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CHARACTERIZATION OF LATTICES OF CONVEX SUBSETS OF POSETS

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Dedicated to Professor Ján Jakubík on the occasion of his seventy-fifth birthday

Systems of convex subsets of partially ordered sets, particularly those of convex sublattices of lattices, have been considered by many authors (see e.g. [1]–[6]). In this note we give necessary and sufficient conditions for a lattice to be isomorphic to the lattice of all convex subsets of a nonempty partially ordered set (Theorem 1.6). Such a lattice will be called a c-lattice. Further, we describe directly irreducible c-lattices and prove that each c-lattice is a direct product of directly irreducible c-lattices (Theorem 2.3).

Let $\mathbb{A} = (A, \leqslant)$ be a partially ordered set. A subset X of A is called convex if $x_1 \leqslant a \leqslant x_2$, $x_1, x_2 \in X$, $a \in A$ imply $a \in X$. Let $\operatorname{Conv} \mathbb{A}$ denote the system of all convex subsets of $\mathbb{A}$. The system $\operatorname{Conv} \mathbb{A}$, ordered by set-inclusion, is a complete lattice. Moreover, it is atomic in the sense that each element of $\operatorname{Conv} \mathbb{A}$ different from the empty set is the join of some atoms. If $X \subseteq A$, the symbol $[X]$ will be used for the least convex subset of $\mathbb{A}$ containing X. The set of all minimal and maximal elements of $\mathbb{A}$ is denoted by Min $\mathbb{A}$ and Max $\mathbb{A}$, respectively.

1. Characterization of $\operatorname{Conv} \mathbb{A}$

In this section we give necessary and sufficient conditions for a lattice to be isomorphic to $\operatorname{Conv} \mathbb{A}$ for a nonempty partially ordered set.

We start with some definitions.

Let $\mathbb{L} = (L, \leqslant)$ be a complete atomic lattice. An element $p \in L$ will be called *totally irreducible* if $p \leqslant \sup M, M \subseteq L$ imply $p \leqslant m$ for some $m \in M$.

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A complete lattice $\mathbb{L}$ will be said to be a *z-lattice* if each $a \in L$ is a join of totally irreducible elements of L.

By a *complete sublattice* of a complete lattice, a sublattice closed under arbitrary joins and meets will be meant.

Let $\mathbb{C} = (C, \leqslant)$ be a complete lattice and let 0 and 1 denote the least and greatest element, respectively. Suppose that $\mathbb{C}$ has a complete sublattice Z which is a z-lattice and contains 0 and 1. Denote by P the set of all totally irreducible elements of Z different from 0. Since $1 \in Z$, it is obvious that for any $c \in C$ the set $\{z \in Z\colon\ z \geqslant c\}$ has a least element. We denote it by $\downarrow c$.

Consider the following conditions:

(i) if $c \in C$, $\{p_i\colon\ i \in I\} \subseteq P$, then $c \wedge \sup\{p_i\colon\ i \in I\} = \sup\{c \wedge p_i\colon\ i \in I\}$;

(ii) if $p \in P$, $\{c_j\colon\ j \in J\} \subseteq C$ and $p \wedge c_j = 0$ for each $j \in J$, then $p \wedge \sup\{c_j\colon\ j \in J\} = 0$;

(iii) if $c, c' \in C$, $\downarrow c \leqslant \downarrow c'$ and the relations $p \in P$, $p \leqslant \downarrow c$, $p \wedge c' = 0$ imply $p \wedge c = 0$, then $c \leqslant c'$;

(iv) if $z_1, z_2 \in Z$, $z_1 \geqslant z_2$, $p \in P$, $p \leqslant z_1$ and c_0 is the greatest element of the set $\{c \in C\colon\ c \leqslant z_1, c \wedge z_2 = 0\}$, then $p \wedge c_0 = 0$ implies $p \leqslant z_2$ and $p \wedge c_0 > 0$ implies $p \leqslant \downarrow c_0$.

