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EIGHTY YEARS OF PROFESSOR ŠTEFAN SCHWARZ

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A prominent Slovak mathematician, Professor Štefan Schwarz, Doctor of Science, an outstanding specialist in the theory of semigroups and in number theory, has reached eighty years of age on May 18, 1994 continuing his creative work in mathematics.

He was born at Nové Mesto nad Váhom. After having completed his studies at Charles University in Prague in 1936, he became Assistant Professor at the Mathematical Institute of the Faculty of Science, Charles University, and later, in 1939, he became member of staff of the newly established Slovak Technical University.

In 1944, he was deportated and emprisoned in a German concentration camp, from where he was deliberated in 1945. Two his sisters died in such camps.

In 1946 he was appointed Associate Professor at the Faculty of Science in Bratislava and since 1947 he has been Full Professor at the Slovak Technical University. He was elected corresponding member and ordinary member (Academician) of the Czechoslovak Academy of Sciences in 1952 and 1960, respectively. In 1953 he also became member of the then established Slovak Academy of Sciences. Since 1966 until 1988 he has been Head of the Mathematical Institute of this Academy. In the years 1965–1970 he was President of the Slovak Academy of Sciences and Vice-President of the Czechoslovak Academy of Sciences.

The scientific papers of Štefan Schwarz concern the following regions: a) theory of semigroups and its applications in other fields of mathematics; b) theory of finite fields; c) number theory; d) non-negative matrices and Boolean matrices.

The results of Štefan Schwarz have been incorporated and reproduced in detail in several monographs; in particular, cf. [1a] and [2a] for the case of algebraic semigroups, [3a] for topological semigroups, [4a] for Boolean matrices and [5a] for the theory of finite fields.

His work was appraised twenty years ago and ten years ago in Czechoslovak Mathematical Journal, Časopis pro pěstování matematiky, Matematický časopis and Mathematica Slovaca; cf., e.g., [1c], [2c], [3c].

The purpose of the present article is to characterize the papers of Štefan Schwarz which have been published after 1984.

For a semigroup S having a minimal left ideal let K be the union of all minimal left ideals of S. Schmetterer proposed the problem of characterizing those semigroups for which Ka is a minimal left ideal of S for each $a \in S$. In [94] Š. Schwarz finds such a characterization; a semigroup has the mentioned property if and only if it can be obtained by a right composition of a special type of semigroups (called U_l-semigroups). He also obtains several results concerning the converse problem: to decide whether a given family of U_l-semigroups admits at least one right composition.

The results of the paper [96] concern the structure of semigroups S such that (i) S contains a universally minimal left ideal L (i.e., the last left ideal L) which is a left group, and (ii) S has an L-homomorphism.

In the paper [92], the author continues in studying the semigroup of binary relations on a finite set which was begun in [72] and [73]. At the same time he shows that there exists a close connection of the question under consideration with the theory of Boolean matrices and the theory of directed graphs. Let $B_n(V)$ be the semigroup of all binary relations on a finite V with n elements, where $n \geqslant 2$. Further let M_n be the set of all $n \times n$ matrices over the Boolean algebra $\{0, 1\}$; M_n is a semigroup under the Boolean matrix multiplication. Finally, let $G_n(V)$ be the set of all directed graphs having the set V as the set of all vertices with allowable loops and simple directed edges. There exists a natural one-to-one correspondence between the sets $B_n(V)$, M_n and $G_n(V)$. By applying this correspondence and by a masterful synthesis of methods belonging to semigroup theory, theory of Boolean matrices and graph theory Š. Schwarz reused to solve a combinatorial problem which was proposed by A. Paz in his monograph [6a] on probabilistic automata. A series of further results in the field under consideration is deduced in [92].

