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Special Incidence Structures of Type (p, n) *

Part II

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Abstract

This paper is the second part of [5]. We examine special incidence structures of type (p, n) in which the conditions $R^i = R^{i+1}$ and $a'_i \not I\, m'_i$ are valid for a certain $i \in \{0, \dots, n-1\}$.

Key words: Incidence structures, independent sets.

2000 Mathematics Subject Classification: 06B05, 08A35

This paper is a continuation of [5]. Thus we use the denotation, the numbering of propositions, theorems, figures and enclosures from [5]. We examine special incidence structures of type (p, n) in which the conditions $R^i = R^{i+1}$ and $a'_i \not I\, m'_i$ are valid for certain $i \in \{0, \dots, n-1\}$. Such incidence structures satisfy the conditions either from Proposition 4 or from Proposition 5 of [5]. In [5] there are all special incidence structures of type (p, n) of the first kind described. In what follows we consider special incidence structures $\mathcal{J}$ of type (p, n) satisfying the conditions from Proposition 5. Hence, accepting the denotation from [5], we assume that $k = l$, $a'_i, a_{i+2}\ I\ b$ and $B^{i+2} = \{b, m_{i+1}\} \cup (Q^i - \{n_k\})$, $B^{i-1} = \{b, m'_i\} \cup (Q^i - \{n_k\})$.

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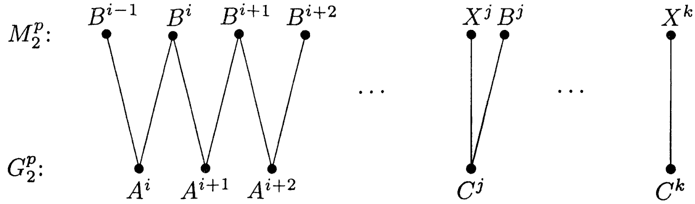
Let $\mathcal{J}_2 = (G_2, M_2, I_2)$ be a substructure of $\mathcal{J}$ where $G_2 = G_1, M_2 = M_1 \cup \{b\}$. If we put $B_j = \{b, m_i', m_{i+1}\} \cup (Q^i - \{n_k, n_j\})$ for each $j \in \{1, \ldots, p-1\} - \{k\}$, then the graph of the incidence structure $\mathcal{J}_2^p$ has a form



Since $\mathcal{J}$ is of type (p, n) there exist $A^{i+3}, A^{i-1} \in G^p$ such that $A^{i+3}\ I^p\ B^{i+2}$ and $A^{i-1}\ I^p\ B^{i-1}$. Furthermore, $A^{i+3}, A^{i-1} \notin G_1^p$ and there exist elements $d \in A^{i+3}$, $e \in A^{i-1}$ such that $d, e \in G - G_1$.

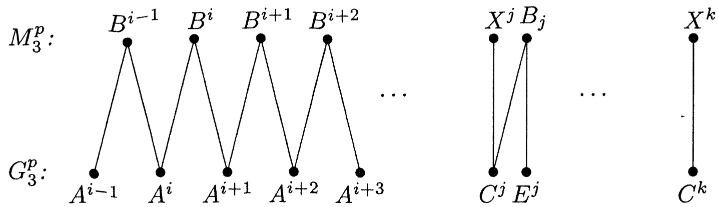
Proposition 6 $A^{i+3} = \{d, a_{i+2}\} \cup (R^i - \{g_u\})$, $A^{i-1} = \{e, a_i'\} \cup (R^i - \{g_{u'}\})$.

Proof From $A^{i+2} = \{a_{i+2}\} \cup R^i$ and $|A^{i+2} \cap A^{i+3}| = p - 1$ we get either $A^{i+3} = \{d\} \cup R^i$ or $A^{i+3} = \{d, a_{i+2}\} \cup (R^i - \{g_u\})$ for a certain $u \in \{1, \ldots, p-1\}$. Similarly, from $A^i = \{a_i'\} \cup R^i$ and $|A^{i-1} \cap A^i| = p - 1$ we obtain either $A^{i-1} = \{e\} \cup R^i$ or $A^{i-1} = \{e, a_i'\} \cup (R^i - \{g_{u'}\})$.