These conditions are not satisfied, in general. Let, e.g., $\mathbb{C}$ be as in Fig. 1, $Z = \{0, p, q, 1\}$. Then $P = \{p, q\}$ and (i) does not hold, while (ii) holds. If $\mathbb{C}$ is as in Fig. 2, $Z = \{0, p, q, 1\}$, then neither (i) nor (ii) is satisfied. On the other hand, if $\mathbb{C}$ is any infinitely distributive complete lattice and Z is any of its complete sublattices which is a z-lattice, both (i) and (ii) are satisfied. So, e.g., a three-element chain with $Z = \{0, 1\}$ satisfies (i), (ii), (iv), while (iii) does not hold. Let $\mathbb{C}$ be as in Fig. 3 with $Z = \{0, 1, p, q, r\}$. Then (i), (ii), (iii) hold but (iv) is not satisfied.

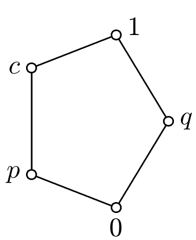

Fig. 1

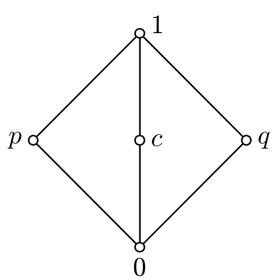

Fig. 2

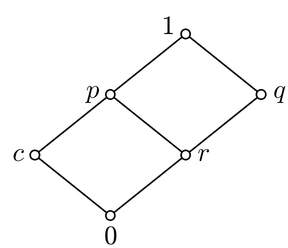

Fig. 3

Lemma 1.1. *Let* $\mathbb{C}$, Z, P *be as above and let the conditions* (i), (ii) *be satisfied. Then for any* $z_1, z_2 \in Z$, $z_1 \geqslant z_2$, *the set* $\{c \in C\colon\ c \leqslant z_1, c \wedge z_2 = 0\}$ *has a greatest element.*

Proof. Evidently $0 \in \{c \in C\colon c \leqslant z_1, c \wedge z_2 = 0\}$. Take $c_0 = \sup\{c \in C\colon c \leqslant z_1, c \wedge z_2 = 0\}$. Evidently $c_0 \leqslant z_1$. If $z_2 = 0$, then $c_0 \wedge z_2 = 0$ holds trivially. Let $z_2 > 0$. Then $z_2 = \sup\{p_i\colon i \in I\}$ for a nonempty subset $\{p_i\colon i \in I\}$ of P. The relation $c \wedge z_2 = 0$ implies $c \wedge p_i = 0$ for each $i \in I$. Thus $p_i \wedge c_0 = 0$ for each $i \in I$ by (ii). Using (i) we obtain $c_0 \wedge z_2 = \sup\{c_0 \wedge p_i\colon i \in I\} = 0$. □

Under the assumptions as in 1.1 let us denote the greatest element of the set $\{c \in C\colon c \leqslant z_1, c \wedge z_2 = 0\}$ by $z_1 - z_2$.

Lemma 1.2. *Let the assumptions of* 1.1 *be satisfied and let, moreover,* (iii) *hold. Then each $c \in C$ can be expressed as $c = z_1 - z_2$ for some $z_1, z_2 \in Z$, $z_1 \geqslant z_2$.*

Proof. Let $c \in C$. Denote $z_1 = {\downarrow}c$, $z_2 = \sup\{p \in P\colon p \leqslant z_1, p \wedge c = 0\}$ (by $\sup \emptyset$ the element 0 is meant). Evidently $z_2 \leqslant z_1, c \leqslant z_1, c \wedge z_2 = 0$, so that $z_1 - z_2 \geqslant c$. Now we are going to show, using (iii), that $z_1 - z_2 \leqslant c$ holds, too. The inequalities $c \leqslant z_1 - z_2 \leqslant z_1$ imply ${\downarrow}c \leqslant {\downarrow}(z_1 - z_2) \leqslant {\downarrow}z_1 = z_1 = {\downarrow}c$, so that ${\downarrow}(z_1 - z_2) = z_1 = {\downarrow}c$. Let $p \in P, p \leqslant {\downarrow}(z_1 - z_2) = z_1$ and $p \wedge c = 0$. Then $p \leqslant z_2$ and consequently $p \wedge (z_1 - z_2) = 0$, since $z_2 \wedge (z_1 - z_2) = 0$. The condition (iii) yields $z_1 - z_2 \leqslant c$. □

Notice that the elements z_1, z_2 in 1.2 are not determined uniquely. E.g., $1 - 1 = 0 - 0 = 0$.

