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Angeliki Kontolatou

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The Cauchy extension of an ordered abelian group realized by r-valuations

A. Kontolatou

Abstract. Our aim is to study the topological Cauchy extension of a directed abelian group realized by a family of r-valuations and topologized by their kernels, considering them as neighborhoods of zero. It is proved that the basic scheme of the realization and the embedding of the given directed topological group into a topological lattice preserve their character, in a sufficient grade, in this Cauchy extension.

1. Introduction

It is our main purpose to see some algebraic consequences of the fact that a po-group which may be realized by a system of r-valuations, becomes a topological group by inducing a topology. That topology is defined by the kernels of the r-valuations of the group, which form a base of zero's neighborhoods and thus this group admits a uniform structure; hence, one of our first tasks is to study the uniform completion of this structure.

Since in our procedure the notions of the *r-valuations*, of the *r-Lorenzen group*, of the *theory of divisors* (or of the *quasi-divisors*, or of the *strong theory of quasi-divisors*) play a central role , the *r-ideals* remain main tools of our paper. We recall in brief the relative notions.

Consider a directed abelian po-group G. By an *r-system* of ideals we mean a map $r : X \to X_r$ (X_r is called an *r-ideal*) from the set of all lower bounded subsets X of G into the power set of G, which satisfies the following conditions:

1) $X \subseteq X_r$
2) $X \subseteq Y_r \Rightarrow X_r \subseteq Y_r$
3) $\{\alpha\}_r = \alpha \cdot G^+ = (\alpha)$, for all $\alpha \in G$
4) $\alpha \cdot X_r = (\alpha \cdot X)_r$, for all $\alpha \in G$

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An r-ideal X_r is said to be *finite* if it is finitely generated, that is, if $X_r = Y_r$, Y finite.

We mention two special ideal systems, the *v-system* which is defined by

$$X_v = \bigcap_{X \subseteq (y),\ y \in G} (y)$$

and the *t-system* which is defined by

$$X_t = \bigcup_{Y \subseteq X,\ Y\ finite} Y_v\ .$$

An r-system is said to be *of finite character* if, for every r-ideal X_r,

$$X_r = \bigcup_{Y \subseteq X,\ Y\ finite} Y_r\ .$$

An o-homomorphism ϕ from a po-group G_1 with an r-system r_1, into a po-group G_2 with an r-system r_2 is an (r_1, r_2)*-morphism* if $\phi(X_{r_1}) \subseteq (\phi(X))_{r_2}$ for any lower bounded subset X. If G_2 is totally ordered (i.e. an o-group) and ϕ is surjective, then ϕ is called an r_1*-valuation* if it is an (r_1, t)-morphism. Sometimes t-valuations will be simply called *valuations*.

By a *defining family* of valuations on G, we understand a set of valuations on G such that

$$G^+ = \bigcap_{w \in W} w^{-1}(G_w^+).$$

A defining family W of valuations defines (c.f. [6], p.47) an ideal system r according to the formula (X is a lower bounded subset of G):

$$(\forall g \in G)\ [g \in X_r] \Leftrightarrow (\forall w \in W)[w(g) \in (w(X))_t].$$

For any lower bounded subset $X \subseteq G$, the set of the subsets X_r,

$$X_r = \cup\{K_r / K \subseteq X,\ \ K \text{ finite}\},$$

where r is the system defined by W, is an ideal system of finite character on G.

The notion of a *theory of divisors* was introduced by Borevich and Shafarevich ([3], ch. 3) as a map h from the group of divisibility G of an integral domain A into a free abelian group $Z^{(I)}$(considered as an l-group with pointwise ordering) satisfying the following three axioms:

1) $(\forall \alpha, \beta\ \in\ G)\ \alpha \leq_G \beta$ if and only if $h(\alpha) \leq h(\beta)$ in $Z^{(I)}$

2) If $h(\alpha) \geq \gamma$ and $h(\beta) \geq \gamma$, then $h(\alpha + \beta) \geq \gamma$

3) If $\gamma \in Z^{(I)}$, then there exist $g_1, \ldots, g_n \in G$ such that $\gamma = h(g_1) \wedge \cdots \wedge h(g_n)$.

