Irena Rachůnková

The first kind periodic solutions of differential equations of the second order

Mathematica Slovaca, Vol. 39 (1989), No. 4, 407--415

Persistent URL: http://dml.cz/dmlcz/129135

Math. Slovaca 39, 1989, No. 4, 407—415

THE FIRST KIND PERIODIC SOLUTIONS OF DIFFERENTIAL EQUATIONS OF THE SECOND ORDER

IRENA RACHŮNKOVÁ

The purpose of this paper is to prove some existence and uniqueness theorems for the problem

(0.1) $$u'' = f(t, u, u')$$

(0.2) $$u(b) - u(a) = A, \qquad u'(b) - u'(a) = B,$$

where $a, b, A, B \in (-\infty, +\infty)$, $a < b$. The problems of such type have been already solved in many works, for example [1—11], [13]. Here, the problem (0.1), (0.2) is solved by means of lower and upper functions and there is used the method of [12]. This approach enables us to find the conditions for the existence of the first kind periodic solutions of (0.1).

1. Notations and definitions

$\mathbb{R} = (-\infty, +\infty)$, $\mathbb{R}_+ = [0, +\infty)$, $T = b - a$, $c_1 = \max\{1, |A/T|\}$; a.e. = almost every, $p_i, q_i \in [1, +\infty]$, $1/p_i + 1/q_i = 1$, $i = 1, \ldots, n$; $AC^1(a, b)$ is the set of all absolutely continuous functions with their first derivatives on [a, b];
$\mathrm{Car}_{\mathrm{loc}}(D)$ is the set of all real functions satisfying the local Carathéodory conditions on D.

Definition. *A function $u \in AC^1(a, b)$ which fulfils* (0.1) *for a.e. $t \in [a, b]$ will be called a solution of the equation* (0.1) *on* $[a, b]$. *Each solution of* (0.1) *on* $[a, b]$ *satisfying* (0.2) *will be called a solution of the problem* (0.1), (0.2). *Each solution of* (0.1) *on $\mathbb{R}$ will be called the first kind T-periodic solution (resp. T-periodic solution) of* (0.1) *if u' (resp. u) is a T-periodic function.*

Definition. *A function $\sigma_1 \in AC^1(a, b)$ will be called a lower function of the problem* (0.1), (0.2) *if*

(1.1) $$\sigma_1''(t) \geqq f(t, \sigma_1, \sigma_1') \text{ for a.e. } t \in (a, b),$$

(1.2) $$\sigma_1(b) - \sigma_1(a) = A, \ \sigma_1'(b) - \sigma_1'(a) \leqq B.$$

A function $\sigma_2 \in AC^1(a, b)$ will be called an upper function of the problem (0.1), (0.2) *if*

$$\sigma_2''(t) \leqq f(t, \sigma_2, \sigma_2') \qquad \text{for a.e. } t \in (a, b), \tag{1.3}$$

$$\sigma_2(b) - \sigma_2(a) = A, \quad \sigma_2'(b) - \sigma_2'(a) \geqq B. \tag{1.4}$$

Throughout the whole paper we suppose that $f: \mathbb{R}^3 \to \mathbb{R}$ is a T-periodic function in its first argument and the restriction of f on $[a, b] \times \mathbb{R}^2$ belongs to $\mathrm{Car}_{\mathrm{loc}}([a, b] \times \mathbb{R}^2)$. We denote $r_i = \max\{|\sigma_1^{(i)}(t)| + |\sigma_2^{(i)}(t)| : a \leqq t \leqq b\}$, $i = 0, 1$, and say that some condition is satisfied on $S(a, b)$ if it is satisfied for a.e. $t \in (a, b)$ and for every $x \in [\sigma_1(t), \sigma_2(t)]$, $|y| \geqq c_1$.

2. The main results

The following two theorems deal with the property (E):

(E)
1. The problem (0.1), (0.2) has at least one solution.
2. If $A = B = 0$, then there exists at least one T-periodic solution of (0.1).
3. If $A \neq 0$, $B = 0$ and f is $|A|$-periodic in its second argument, then there exists at least one first kind T-periodic solution of (0.1).