First let us suppose that $A^{i+3} = \{d\} \cup R^i$. Because of $A^{i+3}\ I^p\ B^{i+2}$ there exists a norming mapping $\alpha : A^{i+3} \to B^{i+2}$ in which $\alpha(d) = m_{i+1}, \alpha(g_k) = b$ and $\alpha(g_j) = n_j$ for $j \neq k$. If $d\ I\ n_k$, then $A^{i+3}\ I^p\ B^{i+1}$ which is a contradiction. Thus $d \not{I}\, n_k$. If $d \not{I}\, m_i'$, then $A^{i+3}\ I^p\ B^{i-1}$ which is a contradiction. If $d\ I\ m_i'$, then $A'\ I^p\ B^i$ where $A' = \{d, a_i'\} \cup (R^i - \{g_k\})$. This is a contradiction again.

In a similar way one can show that $A^{i-1} = \{e\} \cup R^i$ does not hold. Hence $A^{i+3} = \{d, a_{i+2}\} \cup (R^i - \{g_u\})$, $A^{i-1} = \{e, a_i'\} \cup (R^i - \{g_{u'}\})$. □

Proposition 7 *Let A^{i+3}, A^{i-1} be given according to Proposition 6. Then $u = k$ if and only if $u' = k$. If $u = k$, then $d = e$ and for a substructure $\mathcal{J}_3 = (G_3, M_3, I_3)$ of $\mathcal{J}$ where $G_3 = G_1 \cup \{d\}$, $M_3 = M_1 \cup \{b\}$ the $\mathcal{J}_3^p$ has the following graph:*



Furthermore, $E_j = \{d, a_i', a_{i+1}\} \cup (R^i - \{g_k, g_j\})$ for all $j \neq k$.

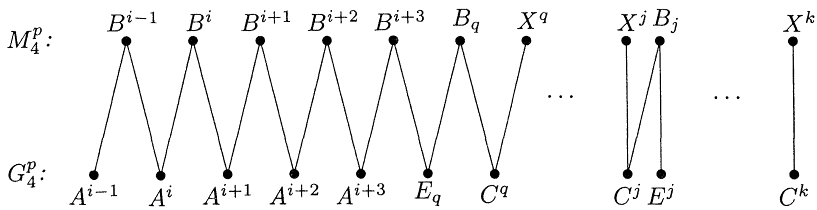
Proof Let $u = k$. There exists a norming mapping $\beta : B^{i+2} \to A^{i+3}$ in which $\beta(b) = d$, $\beta(m_{i+1}) = a_{i+2}$ and $\beta(n_j) = g_j$ for $j \neq k$. If $d \not{I}\, n_k$, then $A^{i+3}\ I^p\ B^{i+1}$ which is a contradiction. Thus $d\ I\ n_k$. If $d \not{I}\, m_i'$, then $A'\ I^p\ B^i$

where $A' = \{d\} \cup R^i$. This is a contradiction again. Hence $d\ I\ m_i'$. It means that $A'\ I^p\ B^{i-1}$ where $A' = \{d, a_i'\} \cup (R^i - \{g_k\})$. Since $A' \neq A^i$ we get $A^{i-1} = \{d, a_i'\} \cup (R^i - \{g_k\}) = \{e, a_i'\} \cup (R^i - \{g_{u'}\})$ and thus $u' = k$, $d = e$. If we put $E_j = \{d, a_i', a_{i+2}\} \cup (R^i - \{g_k, g_j\})$ for $j \neq k$, then $E_j\ I^p\ B_j$. □

Enclosure 11 shows the described situation in the case of $p = 5$ and $k = 2$.

1. First assume that $u = k$. Thus $A^{i+3} = \{d, a_{i+2}\} \cup (R^i - \{g_k\})$ and $A^{i-1} = \{d, a_i'\} \cup (R^i - \{g_k\})$. From $b \neq m_i', m_{i+1}$ we have $B^{i+2} \neq X^j$ and $B^{i-1} \neq X^j$ for all $j \in \{1, \dots, p-1\}$. Since $\mathcal{J}$ is of type (p, n) there exists either B^{i+3} or B^{i-2}.