L. Skula ([11]) introduced the notion of a *theory of divisors* for a partially ordered group (as a very natural generalization of a *theory of divisors* for rings) and K.E. Aubert ([2], p.339) gave a generalization of the notion, introducing the *theory of quasi-divisors*. Recall that a theory of divisors of a directed po-group G is a map h from G into a free abelian group $Z^{(I)}$ satisfying the above axioms (1) and (3). We also recall (c.f. [5], p.222) that a directed po-group $(G, \cdot)$ has a theory of quasi-divisors if there exists a lattice group $(\Gamma, \cdot)$ and a map $h : G \to \Gamma$ such that:

(1) h is an order preserving isomorphism from G into Γ

(2) $(\forall\alpha \in \Gamma_+)(\exists g_1, \dots, g_n \in G^+)[\alpha = h(g_1) \wedge \dots \wedge h(g_n)]$.

K. Gudlaugsson ([2], p.339) demonstrated that a directed po-group G has a theory of quasi-divisors if and only if the monoid $(I_t^f, \times)$ of finitely generated t-ideals is a group. Thus, the so called *r-Lorenzen group* (c.f. [2], p. 330) of G is such a lattice ordered group. A stronger version of a theory of quasi-divisors has been introduced in [9], where an order isomorphism h from a directed group G into a lattice group Γ is called a ***strong theory of quasi-divisors***, if

$$(\forall\alpha, \beta \in \Gamma_+)(\exists\gamma \in \Gamma_+)[\alpha \cdot \gamma \in h(G) \text{ and } \beta \wedge \gamma = 1].$$

It has been proved that any strong theory of quasi-divisors is a theory of quasi-divisors as well.

The basic scheme we work on has as follows: We start with a directed abelian po-group $(G, +, \leq)$. We consider on G an infinite defining family of valuations W, the r-ideal system on G defined by W and $h : G \to \Gamma$ an o-embedding of G into a lattice group Γ (for instance, Γ may be the r-Lorenzen group of G). It is well known that there is a defining family $\hat{W}$ of r-valuations on Γ with the property:

"for every $w \in W$ there is a $\hat{w} \in \hat{W}$ such that, for any $g \in G$, $w(g) = \hat{w}(h(g))$"
and we call $\hat{W}$ an *extension* of W.

On the other hand, as we have already mentioned it, G (Γ as well) is equipped with a topology T_W (resp. $T_{\hat{W}}$), where the $\ker w_i$ (resp. $\ker \hat{w}_i$) constitute a base of zero's neighborhoods on G (resp. on Γ). For any $X \subseteq G$ (the things for Γ are similar), we put

$$cl_{T_W} X = \{g \in G / (\forall I \subseteq W,\ I \text{ finite})(\exists x \in X)(\forall i \in I)\ [w_i(x) = w_i(g)]\},$$

where $cl_{T_W} X$ expresses the closure of X for the topology T_W.

In a recent paper ([10]) J. Močkoř proved a series of statements referred to the sequence:

$$G \xrightarrow{h} \Gamma \xrightarrow{\hat{w}} G_w \longrightarrow 0$$

and to the topologies T_W and $T_{\hat{W}}$, where h is a strong theory of quasi-divisors. Unfortunately, the Cauchy process does not secure the transfer from the given scheme to the extended one of all the desired characteristics of the sets and maps. It is our aim to investigate which properties are extensible.

The second paragraph is devoted to the Cauchy completion of a po-group $(G, +, \leq)$ realized by r-valuations and to the comparison of the uniform topology with the topology of the kernels. We also give some basic characteristics of the extended structures. Similar is our trial in the third paragraph with an emphasis on whether our basic scheme remains unchanged during the completion.

2. The completion by Cauchy systems

2.1. The Cauchy systems on an ordered group realized by r-valuations

Let $(G, +, \leq, T_W)$ be a directed abelian topological group, where T_W is the topology of the kernels of $w \in W$ and W is a defining family of r-valuations w, defined on G

and ranging onto a totally ordered abelian group G_w . Let also be $\mathbf{B}$ a base of the zero's neighborhoods for T_W. It is known that, if $(g_\alpha)_{\alpha\in A}$ is a Cauchy net of the space, then

$$(\forall B \in \mathbf{B})(\exists\alpha_0 \in A)(\forall\alpha,\beta \geq \alpha_0)\ [g_\alpha - g_\beta \in B]$$

and since B is the kernel of an r-valuation w, we have " for every α, $\beta \geq \alpha_0$, $w(g_\alpha) = w(g_\beta)$ ".