Theorem 1. *Let σ_1 be a lower function and σ_2 an upper function of the problem* (0.1), (0.2) *and $\sigma_1(t) \leqq \sigma_2(t)$ for $a \leqq t \leqq b$. Let on the set $S(a, b)$ the inequality*

$$|f(t, x, y)| \leqq \omega(y) \sum_{i=1}^{n} g_i(t)\, h_i(x)\, (1 + |y|)^{1/q_i} \tag{2.1}$$

be satisfied, where $g_i \in L^{p_i}(a, b)$, $h_i \in L^{q_i}(-r_0, r_0)$, $i = 1, \ldots, n$, and $\omega \in C(\mathbb{R})$ is a positive function such that

$$\int_{c_1}^{+\infty} \frac{\mathrm{d}s}{\omega(s)} = \int_{c_1}^{+\infty} \frac{\mathrm{d}s}{\omega(-s)} = +\infty. \tag{2.2}$$

Then (E) is satisfied.

Theorem 2. *Let σ_1, σ_2 satisfy the conditions of Theorem* 1 *and let on the set $S(a, b)$ the inequality*

$$|f(t, x, y)| \leqq \omega(t, |y|) \tag{2.3}$$

be fulfilled, where $\omega \in \mathrm{Car}_{\mathrm{loc}}([a, b] \times \mathbb{R}_+)$ is a non-negative function, non-decreasing with respect to its second variable and

$$\limsup_{\varrho \to +\infty} \frac{1}{\varrho} \int_a^b \omega(t, \varrho)\, \mathrm{d}t < 1. \tag{2.4}$$

Then (E) is satisfied.

Note. For the assertions 1, 2 of (*E*) we can use the following criterions:

1. Let $g_0(t) = (Bt^2 + 2At - B(b + a)t)(2b - 2a)^{-1}$. If there exists $r \in (0, +\infty)$ such that f satisfies for a.e. $t \in (a, b)$ and each $x \in R$

$$(2.5) \qquad (f(t, x + g_0, g_0') - B/(b - a)) \operatorname{sgn} x \geqq 0 \qquad \text{for } |x| \geqq r,$$

then $\sigma_1(t) = g_0(t) - r$ is a lower function and $\sigma_2(t) = g_0(t) + r$ is an upper function of (0.1), (0.2).

2. Let f be continuous on $[a, b] \times \mathbb{R}^2$ and let there exist $c \in (0, +\infty)$ such that $\dfrac{\partial f(t, x, y)}{\partial x} \geqq c$ on $[a, b] \times \mathbb{R}^2$. Then (2.5) is satisfied for $r = \max\{|f(t, g_0, g_0') - B/(b - a)| c^{-1} : a \leqq t \leqq b\}$.

Theorem 3. *Let there exist a non-negative function $h \in L(a, b)$ such that for a.e. $t \in (a, b)$ and every $(x, y) \in \mathbb{R}^2$ there is satisfied the inequality*

$$(2.6) \qquad f(t, x_1, y_1) - f(t, x_2, y_2) + h(t)|y_1 - y_2| > 0 \qquad \text{for } x_1 > x_2.$$

Then the problem (0.1), (0.2) *has not more than one solution.*

3. Lemmas

Lemma 1. *Let $k \in (0, +\infty)$ and $G: [a, b] \times [a, b] \to \mathbb{R}$ be the Green function for the problem*

$$(3.1) \qquad v'' = k^2 . v$$

$$(3.2) \qquad v(b) - v(a) = 0, \quad v'(b) - v'(a) = 0.$$

Then there exists $c_k \in (0, +\infty)$ such that the inequality

$$(3.3) \qquad \left|\frac{\partial G(t, s)}{\partial t}\right| + |G(t, s)| \leqq c_k \qquad \text{for } a \leqq t, s \leqq b$$

is fulfilled.

Proof. It is easy to show that the constant $c_k = 2(k + 1)(e^{km} + 1)e^{2km}/kD$, where $m = \max\{|a|, |b|\}$ and $D = 2(e^{kb} - e^{ka}).(e^{-ka} - e^{-kb})$ satisfies the inequality (3.3).

Lemma 2. (*Conti Theorem*). *Let there exist $h \in L(a, b)$ such that*

$$|f(t, x, y)| \leqq h(t) \qquad \text{for } (t, x, y) \in [a, b] \times \mathbb{R}^2.$$

Then for any $k \in (0, +\infty)$ the problem

$$(3.4) \qquad u'' = k^2 u + f(t, u, u'),$$

(3.5) $$u(b) - u(a) = A\,,\ u'(b) - u'(a) = B$$

has a solution.