Proposition 8 *If B^{i+3} exists, then $B^{i+3} = \{x, b, m_{i+1}\} \cup (Q^i - \{n_k, n_q\})$ where $x \in M - (M_1 \cup \{b\})$. Let $\mathcal{J}_4 = (G_4, M_4, I_4)$ be a substructure of $\mathcal{J}$ with $G_4 = G_1 \cup \{d\}$, $M_4 = M_1 \cup \{b, x\}$. Then $\mathcal{J}_4^p$ has a graph*



Proof Let B^{i+3} exist. Then $B^{i+3} \not\subseteq M_1 \cup \{b\}$ and there exists $x \in B^{i+3}$, $x \notin M_1 \cup \{b\}$. Since $B^{i+2} = \{b, m_{i+1}\} \cup (Q^i - \{n_k\})$ and $|B^{i+2} \cap B^{i+3}| = p - 1$ we get either $B^{i+3} = \{x, b\} \cup (Q^i - \{n_k\})$ or $B^{i+3} = \{x, m_{i+1}\} \cup (Q^i - \{n_k\})$ or $B^{i+3} = \{x, b, m_{i+1}\} \cup (Q^i - \{n_k, n_q\})$. There exists a norming mapping $\alpha : A^{i+3} \to B^{i+3}$ because $A^{i+3}\ I^p\ B^{i+3}$.

a) Assume that $B^{i+3} = \{x, b\} \cup (Q^i - \{n_k\})$. Then $\alpha(a_{i+2}) = x$, $\alpha(d) = b$ and $\alpha(g_j) = n_j$ for $j \neq k$. If $g_k\ I\ x$, then $A^{i+2}\ I^p\ B'$ where $B' = \{x\} \cup R^i$, a contradiction. Thus $g_k \not I\ x$. If at the same time $a_i' \not I\ x$, then $A^{i-1}\ I^p\ B^{i+3}$ which is a contradiction. In the case $a_i'\ I\ x$ we have $A^i\ I^p\ B'$ where $B' = \{x, m_i'\} \cup (Q^i - \{n_k\})$ and this is a contradiction again.

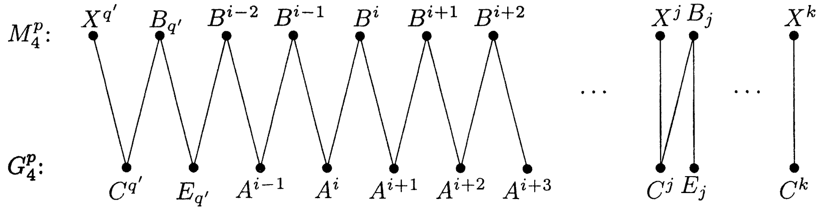
b) Assume that $B^{i+3} = \{x, m_{i+1}\} \cup (Q^i - \{n_k\})$. Then $\alpha(a_{i+2}) = m_{i+1}$, $\alpha(d) = x$ and $\alpha(g_j) = n_j$ for $j \neq k$. If $g_k \not I\ x$, then $A^{i+2}\ I^p\ B^{i+3}$ which is a contradiction. Thus $g_k\ I\ x$. If $a_i' \not I\ x$, then $A^i\ I^p\ B'$ where $B' = \{x\} \cup Q^i$ which is a contradiction. Thus $a_i'\ I\ x$. If $a_{i+1} \not I\ x$, then $A^{i+1}\ I^p\ B'$ where $B' = \{x\} \cup Q^i$ and this is a contradiction. Let $a_{i+1}\ I\ x$. This implies $A^{i-1}\ I^p\ \{x, m_i'\} \cup (Q^i - \{n_k\})$ whence $B^{i-2} = \{x, m_i'\} \cup (Q^i - \{n_k\})$. Therefore, $A'\ I^p\ B^{i+3}, B^{i-2}$ for $A' = \{d, a_{i+2}\} \cup (R^i - \{g_k\})$ which is a contradiction again.

c) According to a), b) we have $B^{i+3} = \{x, b, m_{i+1}\} \cup (Q^i - \{n_k, n_q\})$. Then $\alpha(a_{i+2}) = m_{i+1}$, $\alpha(d) = b$, $\alpha(g_q) = x$ and $\alpha(g_j) = n_j$ for $j \neq k, q$. If $g_k\ I\ x$, then $A^{i+2}\ I^p\ B^{i+3}$ which is a contradiction. Thus $g_k \not I\ x$. Let $a_i'\ I\ x$. Then $C^q\ I^p\ B'$ where $B' = \{x, m_i', m_{i+1}\} \cup (Q^i - \{n_k, n_q\})$ which is a contradiction because $B' \neq X^q, B_q$. Thus $a_i' \not I\ x$. It means that $E_q\ I^p\ B^{i+3}$ and the incidence structure $\mathcal{J}_4^p$ has the graph presented in the proposition. □

The validity of Proposition 8 does not depend on the incidence of elements a_{i+1} and x.