In the usual way we define the completion by Cauchy nets of G and in an evident manner G^* becomes a directed abelian topological group.

If $\{g_\alpha\} \in G^*$ with representative $(g_\alpha)_{\alpha\in A}$, then the family $\mathbf{B}^* = \{u^*/u \in \mathbf{B}\}$, where

$$u^* = \{\{g_\alpha\} \in G^*/(\exists u \in \mathbf{B})(\exists\alpha_0)(\forall\alpha \geq \alpha_0)\ [g_\alpha \in u]\} \tag{1}$$

is a zero's neighborhood in G^*,inducing a topology T_{G^*}.

On the other hand if we equip every G_w, $w \in W$, with the discrete topology, w becomes a continuous function and for any Cauchy net $(g_\alpha)_\alpha$ and any $w \in W$, the net $(w(g_\alpha))_{\alpha\in A}$ is a Cauchy net as well, that is, there is an $\alpha_0 \in A$ such that $w(g_\alpha) = g \in G_w$, for every $\alpha \geq \alpha_0$. We put $w^*(\{g_\alpha\}) = g \in G^*$, thus, we have formed a family W^* of r-valuations, whose the kernels form a base of zero's neighborhoods in G^*, inducing a second topology, say T_{W^*}, on G^*.

2.2. First consequences of the Cauchy completion

We always consider the structures $(G, +, \leq, W)$, G a directed abelian po-group, with the topology T_W (*the topology of the kernels of* W) and $(G^*, +, \leq, W^*)$, with the topologies T_{G^*} and T_{W^*}, as we have described them above.

Proposition 1. The topologies T_{W^*} and T_{G^*} coincide.
Proof. Let $\mathbf{B}$ be a base of the zero's neighborhoods on G for the topology T_W and $\mathbf{B}^*$, $\tilde{\mathbf{B}}$ be, respectively, bases of the zero's neighborhoods on G^* for the topologies T_{G^*}, T_{W^*}. For any $u^* \in \mathbf{B}^*$ and any $\underline{g} = \{g_\alpha\} \in u^*$, there are $u \in \mathbf{B}$ and α_0 such that if $\alpha \geq \alpha_0$, then $g_\alpha \in u$. It means that u is a kernel for an r-valuation $w_i \in W$, hence, for any $\alpha \geq \alpha_0$, $w_i(g_\alpha) = 0$ or $w_i^*(\underline{g}) = 0$, hence the same elements $\underline{g}$ of G^* which form the set u^*, form a zero's neighborhood, element of $\tilde{\mathbf{B}}$; it is the kernel of w_i. The rest are obvious.

From this proposition we take the fruitful result that the topology of G^* is T_1. It is also evident that $\bigcap_{w^*\in W^*} \ker w^* = \{0_{G^*}\}$. Thus we state:

Proposition 2. The family W^* is a defining family of r-valuations on G^*.

Proposition 3. The space (G^*, T_{W^*}) is zero-dimensional.
Proof. Let $\ker w^*$ be a zero's neighborhood for an r-valuation $w^* \in W^*$ and $x \in \ker w^*$. Then, for every $\alpha \in \ker w^*$, we have $w^*(x) = w^*(\alpha) = 0$, hence $x \in \ker w^*$.

2.3. The Cauchy completion of a lattice

We concern (as we have mentioned in the introduction) with a theory of divisors $h : G \to \Gamma$ (or a theory of quasi-divisors, or a strong theory of quasi-divisors), where $(G, +, \leq, W)$ is the given structure, Γ a lattice ordered group, Γ admits (c.f. [6], cor. 1 of th. 5) a family $\hat{W}$ of r-valuations, extensions of the r-valuations of W, which have on G the same values with the r-valuations of W.

