m}^{1+p/m} - \delta \tag{4.16}$$

for any $0 \leqslant t \leqslant T'$. Passing to the limit as $\varepsilon \to 0$ in (4.16), by (4.15) yields

$$J(a(u(t))) \leqslant \nu k |u(t)|_{m+p}^{m+p} - \delta.$$

This, in the same way as in (4.3)—(4.5), implies

$$\|a(u(t))\|^2 + (g(u(t)), a(u(t))) \leqslant \kappa |u(t)|_{m+p}^{m+p} - \delta', \qquad \delta' > 0,$$

hence $a(u(t, u_0)) \in \boldsymbol{B}$ for $0 \leqslant t \leqslant T'$.

However, as it will be seen in the proof of Lemma 2, T' does not depend explicitly on u_0, only on M, and we know that

$$a(u(T', u_0)) \in \boldsymbol{B} \cap L^\infty(D), \quad |u(T', u_0)|_\infty \leqslant \|u\|_{L^\infty(Q_T)}.$$

So we can repeat the above procedure with $u(T', u_0)$ instead of u_0, and after a finite number of steps we obtain that $a(u(t, u_0)) \in \boldsymbol{B}$ on $[0, T]$. However, because T was arbitrary, $0 < T < T_{max}$, we have the desired result in the case of $m \geqslant 1$.

B. The case $0 < m < 1$

Here we put

$$b_\varepsilon(u) = (R_\varepsilon * b)(u) \quad \text{and} \quad a_\varepsilon = b_\varepsilon^{-1} \tag{4.17}$$

and one can see that $b_\varepsilon \to b$ in $C^1(\mathbf{R})$, $a_\varepsilon \to a$ in $C^0(\mathbf{R})$, as $\varepsilon \to 0$. Now, in a similar way as above, we obtain for any ε, $0 < \varepsilon < 1$, the unique solution $u_\varepsilon \in C^{2,1}(\bar{Q}_T)$ of $(\mathrm{I}'_\varepsilon)$ on $[0, T]$, $T < T_{max}$, where $u_{0\varepsilon} \in C_0^\infty(D)$ satisfies (4.11) with our choice of a_ε. We note that the function G need not be regularized in this case as $G \in C^1([0, \infty))$ for any g satisfying (A). The proof of the fact that the analogy of Lemma 2 holds also in this case is postponed to the end of this section.

One can now establish the invariance of the unstable set $\boldsymbol{B}$ just as above. This completes the proof.

P r o o f o f L e m m a 2. Recalling that u_ε is the solution of $(\mathrm{I}'_\varepsilon)$ on $[0, T]$ we claim that there exists $T' \in (0, T]$ such that

$$0 \leqslant u_\varepsilon \leqslant M \quad \text{on } Q_{T'}, \tag{4.18}$$

for all ε, $0 < \varepsilon < 1$. To see this let $\overset{+}{y}$ be the solution of $\overset{+}{y}{}' = (\overset{+}{y} + 1)^p$, $\overset{+}{y}(0) = \|u(t, u_0)\|_{L^\infty(Q_T)} + 1$, which may be solved explicitly and we can see that $\overset{+}{y} \leqslant M$ on $[0, T']$ for some $T' > 0$. So, by the standard comparison theorems (see, e.g. [6, Chapter 2, Theorem 16]), $0 \leqslant u_\varepsilon \leqslant \overset{+}{y}$ on $Q_{T'}$, hence (4.18). Thus the solution u_ε of $(\mathrm{I}'_\varepsilon)$ also satisfies (I_ε) on $Q_{T'}$.

Now, multiplying the equation of (I_ε) by $(U_\varepsilon)_t$ $(U_\varepsilon = a_\varepsilon(u_\varepsilon))$ and performing obvious manipulations we get

$$\int_0^t \left| \left(\int_0^{u_\varepsilon} (a'_\varepsilon(r))^{1/2}\, dr \right)_t \right|_2^2 + J_\eta(U_\varepsilon(t)) = J_\eta(U_\varepsilon(0)) \tag{4.19}$$

for $0 \leqslant t \leqslant T'$. In particular, it follows from (4.18), the construction of a_ε and (4.19) that

