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A POLE ASSIGNMENT TECHNIQUE FOR MULTIVARIABLE SYSTEMS WITH INPUT DELAY

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A design procedure is established for pole allocation in linear multivariable systems with delay in control. A relationship is obtained which permits a straightforward calculation of the feedback matrix to attain prescribed closed-loop poles. An example illustrating the concept involved is included.

1. INTRODUCTION

The pole assignment method for non-delay multivariable systems has received a great deal of attention for designing feedback controllers to achieve desired objectives [1], [2]. Suh and Bien [3] have considered a root locus technique for linear systems with time-delay. Here is an attempt to present a pole assignment method for multi-input systems with input delay. The single-input system with delay is first considered and the results are then extended to multi-input system using unity-rank state feedback matrices.

2. SINGLE-INPUT SYSTEMS WITH INPUT DELAY

Consider a controllable single-input system with input delay described by

$$\dot{x}(t) = A\,x(t) + b\,u(t - \alpha)\,, \tag{1}$$

where x is the n-state vector, u is the scalar input. A and b are constant $n \times n$ and $n \times 1$ matrices, respectively, and α is the constant delay. The transfer-function representation of (1) is given by

$$X(s) = \frac{g(s)\exp(-\alpha s)}{f(s)}\,U(s)\,,$$

where $g(s) = \operatorname{adj}(sI - A)\,b$ is the $n \times 1$ non-delay numerator polynomial vector and $f(s) = |sI - A|$ is the characteristic polynomial of the open-loop system. If state variable feedback $u = v - kx$, where v is the command input and k is the $1 \times n$ state feedback vector, is now applied, the characteristic polynomial of the closed-loop system becomes

$$H(s) = |sI - A + b \exp(-\alpha s) k| .$$

It has been shown that [1], [2]

$$H(s) = |sI - A + \hat{b}k| = |sI - A| + k \operatorname{adj}(sI - A)\, \hat{b} . \tag{2}$$

On substituting $\hat{b} = b \exp(-\alpha s)$ in (2), we obtain

$$H(s) = f(s) + k\, g(s) \exp(-\alpha s) . \tag{3}$$

$H(s) = 0$ is a transcendental equation in s and may have an infinite number of roots. But it is known that the number of zeros of $H(s)$, each of whose real part is greater than any given real number, is finite if $g(s)/f(s)$ is strictly proper rational and all zeros of $H(s)$ except some finite number around the origin lie in the left half of the s-plane [3], [4]. Thus only a finite number of roots near the origin need to be considered. Now for $\lambda_1, \lambda_2, \ldots, \lambda_n$ to be roots of $H(s)$, we require

$$H(\lambda_i) = f(\lambda_i) + k\, g(\lambda_i) \exp(-\lambda_i \alpha) = 0 , \quad i = 1, 2, \ldots, n . \tag{4}$$

From (4) the n elements of the state feedback vector k which positions the n roots at $\lambda_1, \ldots, \lambda_n$ can be found.

3. MULTI-INPUT SYSTEMS WITH DELAY

Consider a cyclic and controllable multi-input system with delay in control described by

$$\dot{x} = A\, x(t) + B\, u(t - \alpha) , \tag{5}$$

where x is the $n \times 1$ state vector, u is the $m \times 1$ control vector, A and B are constant matrices of appropriate dimensions, and α is a constant delay. Taking the Laplace transform from (5) we obtain

$$s\, X(s) = A\, X(s) + B \exp(-\alpha s)\, U(s) .$$

Hence

$$X(s) = [sI - A]^{-1} B \exp(-\alpha s)\, U(s) = \frac{G(s)}{F(s)}\, U(s) ,$$

where $G(s) = \operatorname{adj}[sI - A]\, B \exp(-\alpha s)$ is the $n \times m$ numerator polynomial matrix and $F(s) = |sI - A|$ is the open-loop characteristic polynomial.

On applying state feedback $u = v - Kx$, where v is the $m \times 1$ command input vector and K is the $m \times n$ state feedback matrix, the closed-loop system matrix becomes

$$A_C = A - B \exp(-\alpha s)\, K .$$

The design problem is to determine the state feedback matrix K such that the closed-loop system matrix A_C has n specified eigenvalues $\lambda_1, \dots, \lambda_n$. The closed-loop eigenvalues are roots of the characteristic polynomial

$$H(s) = |sI - A + B\exp(-\alpha s) K| . \quad (6)$$

The $m \times n$ state feedback matrices K considered in this paper are restricted to have unity-rank by predefining them in the dyadic structure $K = qk$, where q and k are $m \times 1$ and $1 \times n$ vectors, respectively. Under this restriction, (6) can be simplified to

$$H(s) = |sI - A + B\exp(-\alpha s)\, qk| = |sI - A + b_1 k| ,$$

where $b_1 = B\exp(-\alpha s)\, q$ is an $n \times 1$ vector. The restriction of the $m \times n$ state feedback matrix K to have unity rank thus reduces the multi-input system to an 'equivalent' single-input system. The design is now carried out as follows:

(1) Choose an $m \times 1$ vector q such that the equivalent single-input system (A, b_1) is controllable.

(2) Find the $1 \times n$ state feedback vector k for this single-input system.

(3) Calculate the $m \times n$ state feedback matrix $K = qk$ for the multi-input system.

A numerical example is now given for illustration.

3.1. Example. Consider the system

$$\dot{x} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 1 & -1 \\ 3 & -5 \end{bmatrix} u(t - 0.1) .$$

Find the unity-rank state feedback matrix $K = qk$ to place two poles at -2, -3. On choosing $q = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, the controllable equivalent single-input system becomes

$$\dot{x} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \mu(t - 0.1) ,$$

where μ is the scalar input for this system. We now find the state feedback vector k to place two poles of this system at -1, -3. The transfer function representation of the system is

$$X(s) = [sI - A]^{-1}\, b\exp(-\alpha s)\, \mu(s)$$
$$= \frac{1}{s^2 - 1}\begin{bmatrix} s + 1 \\ s - 1 \end{bmatrix} \exp(-0.1s)\, \mu(s) .$$

On applying $\mu = kx$, the closed-loop characteristic polynomial becomes

$$H(s) = s^2 - 1 + k_1(s + 1)\exp(-0.1s) + k_2(s - 1)\exp(-0.1s) .$$

To place the poles at -2, -3, we require

$$H(-2) = 0 , \quad 3 - \exp(0.2)\, k_1 - 3\exp(0.2)\, k_2 = 0$$
$$H(-3) = 0 , \quad 8 - 2\exp(0.3)\, k_1 - 4\exp(0.3)\, k_2 = 0 .$$

The feedback gains k_1 and k_2 are found to be

$$k_1 = -6\exp(-0.2) + 12\exp(-0.3)$$
$$k_2 = 3\exp(-0.2) - 4\exp(-0.3).$$

The feedback matrix for the multivariable system is

$$K = qk = \begin{bmatrix} 2k_1 & 2k_2 \\ k_1 & k_2 \end{bmatrix}.$$

4. CONCLUSION

In this paper the pole assignment method for non-delay systems has been extended to linear multivariable systems with input delay. Further work on the extension to multivariable systems with delay in state and output feedback is presently under study.

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