Czechoslovak Mathematical Journal

Surjit Singh Khurana

Positive vector measures with given marginals

Czechoslovak Mathematical Journal, Vol. 56 (2006), No. 2, 613–619

Persistent URL: http://dml.cz/dmlcz/128091

Czechoslovak Mathematical Journal, 56 (131) (2006), 613–619

POSITIVE VECTOR MEASURES WITH GIVEN MARGINALS

Surjit Singh Khurana, Iowa

(Received November 21, 2003)

Abstract. Suppose E is an ordered locally convex space, X_1 and X_2 Hausdorff completely regular spaces and Q a uniformly bounded, convex and closed subset of $M_t^+(X_1 \times X_2, E)$. For $i = 1, 2$, let $\mu_i \in M_t^+(X_i, E)$. Then, under some topological and order conditions on E, necessary and sufficient conditions are established for the existence of an element in Q, having marginals μ_1 and μ_2.

Keywords: ordered locally convex space, order convergence, marginals

MSC 2000: 60B05, 46E10, 28C05, 46G10, 28B05

1. Introduction and notation

In ([7], [5]) some interesting results are proved about the existence of positive vector measures having given marginals when the measures take their values in ordered locally convex spaces and KB Banach spaces. In this paper, these results are extended to more general settings.

All vector spaces are taken over reals. For locally convex spaces and ordered locally convex spaces, we use the notation and results from ([13]).Throughout the paper, E is assumed to be an ordered, quasi-complete locally convex space whose positive cone is normal and whose topology is generated by a family of semi-norms $|\cdot|_p$, $p \in P$ such that $0 \leqslant x \leqslant y$, $p \in P \Rightarrow |x|_p \leqslant |y|_p$ ([13] 3.1, p. 215); being an ordered locally convex space, the positive cone of E is closed ([13] p. 222).

For a completely regular Hausdorff space X, $X^{\sim}$ will always denote its the Stone-Čech compactification of X; $C_b(X)$ and $C(X^{\sim})$ will denote space of all real-valued bounded continuous functions on X and $X^{\sim}$ respectively. Borel subsets of X will be denoted by $\mathcal{B}(X)$.

A positive countably additive bounded mapping $\mu\colon \mathcal{B}(X) \to E$ will be called a measure; for every $p \in P$, this μ gives rise to a submeasure $\dot{\mu}_p\colon \mathcal{B}(X) \to [0, \infty)$,

$\dot{\mu}_p(B) = \sup\{|\mu(A)|_p\colon\ A \in \mathcal{B}(X),\ A \subset B\}$, for every Borel set in X. This submeasure serves the same purpose as the p-semivariation $\|\mu\|_p$, defined in ([10], [11]), as they are connected by the relation $\dot{\mu}_p \leqslant \|\mu\|_p \leqslant 4\dot{\mu}_p$ ([11], p. 158)). Since the measure is positive, $\dot{\mu}_p(B) = |\mu(B)|_p$. The submeasure is increasing, countably subadditive and order sigma-continuous in the sense of ([3] p. 279; also cf. [8]); it will be called a Radon measure if, for each $p \in P$, a Borel set B in X and $c > 0$, there exist a compact K and an open V in X such that $K \subset B \subset V$ and $\dot{\mu}_p(V \setminus K) \leqslant c$. The set of all positive Radon measures: $\mathcal{B}(X) \to E$ will be denoted by $M_t^+(X,E)$. Integration of real-valued function with respect to these measures will be taken in the sense of ([11], [10]).