Under the usual notation, let $GF(q)$ be a finite field with q elements and let S_n be the multiplicative semigroup of all $n \times n$ matrices over $GF(q)$. I. B. Marshall [2b], 1941, I. Niven [3b], 1948 and V. Klein [1b], 1980, dealt with the identity $A^{\kappa} = A^{\kappa+\delta}$, where A runs over the set of all rectangular matrices belonging to S_n, and the integers κ and δ are as small as possible. In [93] Š. Schwarz proved the following theorem concerning singular matrices (by using semigroup-theoretical methods):

For each $n \times n$ matrix A over $GF(q)$ with $1 \leqslant \operatorname{rank}(A) \leqslant h \leqslant n-1$ the relation

$$A^{h+1} = A^{h+1+\lambda(h,q)}$$

is valid, and this result is the best possible. Here, $\lambda(h, q)$ is the least common multiple of h and q.

The powers of non-negative matrices have been investigated by several authors (cf., e.g., the monographs [4a] and [7a]). In particular, the following question has been considered: Suppose that a non-negative $n \times n$ matrix P satisfies the condition

(*) some power of P has a positive column.

What is the least integer k such that P^k has a positive column? D. Isaacson and P. Madsen [4b], 1974, proved that $k \leqslant n^2 - 3n + 3$; the same result is contained in a paper of L. T. Kou [5b], 1984. In his article [95] Š. Schwarz proved a stronger result, namely:

Let P be a non-negative $n \times n$ matrix satisfying the condition (*). Put $L = n^2 - 3n + 3$ and $K = n^2 - 5n + 8$.

(i) If P is primitive, then P^L contains a positive columns. For each $n \geqslant 2$ there are primitive $n \times n$ matrices for which the number L cannot be replaced by a smaller one.

(ii) If P is not primitive, then P^K has a positive column. For each $n \geqslant 3$ there are $n \times n$ matrices for which the number K cannot be replaced by a smaller one.

In the paper [97] the author gives a new proof of the existence of a normal basis for cyclic extensions over any field. This proof is shorter and more transparent then those given until now in the literature. Next, an effective method to find all normal bases is described.

Pei, Wang and Omura [6b] studied normal bases of finite fields having the form $GF(2^m)$ in connection with the coding theory. In [98] Š. Schwarz considers the following question. Let $f(x)$ be an irreducible polynomial of degree n over $F_q = GF(q)$ and let α be a fixed root of $f(x)$; the problem is to find an effective method for deciding whether the roots of $f(x) = 0$ represent a normal basis for the cyclic extension $F_q(\alpha)$ over F_q. A criterion giving such an effective method is described in Schwarz's theorem which is the main result of [98]. This theorem widely generalizes the result of Pei, Wang and Omura; also, the method of proof is different.

As a former student of Charles University Š. Schwarz often remembers on and speaks about his teachers, especially Professors Karel Petr (1868–1950), Bohumil Bydžovský (1880–1969), Vojtěch Jarník (1897–1970), Vladimír Kořínek (1899–1981), and his elder colleagues František Vyčichlo (1905–1958) and Vladimír Knichal (1908–1974).

In June 1993, on the occasion of the celebration (organized in Prague) of the 100th anniversary of birth of Eduard Čech, Professor Štefan Schwarz presented a lecture on this greatest personality of Czech mathematics.

In the name of Czech and Slovak mathematical communities we cordially congratulate Professor Štefan Schwarz to his anniversary, and we do it also in the name of a large number of his thankful former students. We wish him all the best for the next years, good health, further deep and nice mathematical results. We thank him for his scientific, pedagogical and organizatory work.

A. Monographs:

[1a] *E. S. Ljapin*: Pologruppy. Gosizdat fyz. mat. lit., Moskva, 1960, fourth English edition, Amer. Math. Soc., Providence, 1978.
[2a] *A. H. Clifford and G. B. Preston*: The Algebraic Theory of Semigroups. Amer. Math. Soc., Providence, 1961, 1977 (Vol. 1), 1967, 1971 (Vol. 2).
[3a] *A. B. Palman – de Miranda*: Topological Semigroups. Math. Centrum, Amsterdam, 1964.
[4a] *Ki Hang Kim*: Boolean Matrix Theory and Applications. Marcel Dekker Inc., New York, 1982.
[5a] *R. Lidl and H. Niederreiter*: Finite Fields. Adison Wesley Publ. Comp., Reading, Mass., 1983; Russian translation: Izd. Mir, Moskva.
[6a] *A. Paz*: Introduction to Probabilistic Automata. Academic Press, New York, 1971.
[7a] *A. Berman, R. Plemmons*: Nonnegative Matrices in the Mathematical Sciences. Academic Press, New York, London, 1979.