Proof. Put $g_0(t) = (Bt^2 + 2At - B(b+a)t)(2b-2a)^{-1}$ for $a \leqq t \leqq b$, $g(t,x,y) = f(t, x+g_0, y+g_0') + k^2 g_0(t) - B(b-a)^{-1}$ on $[a,b] \times \mathbb{R}^2$ and consider the differential equation

$$v'' = k^2 v + g(t, v, v')\,.$$

Analogously as in the proof of Lemma 3 in [12], denote by $\mathscr{B}$ the Banach space of all functions from $C^1(a, b)$ with a norm

$$\|z\| = \max\{|z(t)| + |z'(t)|:\ a \leqq t \leqq b\} \qquad \text{for } z \in C^1(a, b)$$

and consider the operator $H\colon \mathscr{B} \to \mathscr{B}$ defined by

$$H(z(t)) = \int_a^b G(t, s)\, g(s, z(s), z'(s))\,\mathrm{d}s \qquad \text{for } a \leqq t \leqq b\,,$$

where G is the Green function of the problem (3.1), (3.2). By the Schauder fixed-point theorem, since H is continuous and maps $\mathscr{B}$ into its compact subset, there exists $v \in \mathscr{B}$ such that

$$v(t) = \int_a^b G(t, s)\, g(s, v(s), v'(s))\,\mathrm{d}s\,.$$

Therefore $u = v + g_0$ is a solution of (3.4), (3.5).

Lemma 3. (*A priori estimate*). *Let* $r \in (0, +\infty)$, $g_i \in L^{p_i}(a, b)$ $h_i \in L^{q_i}(-r, r)$, $i = 1, \ldots, n$, *and* $\omega \in C(\mathbb{R})$ *be a positive function satisfying* (2.2). *Then there exists* $r^* \in (c_1, +\infty)$ *such that for any function* $u \in AC^1(a, b)$ *the conditions*

(3.6) $$u(b) - u(a) = A\,, \qquad |u(t)| \leqq r \qquad \textit{for } a \leqq t \leqq b$$

and

(3.7) $$|u''(t)| \leqq \omega(u'(t)) \sum_{i=1}^{n} g_i(t)\, h_i(u(t))\, (1 + |u'(t)|)^{1/q_i} \qquad \textit{for a.e. } t \in (a, b),\ |u'(t)| \geqq c_1$$

imply the estimate

(3.8) $$|u'(t)| \leqq r^* \qquad \textit{for } a \leqq t \leqq b\,.$$

Proof. Lemma 3 can be proved in the same way as Lemma 4 in [12].

Lemma 4 (*A priori estimate*). *Let* $r \in (0, +\infty)$ *and* $\sigma w \in \mathrm{Car}_{\mathrm{loc}}([a, b) \times \mathbb{R}_+)$ *satisfy the conditions of Theorem* 2. *Then there exists* $r^* \in (c_1, +\infty)$ *such that for any function* $u \in AC^1(a, b)$ *the conditions* (3.6) *and*

(3.9) $$|u''(t)| \leqq \sigma w(t, |u'(t)|)$$

for a.e. $t \in (a, b)$, *where* $|u'(t)| \geqq c_1$,
imply the estimate (3.8).

Proof. Let $u \in AC^1(a, b)$ satisfy (3.6) and (3.9). From (3.6) it follows that there exists $a_1 \in (a, b)$ such that $u'(a_1) = A/(b-a)$. Let $\varrho^* = \max\{|u'(t)|: a \leqq t \leqq b\}$ and $t^* \in [a, b]$ be such that $|u'(t^*)| = \varrho^*$. If $\varrho^* > c_1$, then there exists $t_* \in (a_1, t^*)$ (or $t_* \in (t^*, a_1)$) such that

$$|u'(t_*)| = c_1, \ |u'(t)| > c_1 \qquad \text{for } t_* < t < t^* \text{ (or } t^* < t < t_*\text{)}.$$

Integrating (3.9) from t_* to t^* (or from t^* to t_*), we get

$$\varrho^* \leqq c_1 + \int_a^b \omega(t, \varrho^*)\,\mathrm{d}t. \tag{3.10}$$

Since (2.4), there exists $r^* \in (c_1, +\infty)$ such that for any $\varrho > r^*$ the inequality

$$1 > c_1/\varrho + (1/\varrho)\int_a^b \omega(t, \varrho)\,\mathrm{d}t \tag{3.11}$$

holds. By (3.10), (3.11), we have $\varrho^* \leqq r^*$.