Suppose Y is a compact Hausdorff space and assume E is also semi-reflexive. In this case, a positive linear mapping (automatically continuous) $\mu\colon C(Y) \to E$ will give rise to a unique positive Radon measure $\mu\colon \mathcal{B}(Y) \to E$ ([11], p. 163) and conversely. Now suppose that Y is a compactification of X, a completely regular Hausdorff space, and μ is a positive, Radon, E-valued measure on Y. If for each $p \in P$ there exists a sigma-compact $C^{(p)} \subset X$ such that $\dot{\mu}_p(Y \setminus C^{(p)}) = 0$, then μ can be considered to be an element of $M_t^+(X,E)$ by defining, for a Borel set $B \subset X$, $\mu(B) = \mu(B_0)$, where B_0 is any Borel subset of Y such that $B = B_0 \cap Y$ (it is easily verified that it is well-defined). Also an element of $M_t^+(X,E)$ can be considered a Radon measure on Y. The set of positive E-valued Radon measures on a compact Y will be denoted by $M^+(Y,E)$. Considering $M^+(Y,E)$ to be a subspace of $E^{C(Y)}$, we take on $M^+(Y,E)$ the topology induced by the product topology on $E^{C(Y)}$. If X is completely regular Hausdorff space and Y its Stone-Čech compactification, considering $M_t^+(X,E)$ as a subspace of $M^+(Y,E)$, we take on $M_t^+(X,E)$ the topology induced by $M^+(Y,E)$. It is easily verified that this is identical with that induced by $E^{C_b(X)}$ with the product topology, when $M_t^+(X,E)$ is considered a subspace of $E^{C_b(X)}$ (note: for a $\mu \in M_t^+(X,E)$, if we denote it by $\bar{\mu}$, when considered to be an element of $M^+(Y,E)$, and for a function $f \in C_b(X)$, denoting its extenson to Y by $\overline{f}$ we have $\mu(f) = \bar{\mu}(\overline{f})$). For any completely regular space X, on $M_t^+(X,E)$ we always take the topology induced by $E^{C_b(X)}$.

For a compact Hausdorff space Y, we will identify the elements of $M^+(Y,E)$ with weakly compact positive linear maps from $C(Y)$ to E and conversely.

For $i = 1,2$, let X_i be compact Hausdorff spaces and $\lambda \in M^+(X_1 \times X_2, E)$. For $i = 1,2$, We get $\lambda^{(i)} \in M^+(X_i,E)$, defined by $\lambda^{(1)}(B) = \lambda(B \times X_2)$ and $\lambda^{(2)}(B) = \lambda(X_1 \times B))$ for the respective Borel sets B. This is the same as $\lambda^{(1)}(f_1) = \lambda(f_1 \otimes 1)$ and $\lambda^{(2)}(f_2) = \lambda(1 \otimes f_2)$ for $f_i \in C(X_i)$. $\lambda^{(1)}$ and $\lambda^{(2)}$ will be called the marginals of λ.

E', E'' will denote the dual and bidual of E. For an $x \in E$ and $f \in E'$, $\langle x, f\rangle$ will stand for $f(x)$.

2. Main results

The next lemma reduces the discussion of marginals to compact Hausdorff spaces. It generalizes Lemma 5.1 of ([6], p. 31); this lemma is the key result used in the main result of ([7], Theorem 1, p. 3294). It reduces the proof of the Strassen theorem about marginals ([14], Theorem 7) to a simple application of the separation theorem ([13], Theorem 9.2, p. 65).

Lemma 1. *Suppose X_1 and X_2 are Hausdorff completely regular spaces and $\lambda \in M^+((X_1 \times X_2)^\sim, E)$. If the marginals of λ are in $M_t^+(X_i, E)$ $(i = 1, 2)$, then $\lambda \in M_t^+(X_1 \times X_2, E)$.*