Lemma 1.3. *Let the assumptions of* 1.2 *be satisfied and let, moreover,* (iv) *hold. Then the lattice $\mathbb{C} = (C, \leqslant)$ is isomorphic to $(\mathrm{Conv}(P, \leqslant), \subseteq)$ (the partial order in P being inherited from that in C).*

Proof. Let us define a mapping φ from C into the system of subsets of P by $c \in C$, $c = z_1 - z_2$, $z_1, z_2 \in Z$, $z_1 \geqslant z_2 \Longrightarrow \varphi(c) = \{p \in P\colon p \leqslant z_1, p \nleqslant z_2\}$. First we will show that this definition is correct. Let $c \in C$, $c = z_1 - z_2 = z_1' - z_2'$ for some $z_1, z_2, z_1', z_2' \in Z$, $z_1 \geqslant z_2$, $z_1' \geqslant z_2'$. Let $p \in P$, $p \leqslant z_1$, $p \nleqslant z_2$. Using (iv) we obtain $p \wedge c > 0$, $p \leqslant {\downarrow}c$. Obviously ${\downarrow}c \leqslant z_1'$, hence $p \leqslant z_1'$. If $p \leqslant z_2'$ held, we would have $p \wedge c = 0$, since $z_2' \wedge c = 0$, a contradiction. We have proved $\{p \in P\colon p \leqslant z_1, p \nleqslant z_2\} \subseteq \{p \in P\colon p \leqslant z_1', p \nleqslant z_2'\}$. The converse inclusion can be proved analogously.

Notice that if $c = z_1 - z_2$ for some $z_1, z_2 \in Z$, $z_1 \geqslant z_2$, then $\sup\{p \in P\colon p \leqslant z_1, p \wedge c > 0\} = {\downarrow}c$. Namely, we have $z_1 = \sup\{p \in P\colon p \leqslant z_1, p \wedge c = 0\} \vee \sup\{p \in P\colon p \leqslant z_1, p \wedge c > 0\}$, which implies $c = c \wedge z_1 = c \wedge \sup\{p \in P\colon p \leqslant z_1, p \wedge c > 0\} \leqslant \sup\{p \in P\colon p \leqslant z_1, p \wedge c > 0\}$ by (i). Now using (iv) we obtain $\sup\{p \in P\colon p \leqslant z_1, p \wedge c > 0\} \leqslant {\downarrow}c$ and consequently $\sup\{p \in P\colon p \leqslant z_1, p \wedge c > 0\} = {\downarrow}c$.

It is easy to see that $\varphi(c)$ is a convex subset of P. We are going to show that φ is onto. Let Q be any convex subset of P. Set $X = \{x \in P\colon x \leqslant q$ for some $q \in$

$Q\}, Y = X - Q$. Further, let $z_1 = \sup X$, $z_2 = \sup Y$. Obviously $z_1, z_2 \in Z$, $z_1 \geqslant z_2$.We are going to show that $\varphi(z_1 - z_2) = Q$. First, let $p \leqslant z_1$, $p \notin Q$. The relation $p \leqslant z_1$ yields $p \in X$, since p is totally irreducible, so that $p \in Y$. But then $p \leqslant z_2$. Thus $\{p \in P\colon\ p \leqslant z_1, p \nleqslant z_2\} \subseteq Q$. Now let $p \in Q$. Then $p \in X$, which implies $p \leqslant z_1$. Assume that $p \leqslant z_2$. Then $p \leqslant y$ for some $y \in Y$. But as $Y \subseteq X$, there exists $q \in Q$ with $y \leqslant q$. We have $p \leqslant y \leqslant q, p, q \in Q$, which implies $y \in Q$, a contradiction.