Enclosures 12, 13 show the situation for $p = 5, k = 2$ and $q = 1$. There are $a_{i+1}\ I\ x$ at Encl. 12 and $a_{i+1} \not I\ x$ at Encl. 13.

Proposition 9 *If B^{i-2} exists, then $B^{i-2} = \{y, b, m'_i\} \cup (Q^i - \{n_k, n_{q'}\})$ where $y \in M - (M_1 - \{b\})$. If $\mathcal{J}_4 = (G_4, M_4, I_4)$ is a substructure of $\mathcal{J}$ with $G_4 = G_1 \cup \{d\}$ and $M_4 = M_1 \cup \{b, y\}$, then $\mathcal{J}_4^p$ has a graph*



The proof is similar to Proposition 8. In this case $g_k \not I\ y$, $a_{i+2} \not I\ y$.

Remark 4 If B^{i+3} and B^{i-2} exist, then $q \neq q'$ and $x \neq y$.

Theorem 9 *Let $\mathcal{J}_5 = (G_5, M_5, I_5)$ be a substructure of $\mathcal{J}$ with $G_5 = G_1 \cup \{d\}$, $M_5 = M_1 \cup \{b, x, y\}$. If $E_j\ I^p\ B$ where $B \not\subseteq M_5$, then $E_r\ I^p\ B$ for a certain $r \neq q, q', j$ and $B_j \cap B = B \cap B_r$.*

Proof It follows from $E_q\ I^p\ B^{i+3}, B_q$ and $E_{q'}\ I^p\ B^{i-2}, B_{q'}$ that $j \neq q, q'$. Since $B \not\subseteq M_5$ there exists $z \in B$, $z \notin M_5$. It holds $E_j\ I^p\ B_j, B$ and thus $|B \cap B_j| = p - 1$. From $B_j = \{b, m'_i, m_{i+1}\} \cup (Q^i - \{n_k, n_j\})$ we get $B = \{z, m'_i, m_{i+1}\} \cup (Q^i - \{n_k, n_j\})$ or $B = \{z, b, m'_i\} \cup (Q^i - \{n_k, n_j\})$ or $B = \{z, b, m_{i+1}\} \cup (Q^i - \{n_k, n_j\})$ or $B = \{z, b, m'_i, m_{i+1}\} \cup (Q^i - \{n_k, n_j, n_r\})$. There exists a norming mapping $\alpha : E_j \to B$ because $E_j\ I^p\ B$.

a) Let $B = \{z, m'_i, m_{i+1}\} \cup (Q^i - \{n_k, n_j\})$. Then $\alpha(d) = z$, $\alpha(a_{i+2}) = m_{i+1}$, $\alpha(a'_i) = m'_i$ and $\alpha(g_l) = n_l$ for $l \neq k, j$. If $g_k \not I\ z$, then $C^j\ I^p\ B$ which is a contradiction. Hence $g_k\ I\ z$. If $g_j \not I\ z$, then $A^i\ I^p\ B'$ where $B' = \{z, m'_i\} \cup (Q^i - \{n_k\})$. This is a contradiction again. Finally, $g_j\ I\ z$ implies $A^{i+3}\ I^p\ B'$ where $B' = \{z, m_{i+1}\} \cup (Q^i - \{n_k\})$ and this is a contradiction.

b) Let $B = \{z, b, m'_i\} \cup (Q^i - \{n_k, n_j\})$. Then $\alpha(d) = b$, $\alpha(a_{i+2}) = z$, $\alpha(a'_i) = m'_i$ and $\alpha(g_l) = n_l$ for $l \neq k, j$. If $g_j \not I\ z$, then $A^{i-1}\ I^p\ B'$ where $B' = \{z, b, m'_i\} \cup (Q^i - \{n_k, n_q\})$ which is a contradiction. Let $g_k \not I\ z$. Then $A^i\ I^p\ B'$ where $B' = \{z, m'_i\} \cup (Q^i - \{n_k\})$. This is a contradiction again. If $g_k\ I\ z$, then $A^{i+2}\ I^p\ B'$ where $B' = \{z\} \cup Q^i$ which is a contradiction.