Proof. Take a $p \in P$ and fix a $c > 0$. For $i = 1, 2$ let $\mu_i \in M_t^+(X_i, E)$ be the marginals of λ. Let $\varphi\colon (X_1 \times X_2)^\sim \to (X_1^\sim \times X_2^\sim)$ be the extension of the identity mapping $X_1 \times X_2 \to (X_1^\sim \times X_2^\sim)$. Because of this, $C(X_1^\sim \times X_2^\sim)$ can be considered a subspace of $C((X_1 \times X_2)^\sim)$. For $i = 1, 2$ take compacts C_i such that $\mu_i(X_i \setminus C_i) = u_i$ with $p(u_i) < c$. Take $f_i \in C_b(X_i)$, $0 \leqslant f_i \leqslant 1$, $f_i \geqslant \chi_{C_i}$. Denote by $\overline{f}_i$, the extension of f_i to $X_i^\sim$. Then, from $1 - f_1 f_2 = 1 - f_1 + f_1(1 - f_2)$, we have $1 - \overline{f}_1\overline{f}_2 \leqslant (1 - \overline{f}_1) + (1 - \overline{f}_2)$ on $(X_1 \times X_2)^\sim$. This means $\lambda(1 - \overline{f}_1\overline{f}_2) \leqslant \mu_1(X_1 \setminus C_1) + \mu_2(X_2 \setminus C_2) \leqslant u_1 + u_2$. Because of the regularity of λ, taking limits over $\overline{f}_i$ as they decrease to χ_{C_i}, we get $\lambda((X_1 \times X_2)^\sim \setminus C_1 \times C_2) \leqslant u_1 + u_2$. This means $\dot{\lambda}_p((X_1 \times X_2)^\sim \setminus C_1 \times C_2) \leqslant 2c$. This proves $\lambda \in M_t^+(X_1 \times X_2, E)$.

A subset of E will be called absolutely convex if it is convex and circled; also, for a completely regular Hausdorff space X, a $Q \subset M_t^+(X, E)$ will be called uniformly bounded if there is a bounded set $B \subset E$ such that $\lambda(\mathcal{B}) \subset B, \forall \lambda \in Q$, $\mathcal{B}$ being the collection of all Borel subsets of X.

Theorem 2. *Suppose E is a semi-reflexive ordered locally convex space whose positive cone is normal, X_1 and X_2 Hausdorff completely regular spaces and Q a uniformly bounded, convex and closed subset of $M_t^+(X_1 \times X_2, E)$. For $i = 1, 2$, let $\mu_i \in M_t^+(X_i, E)$. Then there exists a $\lambda \in M_t^+(X_1 \times X_2, E)$ such that $\lambda^{(i)} = \mu_i$, $i = 1, 2$ iff for any finite collections $\{f_i\} \subset C_b(X_1), \{g_i\} \subset C_b(X_2)$ and $\{h_i\} \subset E'$ we have $\sum \langle(\mu_1(f_i) + \mu_2(g_i)), h_i\rangle \leqslant \sup\{\sum(\lambda(f_i \otimes 1 + 1 \otimes g_i), h_i), \lambda \in Q\}$.*

Proof. Using the fact that Q is uniformly bounded, take a weakly compact, absolutely convex subset B of E such that $\lambda(\mathcal{B}) \subset B$, $\forall \lambda \in Q$, $\mathcal{B}$ being the collection of all Borel subsets of $X_1 \times X_2$. Now assume Q is a subset of $M^+((X_1 \times X_2)^\sim, E)$ and let $\bar{Q}$ be its closure.

Let U be the closed unit ball of $C((X_1 \times X_2)^\sim)$ and, for $i = 1, 2$, let U_i be the closed unit ball of $C(X_i^\sim)$. Considering $\bar{Q} \subset E^U$ with product topology and with weak topology on E, we see that $\lambda(U) \subset B, \forall \lambda \in \bar{Q}$, and so $\bar{Q}$ is compact and convex. Further the condition of the theorem holds for Q iff it holds for $\bar{Q}$. For $i = 1, 2$, $\{\lambda^{(i)} \colon \lambda \in \bar{Q}\}$ is compact and convex in E^{U_i}. This means $Q_0 = \{(\lambda^{(1)}, \lambda^{(2)}) \colon \lambda \in \bar{Q}\} \subset E^{U_1} \times E^{U_2}$ is compact and convex. If $(\mu_1, \mu_2) = (\lambda^{(1)}, \lambda^{(2)})$ for some $\lambda \in Q$, the condition of the theorem is trivially satisfied.