Lemma 5 (*On the solvability of the problem* (0.1), (0.2)). *Let* σ_1 *be a lower function and* σ_2 *an upper function of the problem* (0.1), (0.2) *and* $\sigma_1(t) \leqq \sigma_2(t)$ *for* $a \leqq t \leqq b$. *Further, let on the set* $S(a, b)$ *the inequality*

$$|f(t, x, y)| \leqq g(t)$$

be valid, where $g \in L(a, b)$.

Then the problem (0.1), (0.2) *has a solution* u *satisfying the condition*

$$\sigma_1(t) \leqq u(t) \leqq \sigma_2(t) \qquad \text{for } a \leqq t \leqq b. \tag{3.12}$$

Proof. Similarly as in the proof of Lemma 8 in [12], we put

$$w_i(t, x, y) = (-1)^i m(x - \sigma_i)[f(t, \sigma_i, \sigma_i') - f(t, \sigma_i, y) + (-1)^i r_0/m], \quad i = 1, 2$$

and

$$f_m(t, x, y) = \begin{cases} f(t, \sigma_1, \sigma_1') - r_0/m & \text{for } x \leqq \sigma_1(t) - 1/m \\ f(t, \sigma_1, y) + w_1(t, x, y) & \text{for } \sigma_1(t) - 1/m < x < \sigma_1(t) \\ f(t, x, y) & \text{for } \sigma_1(t) \leqq x \leqq \sigma_2(t) \\ f(t, \sigma_2, y) + w_2(t, x, y) & \text{for } \sigma_2(t) < x < \sigma_2(t) + 1/m \\ f(t, \sigma_2, \sigma_2') + r_0/m & \text{for } x \geqq \sigma_2(t) + 1/m, \end{cases}$$

where $(t, x, y) \in [a, b] \times \mathbb{R}^2$ and m is a natural number.
Then, by Lemma 2, the problem

$$u'' = (1/m)u + f_m(t, u, u'),$$

$$u(b) - u(a) = A\,, \ u'(b) - u'(a) = B$$

has a solution. First, let us prove that

(3.13) $\qquad \sigma_1(t) - 1/m \leqq u_m(t) \leqq \sigma_2(t) + 1/m \qquad$ for $a \leqq t \leqq b$.

Put $v(t) = (-1)^i(u_m(t) - \sigma_i(t)) - 1/m$ for $a \leqq t \leqq b$, $i \in \{1, 2\}$. Then, by (1.2), (1.4),

(3.14) $\qquad v(b) - v(a) = 0\,, \ v'(b) - v'(a) \leqq 0\,.$

Let $v(t) > 0$ for $t \in I \subset [a, b]$. Then, in view of (1.1), (1.3),

(3.15) $\quad v''(t) = (-1)^i(u''_m(t) - \sigma''_i(t)) \geqq r_0/m + (-1)^i u_m/m \geqq 1/m^2 \quad$ for $t \in I$.

From this it follows according to (3.14) that there exists $t_0 \in (a, b)$ such that

(3.16) $\qquad v(t_0) = 0\,.$

Now, suppose that (3.13) does not hold on $[t_0, b]$, i.e. that for certain $i \in \{1, 2\}$ and $t^* \in (t_0, b)$

$$v(t^*) > 0\,.$$

Let $(\alpha, \beta) \subset (t_0, b)$ be the maximal interval containing t^* in which $v(t) > 0$. Then $v(\alpha) = 0$, $v'(\alpha) \geqq 0$ and, by (3.15), $v''(t) \geqq m^{-2}$ for $\alpha \leqq t \leqq \beta$. Therefore $\beta = b$ and $v(b) > 0$, $v'(b) > 0$. Since (3.14), $v(a) > 0$, $v'(a) > 0$. Let $(a, a_0) \subset (a, t_0)$ be the maximal interval in which $v(t) > 0$. Analogously as above we can prove $a_0 = t_0$, whence $v(t_0) > 0$, which contradicts (3.16). Consequently

(3.17) $\qquad v(t) \leqq 0 \qquad$ for $t_0 \leqq t \leqq b$, and by (3.14), $v(a) \leqq 0$.