B. Scientific articles:

[1b] *A. Klein*: On Fermat's theorem for matrices and the periodic identities of $M_n[GF(q)]$. Archiv der Math. *34* (1980), 399–402.
[2b] *I. B. Marshall*: On the extension of Fermat's theorem to matrices of order n. Proc. Edinburgh Math. Soc. *6* (1939 1941), 85–91.
[3b] *I. Niven*: Fermat's theorem for matrices. Duke Math. J. *15* (1948), 823–826.
[4b] *D. Isaacson and R. Madsen*: Positive columns for stochastic matrices. J. Appl. Prob. *11* (1974), 829–835.
[5b] *L. T. Kou*: Fixed length sources and sink of a graph. Math. Japonica *29* (1984), 411–417.
[6b] *D. Y. Pei, C. C. Wang and J. K. Omura*: Normal basis of finite field $FG(2^m)$. IEEE Trans. of Inform. Theory *IT-32* (1986), 285–287.

C. Other articles:

[1c] *J. Jakubík and M. Kolibiar*: Seventieth anniversary of birthday of Academician Štefan Schwarz. Czechoslovak Math. J. *34(109)* (1984), 410–418.
[2c] *J. Jakubík, B. Kolibiarová and M. Kolibiar*: Seventy years of Academician Štefan Schwarz (in Slovak). Časopis pěst. matem. *109* (1984), 329–334.
[3c] *J. Jakubík and M. Kolibiar*: To the seventieth birthday of Academician Štefan Schwarz (in Slovak). Math. Slovaca *34* (1984), 239–246.

Scientific bibliography of Š. Schwarz

A. Scientific papers

[1] On the reducibility of polynomials over an algebraic number field modulo of a prime ideal. Časopis pěst. mat. fyz. *68* (1939), 112–127. (In German.)
[2] A contribution to the reducibility of a polynomial in the theory of congruences. Věstník Král. české spol. nauk 7 (1939), 1–9. (In French.)
[3] On the number of roots and irreducible factors of polynomials over a finite field. Časopis pěst. mat. fyz. *69* (1949), 128–146. (In French.)
[4] On a theorem of S. Lubelski. Časopis pěst. mat. fyz. *69* (1940), 147–150. (In German.)

[5] A contribution to the arithmetic of finite fields. Prírodovedecká príloha Technického obzoru slovenského I (No 8) (1940), 75–82. (In Slovak.)

[6] A contribution to the theory of congruences. Prírodovedecká príloha Technického obzoru slovenského II (No 8, 9) (1941), 89–92, 95–100. (In Slovak.)

[7] A contribution to the theory of Galois's fields. Prírodovedecká príloha Technického obzoru slovenského III (1942), no. 6, 1–4. (In Slovak.)

[8] Theory of semigroups. Sborník prác Prírodovedeckej fakulty Slovenskej univerzity, č. VI. Bratislava, 1943, pp. 64. (In Slovak.)

[9] A hypercomplex proof of the Jordan-Kronecker's Principle of reduction. Časopis pěst. mat. fys. *71* (1946), 17–21.

[10] A contribution to the reducibility of binomial congruences. Časopis pěst. mat. fys. *71* (1946), 21–32. (In Czech.)

[11] On the extension of the Jordan-Kronecker's Principle of reduction for inseparable polynomials. Časopis pěst. mat. fys. *73* (1947), 61–64.

[12] On Waring's Problem for finite fields. Quart. J. Math., Oxford Ser. *19* (1948), 123–128.

[13] On Generalization of Jordan-Kronecker's Principle of reduction, Věstník Král. české spol. nauk.

[14] On the equation $a_1x_1^k + \ldots + a_kx_k^k + b = 0$ in finite fields. Quart. J. Math., Oxford Ser. *19* (1948), 160–164.