To prove the converse, we consider, for $i = 1, 2$, μ_i to be elements of $M^+(X_i^\sim, E) \subset E^{C(X_i^\sim)}$. If $(\mu_1, \mu_2) \notin Q_0$, then by the separation theorem ([13], 9.2, p. 65), condition of the theorem does not hold (note that $\left(\prod_\alpha E_\alpha\right)' = \oplus_\alpha E'_\alpha$). Thus $(\mu_1, \mu_2) \in Q_0$. So there is a $\lambda \in \bar{Q}$ such that $(\mu_1, \mu_2) = (\lambda^1, \lambda^2)$. By Lemma 1, $\lambda \in M_t^+(X_1 \times X_2, E)$. Since Q is closed, $\lambda \in Q$.

Remark 3. The ordered locally convex space X_σ^*, with weak topology, considered in ([7], Theorem 1, p. 3294), is semi-reflexive, since X is assumed to be barrelled. In the above Theorem 2, we are not necessarily taking weak topology on E.

In an order complete vector lattice E, an order bounded net $\{x_\alpha\}$ is said to be order convergent to x if $y_\beta \downarrow x$ and $z_\beta \uparrow x$, where $y_\beta = \sup\{x_\alpha \colon \alpha \geqslant \beta\}$ and $z_\beta = \inf\{x_\alpha \colon \alpha \geqslant \beta\}$ ([13], p. 238).

In the next theorem we remove the condition that E is semi-reflexive but assume that E is an order complete locally convex vector lattice such that if an order-bounded net $\{x_\alpha\}$ order converges to $x \in E$, then $x_\alpha \to x$, in E. These assumptions on E imply that E is a complete sublattice of E'' ([13], 7.5, p. 239) and order the intervals in E are $\sigma(E, E')$ compact ([1], Theorem 11.13., p. 170). By ([13], 7.5, Corollary 1), if E is an order complete vector lattice whose order is regular and is of minimal type, then E with the order topology ([13], Sec. 6, p. 230) has the above property (in [13], p. 240, examples of these spaces are given).

Theorem 4. *Suppose E is an order complete locally convex vector lattice such that if an order bounded net $\{x_\alpha\}$ order converges to $x \in E$, then $x_\alpha \to x$ in E. Let X_1 and X_2 be Hausdorff completely regular spaces and let $\mu_i \in M_t^+(X_i, E)$ for $i = 1, 2$. Suppose Q is a uniformly bounded, convex and closed subset of $M_t^+(X_1 \times X_2, E)$. Then there exists a $\lambda \in Q$ such that $\lambda^{(i)} = \mu_i$, $i = 1, 2$, iff for any finite collections $\{f_i\} \subset C_b(X_1), \{g_i\} \subset C_b(X_2)$ and $\{h_i\} \subset E'$ we have $\sum \langle(\mu_1(f_i) + \mu_2(g_i)), h_i\rangle \leqslant \sup\{\sum(\lambda(f_i \otimes 1 + 1 \otimes g_i), h_i), \lambda \in Q\}$.*

P r o o f. Let E'' be the bidual of E; E can be considered a subspace of E''. On E'' we take the $\sigma(E'', E')$ topology. Since Q is uniformly bounded, take a compact, absolutely convex subset B of E'' such that $\lambda(\mathcal{B}) \subset B, \forall \lambda \in Q$, $\mathcal{B}$ being the collection of all Borel subsets of $X_1 \times X_2$. Now consider Q to be a subset of $M^+((X_1 \times X_2)^\sim, E'')$ (note that E'' has the $\sigma(E'', E')$ topology) and let $\bar{Q}$ be its closure; the elements of