There holds:

Proposition. The Cauchy completion of a lattice topological group is a lattice topological group as well.
Proof. Let be Γ a lattice ordered abelian group, α^*, β^* two points of its Cauchy completion Γ^* and $(\alpha_n)_n$, $(\beta_n)_n$ two Cauchy sequences in Γ converging respectively to them. Since the r-valuations preserve suprema (c.f. ([2], §4,p.331), $\hat{w}_i(\alpha_n) \vee \hat{w}_i(\beta_n) = \hat{w}_i(\alpha_n \vee \beta_n)$. On the other hand, Γ is topologically dense into Γ^* and the sequence $(\hat{w}_i(\alpha_n \vee \beta_n))_n$ is finally constant, which means that $(\alpha_n \vee \beta_n)_n$ is a Cauchy sequence, thus it converges to a point $\gamma^* \in \Gamma^*$, which is the supremum of α^* and β^*.

2.4.Remark. In general, the completion G^* of a topological group G is not Hausdorff (hence, the uniqueness of f^* is not guaranteed), even if G is Hausdorff itself. However, the space G^*/S, where S is the minimum closed subgroup of G^*, is a Hausdorff complete space. In the case we discuss about, e^* is the neutral element of G^*, the space G^* is T_1 (in fact is $T_{3\frac{1}{2}}$) and since $S = \{e^*\}$, G^*/S is homeomorphic to G^* (c.f. [7], p.110, pr.8.26 and [1], p.243).

2.5. Given a directed abelian topological group $(G, +, \leq, W)$, its Cauchy completion $(G^*, +, \leq, W^*)$ is a directed topological group, too.
In fact it is an easy consequence, after the following:

Lemma. Every Cauchy sequence of G is bounded.

Proposition. The Cauchy extension G^* of G is a directed group.
Proof. Since every Cauchy sequence is bounded, it has a bounded limit, which is an element of G. Thus, given two points of G^*, they are bounded from elements of G, hence, they are bounded from an element of G^*.

2.6. Some more consequences on the Cauchy completion

We always refer to the Cauchy completion of the structure $(G, +, \leq, W)$.

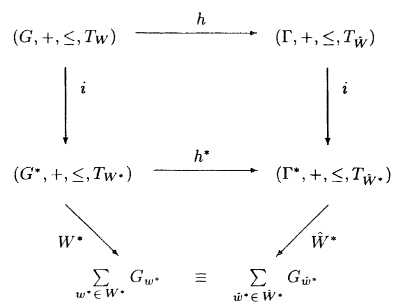
Proposition.
(a) W^* is the infimum semilattice generated by the given W.
(b) $G^*_+ \cap G = G_+$.
(c) The kernels of each w^*, $w^* \in W^*$, are convex sets.
Proof. (a) (c.f. [10]).

(b) It is $G_+ \subseteq G^*_+ \cap G$. On the other hand, every element of G^*_+ is the limit of a Cauchy net, which finally has positive terms. Thus, if $\alpha \in G^*_+ \cap G$, then α is a positive element.

(c) Let be $\ker w^* = H$, $w^* \in W^*$ and $x_1, x_2 \in H$. For every $y \in H$ with $x_1 \le y \le x_2$, we have $0 = w^*(x_1) \le w^*(y) \le w^*(x_2) = 0$. Hence, $y \in H$.

3. The basic scheme

3.1. We describe below the basic scheme to which we refer. We start from the structure $(G, +, \le, W)$ of a directed abelian group realized by a defining family W of r-valuations, which, via a theory of divisors (or a theory of quasi-divisors or a strong theory of quasi-divisors), is corresponded to another structure $(\Gamma, +, \le, \hat{W})$ of a lattice abelian group and we complete these two structures by Cauchy systems, taking the structures $(G^*, +, \le, W^*)$ and $(\Gamma^*, +, \le, \hat{W}^*)$, respectively. Thus we have the diagram:



3.2. We may define the map h^* on G^* as follows: if $g \in G$, we put $h^*(g) = h(g)$. If $\underline{g} = \{g_\alpha\} \in G^* - G$, then $(h(g_\alpha))_\alpha$ is a Cauchy net in Γ (c.f. [7], p.110, prop. 8.26), hence it is an element $\{h(g_\alpha)\} \in \Gamma^*$. We put $h^*(\underline{g}) = \{h(g_\alpha)\} \in \Gamma^*$.