[15] A contribution to the theory of cyclic fields. Sborník prác SVŠT, Bratislava (1948), 7. (In Slovak.)

[16] On the reducibility of binomial congruences an the least bound of an integer belonging to a given exponent (mod p), Časopis pěst. mat. fys..

[17] On universal forms in finite fields, Časopis pěst. mat. fys..

[18] On the generalization of the notion of a group. Časopis pěst. mat. fys. *74* (1949), 95–113. (In Slovak.)

[19] On equations of the form $c_1x_1^k + \ldots + c_sx_s^k = c$ in finite fields. Časopis pěst. mat. fys. *74* (1949), 175–176. (In Slovak.)

[20] On the structure of finite semigroups without zeros. Czechoslovak Math. J. *1 (76)* (1951), 51–65. (In Russian.)

[20a] On the structure of simple semigroups without zero. Czechoslovak Math. J. *1 (76)* (1951), 41–53.

[21] On semigroups having a kernel. Czechoslovak Math. J. *1 (76)* (1951), 259–301. (In Russian.)

[21a] On semigroups having a kernel. Czechoslovak Math. J. *1 (76)* (1951), 229–264.

[22] On the theory of periodic semigroups. Czechoslovak Math. J. *3 (78)* (1953), 7–21. (In Russian.)

[23] On maximal ideals in the theory of semigroups I.. Czechoslovak Math. J. *3 (78)* (1953), 139–153. (In Russian.)

[24] Maximal ideals and the structure of semigroups. Mat.-fyz. čas. Slovenskej akad. vied *3* (1953), 17–39. (In Slovak.)

[25] On maximal ideals in the theory of semigroups II. Czechoslovak Math. J. *3 (78)* (1953), 365–383. (In Russian.)

[26] The theory of characters of commutative semigroups. Czechoslovak Math. J. *4 (79)* (1954), 219–247. (In Russian.)

[27] Characters of commutative semigroups as class funcions. Czechoslovak Math. J. *4 (79)* (1954), 291–295. (In Russian.)

[28] On a Galois connection in the theory of characters of semigroups. Czechoslovak Math. J. *4 (79)* (1954), 296–313. (In Russian.)