$\bar{Q}$ are positive linear mappings from $C((X_1 \times X_2)^{\sim})$ to E''. Let U be the closed unit ball of $C((X_1 \times X_2)^{\sim})$, and for $i = 1, 2$, let U_i be the closed unit ball of $C(X_i^{\sim})$. Considering $\bar{Q} \subset (E'')^U$ with product topology, we see that $\lambda(U) \subset B$, $\forall \lambda \in \bar{Q}$ and so $\bar{Q}$ is compact and convex. Moreover the condition of the theorem holds for Q iff it holds for $\bar{Q}$. For $i = 1, 2$ $\{\lambda^{(i)} \colon \lambda \in \bar{Q}\}$ is compact and convex in $(E'')^{U_i}$.

This means $Q_0 = \{(\lambda^{(1)}, \lambda^{(2)}) \colon \lambda \in \bar{Q}\} \subset (E'')^{U_1} \times (E'')^{U_2}$ is compact and convex. If $(\mu_1, \mu_2) = (\lambda^{(1)}, \lambda^{(2)})$ for some $\lambda \in Q$ the condition of the theorem is trivially satisfied.

To prove the converse, we consider, for $i = 1, 2$, μ_i to be elements of $M^+(X_i^{\sim}, E'') \subset (E'')^{C(X_i^{\sim})}$. If $(\mu_1, \mu_2) \notin Q_0$, then by the separation theorem ([13], 9.2, p. 65) the condition of the theorem does not hold. Thus $(\mu_1, \mu_2) \in Q_0$. So there is a $\lambda \in \bar{Q}$ such that $(\mu_1, \mu_2) = (\lambda^1, \lambda^2)$. Let $\mu_1(1) = v$. This means $v \in E$ and $\lambda(1) = v$. Since E is an ideal in E'' the order interval $[0, v]$ in E'' is contained in E and is weakly compact in E. From the positivity of λ, it follows now for every $f \in U$, $1 \geqslant f \geqslant 0$, that $\lambda(f) \in [0, v]$. This means $\lambda \colon C((X_1 \times X_2)^{\sim}) \to E$ is positive and weakly compact. Thus λ is an E-valued regular Borel measure on $(X_1 \times X_2)^{\sim}$ having marginals μ_1 and μ_2. By Lemma 1, $\lambda \in M_t^+(X_1 \times X_2, E)$. Since Q is closed, this proves $\lambda \in Q$.

In the next theorem we establish the existence of a measure having given marginals, which is partially supported by a given closed set. This comes easily from Hahn-Banach type extension theroems discussed in ([12], Section 1.5, p. 43)

Theorem 5. *Suppose E is an order complete locally convex vector lattice such that if an order bounded net $\{x_\alpha\}$ order converges to $x \in E$, then $x_\alpha \to x$ in E. Let X_1 and X_2 be Hausdorff completely regular spaces. For $i = 1, 2$, let $\mu_i \in M_t^+(X_i, E)$ be such that $\mu_i(X_i) = v \in E$; also take a $\gamma \in E$, $0 < \gamma \leqslant v$, and a non-empty closed subset S of $X_1 \times X_2$. Then there exists a $\lambda \in M_t^+(X_1 \times X_2, E)$ such that $\lambda(S) \geqslant \gamma$ and $\lambda^{(i)} = \mu_i$, $i = 1, 2$, iff for any open subsets $V_i \subset X_i$ $(i = 1, 2)$, the condition $(V_1 \times X_2) \cap S \subset (X_1 \times V_2) \cap S$ implies $\mu_1(V_1) \leqslant \mu_2(V_2) + v - \gamma$.*

Proof. The condition is trivially necessary. For $i = 1, 2$, take $f_i \in C_b(X_i)$ such that $f_1(x) + f_2(y) \geqslant 0$ on $X_1 \times X_2$ and $f_1(x) + f_2(y) \geqslant 1$ on S. We claim that this condition implies that $\mu_1(f_1) + \mu_2(f_2) \geqslant \gamma$. By adjusting constants in f_1 and f_2, we can assume that $f_i \geqslant 0$ on X_i, $i = 1, 2$. To prove the claim, we can replace f_i by $\inf(f_i, 1)$. Thus we assume $0 \leqslant f_i \leqslant 1$.