There holds the following. (We refer to the notation of the above diagram. The first result is independent of whether h is a theory of divisors or of quasi-divisors or is a strong theory of quasi-divisors).

3.3. Proposition. The map h is continuous and moreover is uniformly continuous.
Proof: We prove that for any $A \subseteq G$, $h(clA) \subseteq cl(h(A))$.

Let $y \in h(clA)$. There is an $x \in clA$,hence an $\alpha \in A$ such that $y = h(x)$ and $w(\alpha) = w(x)$ for any $w \in W$. On the other hand, $\hat{w}(h(x)) = w(x) = \hat{w}(h(\alpha)) = w(\alpha)$ and the element $h(\alpha) \in h(A)$, thus $\hat{w}(h(\alpha)) = \hat{w}(h(x)) = \hat{w}(y)$. Hence, h is continuous.

Let now U_Γ be an entourage of the uniform structure of Γ. Then, because of the continuity of h, a neighborhood $U(0)$ corresponds to the neighborhoods $U_\Gamma(0_\Gamma)$ (where 0_Γ is the zero of Γ), such that $h(U(0)) \subseteq U_\Gamma(0_\Gamma)$. At the arbitrary element $h(g) \in \Gamma$ and for the neighborhood $U_\Gamma(h(g))$, we have that

$$h(U(g)) = h(U(0)) + h(g) \subseteq U_\Gamma(h(g)) = U_\Gamma(0_\Gamma) + h(g).$$

Remark: The *θ-continuity* is a principle weaker than that of continuity. More precisely, for $h : G \to \Gamma$, h is θ-continuous in $x_0 \in G$, if

$$(\forall\, V_{h(x_0)})(\exists\, V_{x_0})[h(cl\, V_{x_0}) \subseteq cl\, V_{h(x_0)}]$$

for every $x_0 \in G$, V_{x_0} and $V_{h(x_0)}$ neighborhoods of x_0 and $h(x_0)$, respectively. It is an easy task to be demonstrated directly the θ-continuity of h. On the other hand, for a regular space, continuity and θ-continuity coincide. Here, G is completely regular, that is, we get the continuity of h via the θ-continuity of it.

3.4. Remark. After proposition 1 of section 2.4, every Cauchy net $(g_\alpha)_\alpha$ in X is transferred to a Cauchy net $(k_\alpha)_\alpha$ by h; if $\{g_\alpha\} = g^* \in G^*$, then $\{k_\alpha\}$ defines a $k^* \in \Gamma^*$ and we put

$$h^*(g^*) = k^*.$$

Since Γ contains elements which are not images of G's elements via h, the completion of Γ contains elements not belonging to $h^*(G^*)$.

The r-valuations on Γ^* are defined as the ones on G^*. It is also evident that the set of valuations $\hat{W}^*$ on Γ^* is a defining family as well.

3.5. Proposition. With the notation of our basic scheme,there holds:

The map h^* is uniformly continuous and the completions G^* and Γ^* are unique.
Proof. Since h is continuous and the structures are uniform ones, the first result is concluded from [7], p.152 and the second from [1], p.243.

3.6. Proposition. If h is a strong theory of quasi-divisors, then

$$cl_{\Gamma^*}(h^*(G^*)) = \Gamma^*.$$

Proof. We preserve the notation of the basic scheme. From [10], theor. 2.9, we take that $cl_\Gamma(h(G)) = \Gamma$. On the other hand, $cl_{\Gamma^*}(\Gamma) = \Gamma^*$ and $h^*(G^*) \supseteq h(G)$, hence $cl_{\Gamma^*}(h^*(G^*)) \supseteq cl_{\Gamma^*}(h(G)) \supseteq cl_\Gamma(h(G)) = \Gamma$ and finally $cl_{\Gamma^*}(cl_{\Gamma^*}(h^*(G^*))) \supseteq cl_{\Gamma^*}\Gamma = \Gamma^*$.