[29] Characters of commutative semigroups. Proceedings of the International Congress of Mathematicians 1954, Vol. 1. Amsterdam 1957, p. 438.
[30] On Hausdorff bicompact semigroups. Czechoslovak Math. J. *5 (80)* (1955), 1–23. (In Russian.)
[31] Topological semigroups with one-sided units. Czechoslovak Math. J. *5 (80)* (1955), 153–163. (In Russian.)
[32] Characters of bicompact semigroups. Czechoslovak Math. J. *5 (80)* (1955), 24–28. (In Russian.)
[33] A remark to the theory of bicompact semigroups. Mat.-fyz. čas. Slovenskej akad. vied *5* (1955), 86–89. (In Slovak.)
[34] On magnifying elements in the theory of semigroups. Doklady Akad. Nauk SSSR *102* (1955), no. 4, 697–698. (In Russian.)
[35] On a type of universal forms in discretely valued fields. Acta Sci. Math. (Szeged) *XVII* (1956), no. 1–2, 5–29.
[36] The theory of characters of commutative Hausdorff bicompact semigroups. Czechoslovak Math.J. *6 (81)* (1956), 330–364.
[37] Once more on quadratic polynomials with prime values. Časopis pěst. mat. *81* (1956), 241–243 (with J. Mařík). (In Czech.)
[38] On the reducibility of polynomials over a finite field. Quart J. Math., Oxford Ser. 7 (1956), 110–124.
[39] Semigroups satisfying some weakened forms of the cancellation laws. Mat.-fyz. čas. Slovenskej akad. vied *6* (1956), 149–158. (In Slovak.)
[40] On the existence of invariant measures on certain types of bicompact semigroups. Czechoslovak Math. J. 7 *(82)* (1957), 165–182. (In Russian.)
[41] On the structure of the semigroup of measures on finite semigroups. Czechoslovak Math.J. 7 *(82)* (1957), 358–373.
[42] An elementary semigroup theorem and congruence relation of Rédei. Acta Sci. Math. (Szeged) *XIX* (1958), no. 1–2, 1–4.
[43] On the multiplicative semigroup of conguence classes (mod m). Mat.-fyz. čas. Slovenskej akad. vied *8* (1958), 136–150 (with B. Parízek). (In Slovak.)
[44] Remarks on compact semigroups. Colloq. Math. (Wroclaw) *VI* (1958), 265–270 (with J. Łos).
[45] On totally noncommutative semigroups. Mat.-fyz. čas. Slovenskej akad. vied *9* (1959), 92–100 (with D. Krajňáková). (In Russian.)
[46] On dual semigroups. Czechoslovak Math. J. *10 (85)* (1960), 201–230.
[47] A theorem on normal semigroups. Czechoslovak Math. J. *10 (85)* (1960), 197–200.
[48] Semigroups, in which every proper subideal is a group. Acta Sci. Math. (Szeged) *XXI* (1960), no. 3–4, 125–134.
[49] On a class of polynomials over a finite field. Mat.-fyz. čas. Slovenskej akad. vied *10* (1960), 68–80. (In Russian.)
[50] On the number of irreducible factors of a polynomial over a finite field. Czechoslovak Math. J. *11 (86)* (1961), 213–225. (In Russian.)
[51] Semicharacters of the multiplicative semigroup of integers modulo m. Mat.-fyz. čas. Slovenskej akad. vied *11* (1961), 63–74 (with B. Parízek).
[52] Subsemigroups of simple semigroups. Czechoslovak Math. J. *13 (88)* (1963), 226–239.
[53] Probabilities on non-commutative semigroups. Czechoslovak Math. J. *13 (88)* (1963), 372–426.
[54] Probability measures on non-commutative semigroups. General Topology and its Relations to Modern Analysis and Algebra, Proceedings of the Symposium held in Prague in September 1961. pp. 312–315.

[55] Cyclic matrices and algebraic equations over a finite field.. Mat.-fyz. čas. Slovenskej akad. vied *12* (1962), 38–48 (with K. Horáková). (In Russian.)
[56] A remark on algebraic equations over a finite field. Mat.-fyz. čas. Slovenskej akad. vied *12* (1962), 224–229. (In Russian.)
[57] Convolution semigroup of measures on compact non-commutative semigroups. Czechoslovak Math. J. *14 (89)* (1964), 95–115.
[58] Product decomposition of idempotent measures on compact semigroups. Czechoslovak Math. J. *14 (89)* (1964), 121–124.
[59] Homomorphisms of completely simple semigroups onto a group. Mat.-fyz. čas. Slovenskej akad. vied *12* (1962), 293–300.
[60] On a system of congruences. A remark on the preceding paper of J. Sedláček. Mat.-fyz. čas. Slovenskej akad. vied *13* (1963), 103–104. (In Slovak.)
[61] A semigroup treatment of some theorems on non-negative matrices. Czechoslovak Math. J. *15 (90)* (1965), 212–229.
[62] On the structure of the semigroup of stochastic matrices. Publ. of the Math. Inst. Hungarian Acad. Vol. IX, Series A, Fasc. 3, (1965), 297–311.
[63] On powers of non-negative matrices. Mat.-fyz. časopis Slovenskej akad. vied *15* (1965), 215–228.
[64] A remark to the theory of non-negative matrices. Sibir. Math. J. 6 (1965), 207–211. (In Russian.)
[65] A new approach to some problems in the theory of non-negative matrices. Czechoslovak Math. J. *16 (91)* (1966), 274–284.
[66] New kinds of theorems on non-negative matrices. Czechoslovak Math.J. *16 (91)* (1966), 285–295.
[67] Some estimates in the theory of non-negative matrices. Czechoslovak Math. J. *17 (92)* (1967), 399–407.
[68] A note on the structure of the semigroup of doubly stochastic matrices. Mat. čas. *16* (1967), 308–316.
[69] On the index of imprimitivity of a non-negative matrix. Acta Sci. Math. (Szeged) *28* (1967), 185–189.
[70] Algebraic considerations on powers of stochastic matrices. Mat. čas. *18* (1968), 218–228.
[71] Prime ideals and maximal ideals in semigroups. Czechoslovak Math. J. *19 (94)* (1969), 72–79.
[72] On the semigroup of binary relations on a finite set. Czechoslovak Math. J. *20 (95)* (1970), 632–679.
[73] On a sharp estimate in the theory of binary relations on a finite set. Czechoslovak Math. J. *20 (95)* (1970), 703–714.
[74] On idempotent relations on a finite set. Czechoslovak Math. J. 20 (95) (1970), 696–702.
[75] Any *O*-simple dual semigroup is completely *O*-simple. Semigroup Forum *2* (1971), 90–92.
[76] On the structure of dual semigroups. Czechoslovak Math. J. *21 (96)* (1971), 461–483.
[77] The semigroup of fully indecomposable relations and Hall relations. Czechoslovak Math. J. *23 (98)* (1973), 151–163.
[78] A note on small categories with zero. Acta Sci. Math. (Szeged) *35* (1973), 161–164.
[79] Summs of powers of binary relations. Mat. čas. *24* (1974), 161–171. (In Russian.)
[80] Circulant Boolean relation matrices. Czechoslovak Math. J. *24 (99)* (1974), 252–253.
[81] The ideal structure of *C*-semigroups. Czechoslovak Math. J. *27* (1977), 313–338.
[82] The semigroup of circulant Boolean matrices. Czechoslovak Math. J. *26* (1976), 632–635 (with Kim H. Butler).
[83] A counting theorem in the semigroup of circulant Boolean matrices. Czechoslovak Math. J. *27* (1977), 504–510.