Supposing that (3.13) does not hold on $[a, t_0]$, we obtain a contradiction similar to (3.16). Hence u_m satisfies (3.13) on $[a, b]$.
Finally, since the sequences $(u_m)_1^\infty$ and $(u'_m)_1^\infty$ are uniformly bounded and equicontinuous on $[a, b]$, by the Arzelà-Ascoli lemma we can suppose without loss of generality that they are uniformly converging on $[a, b]$. Consequently the function $u(t) = \lim_{m \to \infty} u_m(t)$ for $a \leqq t \leqq b$ is a solution of the problem (0.1), (0.2) and satisfies the condition (3.12).

4. Proofs of Theorems

Proof of Theorem 1. Let r^* be the constant constructed by Lemma 3 for $r = r_0$. Put $\varrho_0 = r^* + r_0 + r_1$,

$$\chi(\varrho_0, s) = \begin{cases} 1 & \text{for } 0 \leqq s \leqq \varrho_0 \\ 2 - s/\varrho_0 & \text{for } \varrho_0 < s < 2\varrho_0 \\ 0 & \text{for } s \geqq 2\varrho_0 . \end{cases}$$

(4.1) $\tilde{f}(t, x, y) = \chi(\varrho_0, |x| + |y|) f(t, x, y) \qquad \text{for } (t, x, y) \in [a, b] \times \mathbb{R}^2$

and consider the equation

$$u'' = \tilde{f}(t, u, u'). \tag{4.2}$$

Since $\max\{|\sigma_i(t)| + |\sigma_i'(t)|: a \leqq t \leqq b\} < \varrho_0$, $i = 1, 2$, σ_1 is a lower function and σ_2 an upper function of the problem (4.2), (0.2). Moreover $|\tilde{f}(t, x, y)| \leqq g(t)$ for $(t, x, y) \in [a, b] \times \mathbb{R}^2$, where $g(t) = \sup\{|f(t, x, y)| : |x| + |y| \leqq 2\varrho_0\} \in L(a, b)$. Therefore, by Lemma 5, the problem (4.2), (0.2) has a solution u satisfying (3.12). Clearly u fulfils (3.6) for $r = r_0$ and (3.7) and so, by Lemma 3, the estimate (3.8) is valid. Therefore

$$|u(t)| + |u'(t)| \leqq \varrho_0 \qquad \text{for } a \leqq t \leqq b. \tag{4.3}$$

In view of (4.1), (4.2) and (4.3), u is a solution of the problem (0.1), (0.2).
Now, let $A = B = 0$ and $u^* : \mathbb{R} \to \mathbb{R}$ be the T-periodic extension of u. Then u^* is a T-periodic solution of (0.1).
Finally, let $A \neq 0$, $B = 0$ and f be $|A|$-periodic in its second argument. Let $u^* : \mathbb{R} \to \mathbb{R}$ be defined by $u_*(t) = u(t) + nA$ for $t \in [a + n(b - a), b + n(b - a)]$, $n = 0, \pm 1, \pm 2, \ldots$. Then u_*' is a T-periodic function and u_* satisfies (0.1) for a.e. $t \in R$. Therefore u_* is the first kind T-periodic solution of (0.1) and we have proved Theorem 1.

Theorem 2 can be proved analogously as Theorem 1 only instead of Lemma 3 we use Lemma 4.

P r o o f o f T h e o r e m 3. Let us assume that the problem (0.1), (0.2) has two solutions u_1, u_2. Put $v = u_1 - u_2$ on $[a, b]$. Then

$$v(a) = v(b), \quad v'(a) = v'(b). \tag{4.4}$$

Let us suppose that $v(a) \neq 0$. Without loss of generality we may consider that

$$v(a) > 0. \tag{4.5}$$

Since (4.4), there exists $t_0 \in (a, b)$ such that $v'(t_0) = 0$. Now, let $v(t) > 0$ for $a \leqq t \leqq b$. Then, by (2.6), $v''(t) + \tilde{h}(t) v'(t) > 0$ for a.e. $t \in (a, b)$, where $\tilde{h} = h \operatorname{sgn} v'$. Therefore the inequality

$$\left(\exp\left(\int_a^t \tilde{h}(s)\, ds\right) v'(t)\right)' > 0 \tag{4.6}$$