(4.20) $$0 \leqslant U_\varepsilon \leqslant M' \quad on\ Q_{T'}, \sup_{0 \leqslant t \leqslant T'} \|U_\varepsilon(t)\|^2, \int_0^{T'} |(U_\varepsilon)_t|_2^2 \leqslant C$$

for all ε, $0 < \varepsilon < 1$, where the positive constants M', C do not depend on ε. To see the existence of the time derivative of U_ε we use the fact that

$$(U_\varepsilon)_t = \left(\int_0^{u_\varepsilon} (a'_\varepsilon(r))^{1/2}\, dr\right)_t (a'_\varepsilon(u_\varepsilon))^{1/2}.$$

Now, in a standard way (see, e.g. [1, Theorem 13]) we obtain a function $U \in C([0, T]; L^2(D))$ such that

(4.21) $$U_\varepsilon \to U \quad \text{in } C([0, T']; L^2(D)) \quad \text{as } \varepsilon \to 0$$

(through a subsequence), but by the uniform boundedness of U_ε, $U_\varepsilon \to U$ also in $C([0, T']; L^{1+p/m}(D))$. Now, using the estimates (4.18), (4.20), the properties of a_ε, a and the uniqueness of Problem (I) (Proposition 2), it is not difficult to demonstrate that $U = a(u)$, where u is the solution of Problem (I). Further, as $U_\varepsilon(t) \in C^{2,1}(\bar{Q}_T)$ and $U_\varepsilon(0) \in \boldsymbol{B}_\eta$, we obtain by similar arguments as above (see (4.8) and what follows) that $U_\varepsilon(t) \in \boldsymbol{B}_\eta$ for $0 \leqslant t \leqslant T'$, $\eta = \eta(\varepsilon)$. This completes the proof of Lemma 2.

Proof of the analogy of Lemma 2 for $0 < m < 1$: In the same way as in the case of $m \geqslant 1$ we can show that there exists $T' \in (0, T]$ such that $0 \leqslant u_\varepsilon \leqslant M$ on $Q_{T'}$. To obtain appropriate apriori estimates, we rewrite Problem (I_ε) putting $U_\varepsilon = a_\varepsilon(u_\varepsilon)$ into

(4.22) $$\begin{aligned} &(b_\varepsilon(U_\varepsilon))_t = \Delta U_\varepsilon + U_\varepsilon^{p/m} - G(U_\varepsilon) \quad \text{in } Q_{T'}, \\ &U_\varepsilon(x, 0) = a_\varepsilon(u_{0\varepsilon}) \quad \text{in } D,\ U_\varepsilon(x, t) = 0 \quad \text{on } S_{T'}. \end{aligned}$$

Now, in the same way as above we obtain from (4.22)

(4.23) $$\int_0^{T'} \left|\left(\int_0^{U_\varepsilon} (b'_\varepsilon(r))^{1/2}\, dr\right)_t\right|_2^2, \quad \sup_{0 \leqslant t \leqslant T'} \|U_\varepsilon(t)\|^2 \leqslant C,$$

where the positive constant C does not depend on ε. As

$$(u_\varepsilon)_t = \left(\int_0^{U_\varepsilon} (b'_\varepsilon(r))^{1/2}\, dr\right)_t (b'_\varepsilon(U_\varepsilon))^{1/2},$$

it follows from (4.23), the properties of b_ε and the uniform boundedness of U_ε that

(4.24) $$\sup_{0 \leqslant t \leqslant T'} \|u_\varepsilon(t)\|^2, \quad \int_0^{T'} |(u_\varepsilon)_t|_2^2 \leqslant C', \quad \text{and } 0 \leqslant u_\varepsilon \leqslant M,$$

hence there exists a $v \in C([0, T']; L^2(D))$ such that $u_\varepsilon \to v$ in $C([0, T']; L^2(D))$ as $\varepsilon \to 0$ (through a subsequence). Again, it is not difficult to show that v is a solution of Problem (I), so $v = u$. To demonstrate (4.15), it is sufficient to show that $a(u_\varepsilon) \to a(u)$ in $C([0, T']; L^{1+p/m}(D))$, as $a_\varepsilon \to a$ uniformly on compact subsets of $\mathbf{R}$. But $|a(u_\varepsilon) - a(u)| \leqslant a(|u_\varepsilon - u|)$ and (4.15) follows easily. The invariance of the set $\boldsymbol{B}_\eta = \boldsymbol{B}$ may be proved as above.