3.7. Remarks. (1) Although we have proved that, in the case where h is a strong theory of quasi divisors, $h^*(G^*)$ is dense into Γ^* , h^* is not, in general, a strong theory of quasi divisors, a result which we should expect after the theorem 2.9 of [10]. The reason is that such a result is in valid, if the defining families W^* and

$\hat{W}^*$ of G^* and Γ^* respectively, are of finite character. But such a conclusion is not always true as the following example shows:

Let us consider the defining family of p-adic valuations $\{w_p\}_{p\in P}$, P the set of prime numbers of the group $(Q \setminus \{0\}, .)$. This defining family is of finite character, but the family of the corresponded extended p-adic valuations is not of finite character.

More precisely: Considering the p-adic valuations as r-valuations on Q^*, any element of Q^*, has values different of 0 for a finite number of these valuations are of finite character. It is an easy task to find a Cauchy sequence for which the family of p-valuations is not of finite character. Consider, for instance,the sequence $(p^a.\beta)_\beta$ of numbers, where a is a fixed positive number, p prime and β runs through the set of prime numbers. This sequence is a Cauchy one,defines a new element, say a^* (not a real number), for which the family of p-adic valuations is not of finite character, since, for infinitely many of them, a takes value different of 0.

(2) In a structure $(G, +, \leq, W)$ of a directed abelian group endowed with an r-system x of finite character and W a defining family of x-valuations of G, we say, as usual, that W satisfies the *PWAT (Positive weak approximation theorem)* (c.f. [8], p. 84) if:

for any $w_1, w_2, \ldots, w_n$ in W and any "compatible system" $(g_1, g_2, \ldots, g_n) \in \prod_i G^+_{w_i}$, there is a $g \in G^+$ such that $w_i(g) = g_i$, $i \in \{1, 2, \ldots, n\}$.

It is known (see for instance [8], p.84) that the first approximation theorem was the following one, due to T.Nakano: *If Γ is an l-group and $\hat{W}$ a defining family of t-valuations of Γ, then W satisfies the PWAT.*

Let us, now, consider our basic scheme and assume that the o-homomorphism h of $(G, +, \leq, W)$ into $(\Gamma, +, \leq, \hat{W})$ is a strong theory of quasi-divisors. After Nakano's theorem, if $(g_1, g_2, \ldots, g_n) \in \prod_i G^+_{\hat{w}_i}$ is a compatible family for the x-valuations $\hat{w}_1, \hat{w}_2, \ldots, \hat{w}_n$, elements of $\hat{W}$, then there is a $k \in \Gamma$ such that $\hat{w}_i(k) = g_i$.

On the other hand, since h is a strong theory of quasi-divisors, the defining family W of x-valuations of G fulfils the PWAT as well, that is, for the above compatible system and x-valuations, there is an $s \in G$ such that $w_i(s) = g_i$, $i \in \{1, 2, \ldots, n\}$ and $\hat{w}_i$ are the above extensions of w_i.

Next, consider the structure $(G^*, +, \leq, W^*)$, again in our basic scheme and a compatible family $(g_1, g_2, \ldots, g_n) \in \prod_i G^+_{w_i}$ corresponded to $w_1^*, w_2^*, \ldots, w_n^*$, respectively. If $(g_1, \ldots, g_n)$ is compatible with respect to $w_1^*, \ldots, w_n^*$ then it is compatible with respect to $w_1, \ldots, w_n$ as well, and there exists $g \in G \subseteq G^*$ such that $w_i^*(g) = w_i(g) = g_i$ for all i.

(3) If $g_1, g_2, \ldots, g_n$ is a compatible family referred to the t-valuations $\hat{w}_1^*, \hat{w}_2^*, \ldots, \hat{w}_n^*$, elements of $\hat{W}^*$, then, from the above relation, $\hat{w}_i^* \circ h^*(s) = w_i^*(s) = g_i$,hence the defining family $\hat{W}^*$ fulfils the PWAT as well.

(4) Although the defining family $\hat{W}^*$ fulfils the PWAT we can't say that the strong theory of quasi-divisors h is extended to another strong theory of quasi-divisors h^*. Such a result is true when the h^*-image of G^* is dense into Γ^* (compare with the above first remark), but at the same time it must $\hat{W}^*$ be of finite character.

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Angeliki Kontolatou, Associate Professor, Department of Mathematics, University of Patras, Patras 26500, Greece
E-mail address: Kontolat@math.upatras.gr