is satisfied for a.e. $t \in (a, b)$. Integrating (4.6) from a to t_0 and from t_0 to b, we get $v'(a) < 0$ and $v'(b) > 0$, which contradicts (4.4). Therefore there exists $t_1 \in (a, b)$ such that

$$v(t_1) = 0. \tag{4.7}$$

In view of (4.4), (4.5), (4.7), there exist a_1, $b_1 \in (a, b)$ such that $v(t) > 0$ for $t \in [a, a_1) \cup (b_1, b]$ and $v(a_1) = v(b_1) = 0$. Then (4.6) holds on $[a, a_1) \cup (b_1, b]$ and integrating it from a to a_1 and from b_1 to b, we get (as above) the contradiction to (4.4). Hence

$$v(a) = v(b) = 0. \tag{4.8}$$

Let there exists $\tilde{t} \in (a, b)$ such that $v(\tilde{t}) > 0$ and let $(\alpha, \beta) \subset (a, b)$be the maximal interval containing $\tilde{t}$ in which $v(t) > 0$. Then, by (4.8), $v'(\alpha) \geqq 0$, $v'(\beta) \leqq 0$. Moreover (4.6) holds on (α, β). Integrating (4.6) from α to β, we get $0 \geqq v'(\beta) - v'(\alpha) > 0$. This contradiction proves Theorem 4.

REFERENCES

[1] BATES, P. W.—WARD, Y. R.: Periodic solutions of higher order systems. Pacific J. Math. 84, 1979, 275—282.

[2] CONTI, R.: Recent trends in the theory of boundry value problems for ordinary differential equations. Boll. Unione Mat. Ital., 22, 3, 1967, 135—178.

[3] GAINES, R. E.—MAWHIN, J. L.: Coincidence Degree and Nonlinear Differential Equations. Berlin—Heidelberg—New York, Springer-Verlag, 1977, 262 p.

[4] GEGELIA, G. T.: O krajevych zadačach tipa periodičeskoj dlja obvyknovennych differencialnych uravnenij. Trudy IPM, Tbilisi, 1986, 17, 60—93.

[5] GREGUŠ, M.—ŠVEC, M.—ŠEDA, V.: Obyčajné diferenciálne rovnice. Alfa Bratislava, 1985, 374 p.

[6] HARTMAN, P.: Ordinary Differential Equations (in Russian). Mir, Moscow, 1970, 720 p.

[7] KIBENKO, A. V.—KIPNIS, A. A.: O periodičeskih rešenijach nelinejnych differencialnych uravnenij tretjego porjadka. Prikladnyj analiz, Voroněž, 1979, 70—72.

[8] KIGURADZE, I. T.: Nekotoryje singularnyje krajevyje zadači dlja obvyknovennych differencialnych uravnenij. Tbilisi, 1975, 352 p.

[9] KIGURADZE, I. T.—PŮŽA, B.: O někotorych krajevych zdačach dlja sistem obvyknovennych differencialnych uravnenij. Diff. ur., 12, 1976, 2139—2148.

[10] KIGURADZE, I. T.: Krajevyje zadači dlja sistem obvyknovennych diferencialnych uravnenij. Itogi nauki i techn., Sovr. pr. mat., 30, Moscow, 1987, 71—91.

[11] KRASNOSELSKIJ, M. A.: Operator sdviga po trajektorijam differencialnych uravnenij. Moscow, Nauka, 1966, 331 p.

[12] RACHŮNKOVÁ, I.: On a certain four-point problem, preprint.

[13] SANSONE, G.: Equazioni differenziali nel campo reale I, II (in Russian). IL, Moscow, 1954, 346 p., 415 p.

Received February 11, 1988

Katedra matematické analýzy
a numerické matematiky PřF UP
Gottwaldova 15
771 46 Olomouc

О ПЕРИОДИЧЕСКИХ РЕШЕНИЯХ ПЕРВОГО РОДА ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ ВТОРОГО ПОРЯДКА

Irena Rachůnková

Резюме

В статье доказаны достаточные условия для существования и единственности решения задачи

$$u'' = f(t, u, u'),\ u(b) - u(a) = A,\ u'(b) - u'(a) = B,\ a,\ b,\ A,\ B \in (-\infty, \infty),\ a < b.$$

В случае $A \neq 0$, $B = 0$ показаны условия для существования решения u уравнения $u'' = f(t, u, u')$ такого, что u' периодическая функция.