Define, for any real t, $0 \leqslant t \leqslant 1$, $U_{1,t} = \{x \in X_1 \colon 1 - f_1(x) > t\}$, $U_{2,t} = \{y \in X_2 \colon f_2(y) > t\}$. By the given condition, $\mu_1(U_{1,t}) \leqslant \mu_2(U_{2,t}) + v - \gamma$. Integrating, we get $\int_0^1 \mu_1(U_{1,t})\,\mathrm{d}t \leqslant \int_0^1 \mu_2(U_{2,t})\,\mathrm{d}t + v - \gamma$ ([4], p. 392) and so $\mu_1(1 - f_1) \leqslant \mu_2(f_2) + v - \gamma$. This implies that $\mu_1(f_1) + \mu_2(f_2) \geqslant \gamma$ and so the claim is proved.

Consider $\mu_i \in M^+(X_i^\sim, E)$. Let $F = \{f \in C((X_1 \times X_2)^\sim)\colon f = f_1 + f_2,\ f_i \in C(X_i^\sim),\ i = 1,2\}$ (note that $C(X_1^\sim \times X_2^\sim)$ can be considered a subspace of $C((X_1 \times X_2)^\sim)$. F is a majorizing ([12], p. 47) subspace of $C((X_1 \times X_2)^\sim)$. Define $T_0\colon F \to E$, $T_0(f_1 + f_2) = \mu_1(f_1) + \mu_2(f_2)$. T_0 is a well-defined positive linear operator on F. Define $\theta\colon C((X_1 \times X_2)^\sim) \to E$, $\theta(f) = \inf\{T_0(g);\ g \in F,\ g \geqslant f\}$. It is easily verified that θ is monotone and sublinear and $\theta(f) = T_0(f), \forall f \in F$ ([12], p. 47, Corollary 1.5.9). Let $K = \{f \in C((X_1 \times X_2)^\sim),\ f \geqslant 0,\ f_{|S} \geqslant 1\}$. K is convex. Define $\tau\colon K \to E$, $\tau(k) = \gamma, \forall k \in K$. It is a obvious that τ is concave and $\tau(f) \leqslant \theta(f)$ on K. As in ([12], Lemma 1.51, p. 44), define $\varrho\colon C((X_1 \times X_2)^\sim) \to E$, $\varrho(f) = \inf\{\theta(f + tk) - t\tau(k)\colon t \in [0,\infty),\ k \in K\}$. As in ([12], Lemma 1.51, p. 44), ϱ is sublinear and $\varrho \leqslant \theta$. We claim that $T_0 \leqslant \varrho$ on F: fix an $f \in F$ and take a $k \in K$ and a $t \in (0,\infty)$. For any $g \in F$ with $g \geqslant f + tk$ we have $\frac{g-f}{t} \geqslant k$ and so $T_0(\frac{g-f}{t}) \geqslant \gamma$. This means $T_0(g) - t\tau(k) \geqslant T_0(f)$, $\forall t \in [0,\infty)$. This proves the claim.