c) Let $B = \{z, b, m_{i+1}\} \cup (Q^i - \{n_k, n_j\})$. Then $\alpha(d) = b$, $\alpha(a_{i+2}) = m_{i+1}$, $\alpha(a'_i) = z$ and $\alpha(g_l) = n_l$ for $l \neq k, j$. If $g_j \not I\ z$, then $A^{i+3}\ I^p\ B'$ where $B' = \{z, b, m_{i+1}\} \cup (Q^i - \{n_k, n_q\})$ which is a contradiction. Hence $g_j\ I\ z$. If $g_k \not I\ z$, then $A^{i+2}\ I^p\ B'$ where $B' = \{z, m_{i+1}\} \cup (Q^i - \{n_k\})$ and this is a contradiction again. If $g_k\ I\ z$, then $A^i\ I^p\ B'$ where $B' = \{z\} \cup Q^i$ which is a contradiction.

d) According to a)–c) we obtain $B = \{z, b, m'_i, m_{i+1}\} \cup (Q^i - \{n_k, n_j, n_r\})$. Then $\alpha(d) = b$, $\alpha(a_{i+2}) = m_{i+1}$, $\alpha(a'_i) = m'_i$, $\alpha(g_r) = z$ and $\alpha(g_l) = n_l$ for $l \neq k, j, r$. Let $g_k \ I \ z$. Then $C^j \ I^p \ B'$ where $B' = \{z, m'_i, m_{i+1}\} \cup (Q^i - \{n_k, n_j\})$ which is a contradiction because $B' \neq X^j, B_j$. Thus $g_k \not{I} \ z$. Let $g_j \ I \ z$. Then $A^{i+3} \ I^p \ B'$ where $B' = \{z, b, m_{i+1}\} \cup (Q^i - \{n_k, n_r\})$ which is a contradiction. Hence $g_k, g_j \not{I} \ z$ which yields $E_r \ I^p \ B$. If $r = q$, then from $B \neq B^{i+3}, B_q$ we obtain a contradiction. Thus $r \neq q$. Similarly $r \neq q'$. It is easy to see that $B_j \cap B = \{b, m'_i, m_{i+1}\} \cup (Q^i - \{n_k, n_j, n_r\}) = B \cap B_r$. □

Theorem 10 *If $C^k \ I^p \ B$, then $B = X^k$.*

Proof Let us suppose that $C^k \ I^p \ B$ and $B \neq X^k$. Then $B \not\subseteq M_1 \cup \{b, x, y\}$ and thus there exists $v \in B$, $v \notin M_1 \cup \{b, x, y\}$. It follows from $C^k \ I^p \ X^k, B$ that $|B \cap X^k| = p - 1$. Hence $B = \{v, m'_i\} \cup (Q^i - \{n_k\})$ or $B = \{v, m_{i+1}\} \cup (Q^i - \{n_k\})$ or $B = \{v, m'_i, m_{i+1}\} \cup (Q^i - \{n_k, n_r\})$. There exists a norming mapping $\alpha : C^k \to B$.

a) Let $B = \{v, m'_i\} \cup (Q^i - \{n_k\})$. Then $\alpha(a_{i+2}) = v$, $\alpha(a'_i) = m'_i$ and $\alpha(g_j) = n_j$ for $j \neq k$. If $g_k \not{I} \ v$, then $A^i \ I^p \ B'$ where $B' = \{v, m'_i\} \cup (Q^i - \{n_k\})$ which is a contradiction because of $B' \neq B^i, B^{i-1}$. If $g_k \ I \ v$, then $A^{i+2} \ I^p \ B'$ where $B' = \{v\} \cup Q^i$. This is a contradiction again.

b) If $B = \{v, m_{i+1}\} \cup (Q^i - \{n_k\})$, we obtain a contradiction similarly to the case a).

c) Let $B = \{v, m'_i, m_{i+1}\} \cup (Q^i - \{n_k, n_r\})$. Then $\alpha(a_{i+2}) = m_{i+1}$, $\alpha(a'_i) = m'_i$, $\alpha(g_r) = v$ and $\alpha(g_j) = n_j$ for $j \neq k, r$. If $g_k \ I \ v$, then $A^i \ I^p \ B'$ where $B' = \{v, m'_i\} \cup (Q^i - \{n_r\})$ which is a contradiction. If $g_k \not{I} \ v$, then $C^r \ I^p \ B'$ which is a contradiction again.