It remains to prove that if $c, c' \in C$, then

$$c \leqslant c' \text{ if and only if } \varphi(c) \subseteq \varphi(c').$$

Let $c, c' \in C$. Take $z_1 = \downarrow c$, $z_1' = \downarrow c'$, $z_2 = \sup\{p \in P\colon\ p \leqslant z_1, p \wedge c = 0\}$, $z_2' = \sup\{p \in P\colon\ p \leqslant z_1', p \wedge c' = 0\}$. We know that $c = z_1 - z_2$, $c' = z_1' - z_2'$. Now suppose that $c \leqslant c'$. Then evidently $z_1 \leqslant z_1'$. Take any $p \in P$ with $p \leqslant z_1$, $p \nleqslant z_2$. We have $p \leqslant z_1'$, $p \wedge c > 0$ and consequently $p \wedge c' > 0$, which implies $p \nleqslant z_2'$. We have proved $\varphi(c) \subseteq \varphi(c')$. Conversely, let $\varphi(c) \subseteq \varphi(c')$. First we will show that $z_1 \leqslant z_1'$. As we have noticed, we have $\sup\{p \in P\colon\ p \leqslant z_1, p \wedge c > 0\} = \downarrow c = z_1$, $\sup\{p \in P\colon\ p \leqslant z_1', p \wedge c' > 0\} = \downarrow c' = z_1'$. Since $\{p \in P\colon\ p \leqslant z_1, p \wedge c > 0\} = \{p \in P\colon\ p \leqslant z_1, p \nleqslant z_2\} \subseteq \{p \in P\colon\ p \leqslant z_1', p \nleqslant z_2'\} = \{p \in P\colon\ p \leqslant z_1', p \wedge c' > 0\}$, we have $z_1 \leqslant z_1'$. Further, $\varphi(c) \subseteq \varphi(c')$ implies also that if $p \leqslant z_1$, $p \wedge c' = 0$, then $p \wedge c = 0$. Using (iii) we infer $c \leqslant c'$. The proof is complete. $\square$

Now we are going to prove the converse.

Let $\mathbb{A} = (A, \leqslant)$ be any partially ordered set. Let us recall that $\mathcal{C} = (\operatorname{Conv} \mathbb{A}, \subseteq)$ is a complete lattice, $\emptyset$ is its least, A the greatest element. If $\{C_i\colon\ i \in I\} \subseteq \operatorname{Conv} \mathbb{A}$, then $\bigwedge\{C_i\colon\ i \in I\} = \bigcap\{C_i\colon\ i \in I\}$, $\bigvee\{C_i\colon\ i \in I\} = [\bigcup\{C_i\colon\ i \in I\}]$. Consider the system $\mathcal{Z}$ of all $Z \subseteq A$ which are down-closed, i. e. fulfil the condition

$$x \leqslant y, y \in Z \Longrightarrow x \in Z.$$

It is easy to see that $\mathcal{Z} \subseteq \operatorname{Conv} \mathbb{A}$ and that $(\mathcal{Z}, \subseteq)$ is a complete sublattice of $\mathcal{C}$ containing $\emptyset$ and A. By the way, if $\{Z_i\colon\ i \in I\} \subseteq \mathcal{Z}$, then $\vee\{Z_i\colon\ i \in I\} = \cup\{Z_i\colon\ i \in I\}$. It si also easy to verify that nonempty totally irreducible elements of $(\mathcal{Z}, \subseteq)$ are just the sets $(a\rangle = \{x \in A\colon\ x \leqslant a\}$ for all possible $a \in A$ and that each $Z \in \mathcal{Z}$ is the join of all $(z\rangle$, $z \in Z$. So we have proved

Lemma 1.4. *The complete sublattice* $(\mathcal{Z}, \subseteq)$ *of* $(\operatorname{Conv} \mathbb{A}, \subseteq)$ *is a z-lattice.*

Denote by $\mathcal{P}$ the system of all $(a\rangle$, $a \in A$.

Now it is clear that

(1) if $C \in \operatorname{Conv} \mathbb{A}, \{a_i\colon\ i \in I\} \subseteq A$, then $C \cap (\cup\{(a_i\rangle : i \in I\}) = \vee\{C \cap (a_i\rangle : i \in I\}$; and

(2) if $a \in A$, $\{C_j\colon\ j \in J\} \subseteq \operatorname{Conv}\mathbb{A}$, $(a\rangle \cap C_j = \emptyset$ for each $j \in J$, then $(a\rangle \cap (\bigvee\{C_j\colon j \in J) = \emptyset$.