5. Proofs of Lemma 1 and Theorem 2

We start with the proof of Lemma 1. Computing $\sup_{0 \leqslant \lambda < \infty} J(\lambda w)$ for our choice of data we obtain

$$(5.1) \qquad \boldsymbol{d} = \frac{m-q}{2(m+q)} \inf_{w \in Q} \left(\frac{\alpha^{m/(m+q)} |w|_{1+q/m}}{(|w|_2^2 - \|w\|^2)^{1/2}} \right)^{\frac{2(m+q)}{m-q}} = : \inf_{w \in Q} \Phi(w),$$

where $Q = \{w \in \overset{+}{H}{}_0^1 : |w|_2^2 > \|w\|^2\}$. The set Q is nonempty due to the assumption $\lambda_1 < 1$. Now, because $\Phi(\lambda w) = \Phi(w)$ for any $0 < \lambda < \infty$ and $w \in Q$, it follows from (5.1) that

$$(5.2) \qquad \boldsymbol{d} = \frac{m-q}{2(m+q)} \alpha^{2m/(m-q)} \inf_{\xi \in Q_1} \left(\frac{|\xi|_{1+q/m}}{(|\xi|_2^2 - 1)^{1/2}} \right)^{\frac{2(m+q)}{m-q}},$$

where $Q_1 = \{\xi \in \overset{+}{H}{}_0^1 : |\xi|_2 > \|\xi\| = 1\}$. To see that $\boldsymbol{d}$ is positive we use the Nirenberg–Gagliardo inequality (see, e.g. [7, Theorem 9.3])

$$(5.3) \qquad |\xi|_2 \leqslant c \|\xi\|^{\Theta} |\xi|_{1+q/m}^{1-\Theta},$$

where c is positive and $\Theta = N(1 - q/m)/(2 + 2qm^{-1} + N(1 - qm^{-1}))$.
For $\xi \in Q_1$, (5.3) yields

$$|\xi|_{1+q/m} \geqslant c' |\xi|_2^{1 + \frac{N(m-q)}{2(m+q)}} > c' |\xi|_2,$$

hence $\boldsymbol{d} > 0$, and the proof of Lemma 1 is finished.
Now we claim that the set $\boldsymbol{B}$ is nonempty and invariant. To see this we proceed similarly as in the proof of Theorem 1 and we omit the details. Next, putting $u^m(t)$ into (2.1) we obtain

$$(5.4) \qquad \frac{1}{m+1} \frac{d}{dt} (|u(t)|_{m+1}^{m+1}) + \|u^m(t)\|^2 - |u^m(t)|_2^2 = -\alpha |u(t)|_{m+q}^{m+q}$$

for a.e. $t \in [0, T_{max})$. On the other hand, according to (2.3), (5.1) and the fact that $\boldsymbol{B}$ is invariant, we have

$$J(u^m(t)) \leqslant \nu \frac{\alpha(m-q)}{2(m+q)} |u(t)|_{m+q}^{m+q}, \quad \text{hence}$$

$$\|u^m(t)\|^2 - |u^m(t)|_2^2 \leqslant -\frac{\alpha(2m - \nu m + \nu q)}{m+q} |u(t)|_{m+q}^{m+q}, \tag{5.5}$$

where $\nu = 0$ if $J(u_0^m) \leqslant 0$ or $\nu \in (0, 1)$ if $0 < J(u_0^m) < \boldsymbol{d}$ and such that $J(u_0^m) \leqslant \nu \boldsymbol{d}$. If we denote $y(t) = |u(t)|_{m+1}^{m+1}$, (5.4), (5.5) and the Hölder inequality yield

$$y'(t) - \alpha(1-\nu)(m-q)(m+1)(m+q)^{-1}|D|^{(1-q)/(1+m)} y^{(m+q)/(m+1)}(t) \geqslant 0$$

for a.e. $t \in [0, T_{max})$. As $(m+q)(m+1)^{-1} > 1$, (3.11) follows easily.