[84] Semigroups containing maximal ideals. Math. Slovaca *28* (1978), 157–168.
[85] Intersections of maximal ideals in semigroups. Semigroup Forum *12* (1976), 367–372.
[86] A theorem on binary relations and infinite regular languages. Semigroup Forum *17* (1979), 307–316.
[87] The Euler-Fermat Theorem for the semigroup of circulant Boolean matrices. Czechoslovak Math. J. *30* (1980), 135–141.
[88] Infinite products on doubly stochastic matrices. Acta Math. Univ. Comenian. *39* (1980), 131–150.
[89] The role of semigroups in the elementary theory of numbers. Math. Slovaca *31* (1981), 369–395.
[90] An unconventional problem in the elementary theory of numbers. Czechoslovak Math. J. *31* (1981), 159–169.
[91] Extensions of Bauer's identical congruences. Math. Slovaca *33* (1983), 209–224.
[92] Common consequents in directed graphs. Czechoslovak Math. J. *35 (110)* (1985), 212–247.
[93] Fermat's Theorem for matrices revisited. Math. Slovaca *35* (1985), 343–347.
[94] Right compositions of semigroups. Math. Slovaca *36* (1988), 3–14.
[95] A combinatorial problem arising in finite Markov chains. Math. Slovaca *36* (1986), 199–210.
[96] Semigroups with a universally minimal left ideal. Acta Sci. Math. (Szeged) *52* (1988), 21–28.
[97] Construction of normal bases in cyclic extensions of a field. Czechoslovak Math. J. *38* (1988), 291–312.
[98] Irreducible polynomials over finite field with linearly independent roots. Math. Slovaca *38* (1988), 147–158.
[99] Powers of subsets of finite semigroups. To appear.

B. Books

[1] On equations. 1st ed., Prague, 1940, pp. 96. (In Czech.)
[2] On equations. 3rd ed., Prague, 1947, pp. 160. (In Czech.)
[3] Algebraic numbers. Přírodovědecké nakl., Prague, 1950, pp. 292. (In Slovak.)
[4] The theory of solutions of equations. Nakl. ČSAV, Prague, 1958, pp. 348. (In Slovak.)
[5] The theory of solutions of equations. Vydavatelstvo SAV, 1st ed., Bratislava, 1967, 1968, pp. 456. (In Slovak.)