If $C \in \operatorname{Conv}\mathbb{A}$, then evidently $\downarrow C = \{x \in A\colon$ there exists $c \in C$ with $x \leqslant c\}$. If $Z_1, Z_2 \in \mathcal{Z}$, $Z_1 \supseteq Z_2$, then the greatest element of the system $\{C \in \operatorname{Conv}\mathbb{A}\colon C \subseteq Z_1, C \cap Z_2 = \emptyset\}$ is $Z_1 - Z_2$ (in the set theoretical meaning). The following can be proved easily:

(3) if $C, C' \in \operatorname{Conv}\mathbb{A}$, $\downarrow C \subseteq \downarrow C'$ and $a \in A$, $(a\rangle \subseteq \downarrow C$, $(a\rangle \cap C' = \emptyset$ imply $(a\rangle \cap C = \emptyset$, then $C \subseteq C'$; and

(4) if $Z_1, Z_2 \in \mathcal{Z}$, $Z_1 \supseteq Z_2$, $a \in Z_1$, then $(a\rangle \cap (Z_1 - Z_2) = \emptyset$ implies $(a\rangle \subseteq Z_2$ and $(a\rangle \cap (Z_1 - Z_2) \neq \emptyset$ implies $(a\rangle \subseteq \downarrow(Z_1 - Z_2)$.

The above results can be summarized as follows:

Lemma 1.5. *If $\mathbb{A} = (A, \leqslant)$ is a partially ordered set, $\mathcal{C} = (\operatorname{Conv}\mathbb{A}, \subseteq)$, $\mathcal{Z}$ and $\mathcal{P}$ are as above, then the conditions* (i)–(iv) *are satisfied.*

Combining 1.3 and 1.5 we obtain the following theorem.

Theorem 1.6. *Let $\mathbb{C} = (C, \leqslant)$ be a complete lattice,* $\operatorname{card} C \geqslant 2$. *Then $\mathbb{C}$ is isomorphic to* $(\operatorname{Conv}\mathbb{A}, \subseteq)$ *for a partially ordered set $\mathbb{A}$ if and only if $\mathbb{C}$ has a complete sublattice Z containing the least and the greatest elements of C, which is a z-lattice, with the conditions* (i)–(iv) *being satisfied.*

2. Direct decomposition

If a lattice $\mathbb{L} = (L, \wedge, \vee, \leqslant)$ is isomorphic to $\operatorname{Conv}\mathbb{A}$ for a nonempty partially ordered set $\mathbb{A}$, we will refer to it as a *c-lattice.*

Theorem 2.1. *The direct product of any nonempty system of c-lattices is a c-lattice.*

Proof. Let $\{\mathbb{A}_i\colon\ i \in I\}$ be any nonempty system of partially ordered sets. Let $\mathbb{A}$ be their cardinal sum. It is easy to see that the mapping $X (\in \operatorname{Conv}\mathbb{A}) \mapsto (X \cap A_i)_{i \in I}$ is an isomorphism of the lattice $\operatorname{Conv}\mathbb{A}$ onto the direct product of the lattices $\operatorname{Conv}\mathbb{A}_i$ $(i \in I)$. □

Let $\mathbb{A} = (A, \preceq)$ be any partially ordered set. Denoting by S the set of all couples $(u, v) \in A \times A$ such that $u \in \operatorname{Min}\mathbb{A}$, $v \in \operatorname{Max}\mathbb{A}$ and v covers u, define

$$a \preceq_c b (a, b \in A) \Leftrightarrow a \preceq b, \quad (a, b) \notin S.$$

It is easy to see that $\preceq_c$ is a partial order in A and $\mathrm{Conv}(A,\preceq) = \mathrm{Conv}(A,\preceq_c)$. The order $\preceq_c$ will be said to be the *c-order corresponding to* $\preceq$.

Theorem 2.2. *Let $\mathbb{A} = (A,\preceq)$ be any partially ordered set. The lattice* $\mathrm{Conv}\,\mathbb{A}$ *is directly irreducible if and only if the partially ordered set $(A,\preceq_c)$ is connected.*

Proof. If $(A,\preceq_c)$ is disconnected, then there exist nonempty subsets B, C of A such that $(A,\preceq_c)$ is the cardinal sum of $(B,\preceq_c)$ and $(C,\preceq_c)$. But then $\mathrm{Conv}\,\mathbb{A} = \mathrm{Conv}(A,\preceq_c)$ is isomorphic to $\mathrm{Conv}(B,\preceq_c)\times\mathrm{Conv}(C,\preceq_c)$, so that $\mathrm{Conv}\,\mathbb{A}$ is directly reducible.