Now, by the same arguments as we get (5.2) from (5.1), we can obtain

$$\boldsymbol{d} = \inf_{w \in Q_2} \frac{(m-q)\,\alpha^{m/(m+q)}}{2(m+q)} \left(\frac{|w|_{1+q/m}}{(1 - \|w\|^2)^{1/2}} \right)^{\frac{2(m+q)}{m-q}} = : \inf_{w \in Q_2} \chi(w), \tag{5.6}$$

where $Q_2 = \{w \in \overset{+}{H}{}_0^1 \colon\ 1 = |w|_2 > \|w\|\}$. Hence it follows that there exists $\{w_n\} \subset Q_2$ such that $\chi(w_n) \to \boldsymbol{d}$ as $n \to \infty$. As $\|w_n\| < 1$ for all n, there exists $w_0 \in \overset{+}{H}{}_0^1$ such that $w_n \to w_0$ weakly in H_0^1, as well as $w_n \to w_0$ strongly in $L^2(D)$ and $L^{1+q/m}(D)$, as $n \to \infty$ (through a subsequence) and $|w_0|_2 = 1$, $\|w_0\| \leqslant 1$. We claim that $w_0 \in Q_2$. To see this let us note that

$$\boldsymbol{d} = \lim_{n \to \infty} \chi(w_n) = : \frac{(m-q)\,\alpha^{m/(m+q)}}{2(m+q)} \lim_{n \to \infty} \frac{A_n}{B_n}$$

and that $\lim_{n \to \infty} A_n$ exists, is positive and finite. But then $\lim_{n \to \infty} B_n$ also exists and is not equal to zero, hence $\|w_0\|^2 < 1$.
This implies that

$$\boldsymbol{d} = \Phi(w_0). \tag{5.7}$$

Now let us compute $D\Phi(w_0, \varphi) = \zeta^{2m/(q-m)}(-(\nabla w_0, \nabla \varphi) + (w_0 - \zeta a w_0^{q/m}, \varphi))$, where $\zeta = (|w_0|_2^2 - \|w_0\|^2)/\alpha |w_0|_{1+q/m}^{1+q/m}$ and $\varphi \in H_0^1$, so it follows from (5.7) and (5.1) that $D\Phi(w_0, \varphi) = 0$ for all $\varphi \in H_0^1$, hence

$$\Delta w_0 + w_0 - \zeta a w_0^{q/m} = 0 \quad \text{in a weak sense.}$$

Since $w_0 \in H_0^1$, the equation holds classically. Putting $v = \zeta^{1/(q-m)} w_0^{1/m}$, the equation above may be rewritten into

$$\Delta(v^m) + v^m - a v^q = 0 \quad \text{in } D,\ v = 0 \quad \text{on } \partial D,$$

hence $v \in E$. This completes the proof of Theorem 2.

Proof of Corollary 1. First, $\lambda_1 < 1$ implies that $J(u_0^m) < 0$. Then there exists ε such that $J(u_0^m) + \varepsilon m(m+q)^{-1} |u_0|_{m+q}^{m+q} \leqslant 0$ ($\varepsilon > 0$). If we denote $u^\varepsilon(t, u_0)$, the solution of (I) with the absorptive term εu^q for $u \geqslant 0$, $u^\varepsilon(t, u_0)$ is a sub-solution of Problem (I) with $g \equiv 0$ and, by Theorem 2, $u^\varepsilon(t, u_0)$ blows up in a finite time and so does $u(t, u_0)$.

6. A final example

In this section we consider the case $N = 1$ ($D = (-L, L)$, $L > 0$), $m = p > 1$ and g as in Theorem 2. We first describe the set $E = E(L)$ and after this the number $\boldsymbol{d}$ is determined.

Lemma 3.

(i) *If* $0 < L \leqslant \pi/2$, *then* $E(L) = \{0\}$.

(ii) *If* $\pi/2 < L \leqslant L_1$, $L_1 = \pi m/(m-q)$, *then* $E(L) = \{0, v(\cdot, L)\}$, *where* $v(\cdot, L)$ *denotes the unique nontrivial stationary solution to Problem* (I), *positive in* $(-L, L)$.

(iii) *If* $L_1 < L$, *then* $E(L)$ *consists of the trivial solution and of continua of solutions generated by* $v(\cdot, L_1)$.