As proved above, the mapping $T_0\colon F \to E$ satisfies the condition $T_0 \leqslant \varrho$. By ([12], Theorem 1.5.4, p. 45), it can be extended to a linear mappling $\lambda\colon C((X_1 \times X_2)^\sim) \to E$ such that $\lambda \leqslant \varrho$. This means ([12] Lemma 1.51., p. 44), $\lambda \leqslant \theta$ and, on K, $\lambda \geqslant \tau$. Now we will prove that λ is positive. Take an $f \leqslant 0$. Now $\lambda(f) \leqslant \theta(f) \leqslant \theta(0) = 0$ (note that θ is monotone). This proves that λ is positive. Since the order intervals in E are weakly compact, we prove that $\lambda\colon C((X_1 \times X_2)^\sim) \to E$ is a positive, weakly compact operator and so λ is an E-valued regular Borel measure on the compact Hausdorff space $(X_1 \times X_2)^\sim$ having marginals in $M_t^+(X_i, E)$ $(i = 1,2)$. By Lemma 1, $\lambda \in M_t^+(X_1 \times X_2, E)$. To prove $\lambda(S) \geqslant \gamma$, note $\lambda \geqslant \tau$ on K.

Remark 6. The assumptions made on E in the above theorem are satisfied when E is an order complete Banach lattice which is a KB-space ([12], Theorem 2.4.12, p. 92). So, in our setting, the above theorem is a generalization of ([5], Theorem 2).

We are very thankful to the referee for pointing out some typographical errors and also making some very useful suggestions which have improved the paper.

References

[1] *C. D. Aliprantis and O. Burkinshaw*: Positive Operators. Academic Press, 1985. Zbl 0608.47039

[2] *J. Diestel and J. J. Uhl*: Vector Measures. Amer. Math. Soc. Surveys, Vol. 15, Amer. Math. Soc., 1977. Zbl 0369.46039

[3] *L. Drewnowski*: Topological rings of sets, continuous set functions, integration I, II. Bull. Acad. Polon. Sci. Ser. Math. Astronom. Phys. *20* (1972), 269–276. Zbl 0249.28004

[4] *E. Hewitt and K. Stromberg*: Real and Abstract Analysis. Springer-Verlag, 1965. Zbl 0137.03202

[5] *A. Hirshberg and R. M. Shortt*: A version of Strassen's theorem for vector-valued measures. Proc. Amer. Math. Soc. *126* (1998), 1669–1671. Zbl 0904.28011

[6] *Hoffmann-Jorgensen*: Probability in Banach spaces. vol. 598, Lecture Notes in Math., Springer-Verlag, 1977, pp. 1–186. Zbl 0385.60006

[7] *Jun Kawabe*: A type of Strassen's theorem for positive vector measures in dual spaces. Proc. Amer. Math. Soc. *128* (2000), 3291–3300. Zbl 0956.28011

[8] *S. S. Khurana*: Extension and regularity of group-valued Baire measures. Bull. Acad. Polon. Sc., Ser. Math. Astro. Phys. *22* (1974), 891–895. Zbl 0275.28012

[9] *S. S. Khurana*: Topologies on spaces of continuous vector-valued functions. Trans. Amer. Math. Soc. *241* (1978), 195–211. Zbl 0335.46017

[10] *I. Kluvanek and G. Knowles*: Vector Measures and Control Systems. North-Holland, 1976. Zbl 0316.46043

[11] *D. R. Lewis*: Integration with respect to vector measures. Pac. J. Math. *33* (1970), 157–165. Zbl 0195.14303

[12] *Peter Meyer-Nieberg*: Banach Lattices. Springer-Verlag, 1991. Zbl 0743.46015

[13] *H. H. Schaefer*: Topological Vector Spaces. Springer Verlag, 1986. Zbl 0435.46003

[14] *V. Strassen*: The existence of probability measures with given marginals. Ann. Math. Statist. *36* (1965), 423–439. Zbl 0135.18701

[15] *V. S. Varadarajan*: Measures on topological spaces. Amer. Math. Soc. Transl. *48* (1965), 161–228. Zbl 0152.04202

Author's address: Department of Mathematics, Univerisity of Iowa, Iowa City, Iowa 52242, e-mail: `skhurana@blue.weeg.uiowa.edu`.