Conversely, let $\mathrm{Conv}\,\mathbb{A}$ be directly reducible, i.e. there exist lattices $\mathbb{L}_1, \mathbb{L}_2$, each containing at least two elements, and an isomorphism $\varphi\colon \mathrm{Conv}\,\mathbb{A} \to \mathbb{L}_1 \times \mathbb{L}_2$. Evidently $\mathbb{L}_1$, $\mathbb{L}_2$ are complete atomic lattices. φ maps atoms of the lattice $\mathrm{Conv}\,\mathbb{A}$ into atoms of the direct product $\mathbb{L}_1\times\mathbb{L}_2$. Set $A_1 = \{a \in A\colon \varphi(\{a\}) = (p,0)$ for an atom p of $\mathbb{L}_1\}$, $A_2 = \{a\in A\colon \varphi(\{a\}) = (0,q)$ for an atom q of $\mathbb{L}_2\}$. Evidently $A_1, A_2 \neq \emptyset$, $A_1\cup A_2 = A$. Let $\mathbb{A}_1$ and $\mathbb{A}_2$ be A_1 and A_2, respectively, with the order inherited from $(A,\preceq_c)$. The aim is to show that $(A,\preceq_c)$ is the cardinal sum of $\mathbb{A}_1$ and $\mathbb{A}_2$, which will imply that $(A,\preceq_c)$ is disconnected. We have to prove that if $a_1\in A_1$, $a_2\in A_2$, then a_1, a_2 are incomparable in $(A,\preceq_c)$. Let $a_1\in A_1$, $a_2\in A_2$, $\varphi(\{a_1\}) = (p_1,0), \varphi(\{a_2\}) = (0,q_1)$. As $\varphi([\{a_1,a_2\}]) = \varphi(\{a_1\}\vee\{a_2\}) = \varphi(\{a_1\})\vee\varphi(\{a_2\}) = (p_1,0)\vee(0,q_1) = (p_1,q_1)$ and $(p_1,0),(0,q_1)$ are the only atoms in $\mathbb{L}_1\times\mathbb{L}_2$ which are less than (p_1,q_1), the elements a_1, a_2 are incomparable or one of them covers the other in $(A,\preceq_c)$. Assume, e.g., that a_2 covers a_1. By the definition of $\preceq_c$ there exists $a\in A$ such that either $a\preceq_c a_1$, $a\neq a_1$, or $a_2\preceq_c a$, $a\neq a_2$. Let, e.g., the first possibility occur. Then $\{a_1\}\subset[\{a,a_2\}]$, which implies $(p_1,0) = \varphi(\{a_1\}) < \varphi([\{a,a_2\}]) = \varphi(\{a\})\vee\varphi(\{a_2\})$. Now $\varphi(\{a\})$ is of the form $(p,0)$ or $(0,q)$, so that $(p_1,0) < (p,q_1)$ or $(p_1,0) < (0,q_1\vee q_2)$, respectively. The first inequality implies $p_1 = p$, which contradicts $a\neq a_1$. The latter case is also impossible. So a_1, a_2 are incomparable and the proof is complete. $\square$

Theorem 2.3. *Every c-lattice is the direct product of directly irreducible c-lattices.*

Proof. Let $\mathbb{A} = (A,\preceq)$ be any partially ordered set, $\preceq_c$ the c-order corresponding to $\preceq$. Let $\mathbb{A}_i = (A_i,\preceq_c)$ $(i\in I)$ be maximal connected subsets of $(A,\preceq_c)$. Then the lattice $\mathrm{Conv}\,\mathbb{A} = \mathrm{Conv}(A,\preceq_c)$ is isomorphic to the direct product of $\mathrm{Conv}\,\mathbb{A}_i$ and all $\mathrm{Conv}\,\mathbb{A}_i$ are directly irreducible c-lattices by 2.2. $\square$

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