Theorem 2 states that blowing up may occur if $L > \pi/2$. In this case, using Lemma 3, we obtain

Theorem 3. *If* $\pi/2 < L \leqslant L_1$, *then* $\boldsymbol{d} = J(v(\cdot, L))$. *If* $iL_1 \leqslant L < (i+1)L_1$ *for some positive integer* i, *then for any* $w \in E(L)$ *it holds that* $J(w) = jJ(v(\cdot, L_1))$ *for some* $j \in \{1, 2, \ldots, i\}$, *hence*

$$\boldsymbol{d} = J(v(\cdot, L_1)).$$

P r o o f o f L e m m a 3. Denote $F(u) = (2m)^{-1} u^{2m} - (m+q)^{-1} a u^{m+q}$, $\kappa = (2am/(m+q))^{1/(m-q)}$ (κ is the unique root of F in $(0, \infty)$) and for $v \in [\kappa, \infty)$ define

$$T(v) = \sqrt{\frac{m}{2}} \int_0^v \frac{s^{m-1}}{\sqrt{F(v) - F(s)}}\, ds. \tag{6.1}$$

In the same way as in [1] (see also [4]), it may be demonstrated that the following proposition holds.

Proposition 3 ([1]). *A function* v, $v > 0$ *in* $(-L, L)$, *belongs to* $E(L)$ *if and only if*

$$\sqrt{\frac{m}{2}} \int_{v(x)}^{v} \frac{s^{m-1}}{\sqrt{F(v) - F(s)}}\, ds = |x|,$$

where $v \in [\kappa, \infty)$ *and* $L \in (0, \infty)$ *are related by the equation* $T(v) = L$.

Now Lemma 3 follows from the next proposition.

Proposition 4.

(i) $T \in C([\kappa, \infty)) \cap C^1((\kappa, \infty))$, $T(\kappa) = \pi m/(m-q)$,

(ii) $T'(v) < 0$ *for* $v \in (\kappa, \infty)$,

(iii) $T(v) \to \pi/2$ *as* $v \to \infty$.

Proposition 4 may be proved by direct computations and we indicate only the proofs of (ii) and (iii).

(ii)

$$T'(v) = \sqrt{\frac{m}{2}} \int_0^v \frac{\Theta(v) - \Theta(s)}{\sqrt{F(v) - F(s)}}\, ds, \quad \text{where} \quad \Theta(s) = \alpha(q - m)(q + m)^{-1} s^{m+q},$$

i.e. Θ is decreasing on $[0, v]$.

(iii) Putting $s = vy$ in (6.1) we obtain

$$T(v) = \sqrt{\frac{m}{2}} \frac{v^m}{\sqrt{F(v)}} \int_0^1 \frac{y^{m-1}}{\sqrt{1 - F(vy)(F(v))^{-1}}}\, dy$$

and one can see that the integrand has the integrable majorant $y^{m-1}(1 - y^{2m})^{-1/2}$ and converges pointwise to it as $v \to \infty$, hence the conclusion.

To see that $v(\cdot, L_1)$ generates families of nonnegative stationary solutions to Problem (I) on $(-L, L)$ with $L > L_1$ let us note that

$$v(\pm L_1, L_1) = (v^m)_x(\pm L_1, L_1) = 0$$

as $F(\kappa) = 0$. So we can, e.g., extend v as zero on intervals larger than $(-L_1, L_1)$ (for further details see [1]).

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Katedra matematickej analýzy
Matematicko-fyzikálnej fakulty UK
Mlynská dolina
842 15 Bratislava

Ústav aplikovanej matematiky
a výpočtovej techniky UK
Mlynská dolina
842 15 Bratislava

ОДИН РЕЗУЛЬТАТ О НЕСУЩЕСТВОВАНИИ ГЛОБАЛЬНЫХ РЕШЕНИЙ ДЛЯ УРАВНЕНИЙ НЕЛИНЕЙНОЙ ДИФФУЗИИ

M. Fila—J. Filo

Резюме

В статье с помощью функционала Ляпунова охарактеризовано одно множество начальных условий, для которых L^∞-норма решения задачи Дирихле стремится к бесконечности в конечном времени.