It follows from a), b), c) that $B = X^k$. □

Let us suppose that there does not exist the set B^{i-2}. Then the set B^{i+3} exists and the incidence structure $\mathcal{J}_4^p$ has the graph from Proposition 8.

At the same time $A^{i-1} = A^0$, thus $i = 1$. Let us put $L = \{1, \ldots, p-1\}$ again and let $L' = L - \{k, q\}$. If $E_j \in G_4^p$, $j \neq q$, then there exists a set $B \subseteq M - M_4$ such that $E_j \ I^p \ B$. By Theorem 9, there exists $E_r \subseteq G^p$ (where $r \in L'$, $r \neq j$) such that $E_r \ I^p \ B$.

Let us put $Y_j := B$ and $r = \xi(j)$. By this a bijective mapping ξ of the set L' is assigned which is involutory. Let $\varphi : L \to L$ is the mapping from Theorem 7. If we put $\varphi(q) = q_2$, then a set $A_q \in G^p$ exists such that $A_q \ I^p \ X^q, X^{q_2}$. If $q_2 \neq k$, then $C^{q_2} \ I^p \ B_{q_2}$ and $E_{q_2} \ I^p \ B_{q_2}$. (See Figure 5 where the graph of the substructure $\mathcal{J}_1^p$ is emphasized.)

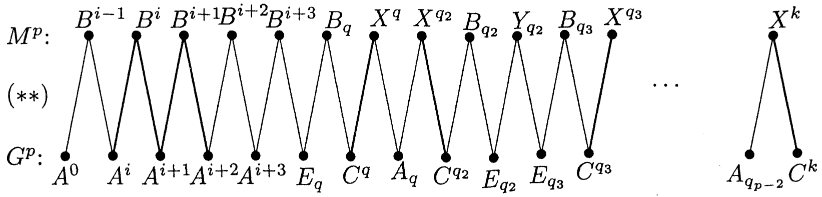


Figure 5

There exists $Y_{q_2} \in M^p$ such that $E_{q_2}\ I^p\ Y_{q_2}$ and, by Theorem 9, there exists $E_{q_3} \in M^p$, $E_{q_3}\ I^p\ Y_{q_2}$ where $\xi(q_2) = q_3$. Then $E_{q_3}\ I^p\ B_{q_3}, C^{q_3}\ I^p\ B_{q_3}, X^{q_3}$ and $A_{q_3} \in G^p$ exists with $A_{q_3}\ I^p\ X^{q_3}$ and $A_{q_3}\ I^p\ X^{q_4}$ where $\varphi(q_3) = q_4$. If $q_4 \neq k$, then we proceed in the same way until we get $E_{q_{p-2}}\ I^p\ B_{q_{p-2}}, C^{q_{p-2}}\ I^p\ X^{q_{p-2}}$, $A_{q_{p-2}}\ I^p\ X^{q_{p-2}}, X^{q_{p-1}}$ and $C^{q_{p-1}}\ I^p\ X^{q_{p-1}}$ where $q_{p-1} = k$ and $C^{q_{p-1}} = A_n$ by Theorem 10.

If the set B^{i-2} exists and B^{i+3} does not, then $A^{i+3} = A^n$ and we proceed analogously to the previous case, using q' instead of q.

Let us assume that there exist both sets B^{i-2}, B^{i+3}. Then we put $L'' = L - \{k, q, q'\}$. Consider mappings $\varphi : L \to L$, $\xi : L'' \to L''$ described in the first case. Let us put

$$\underbrace{\varphi\xi\varphi\ldots\xi\varphi}_{l}(q) = q_{l+1} \quad \text{and} \quad \underbrace{\varphi\xi\varphi\ldots\xi\varphi}_{r}(q') = q'_{r+1}$$

for $l \in \{1, \ldots, u\}$ and $r \in \{1, \ldots, v\}$ where $u + v + 2 = p - 1$. Then either $k = u + 1$ or $k = v + 1$. Suppose that $k = u + 1$. Then, by Theorem 10, $C^k = A^n$.

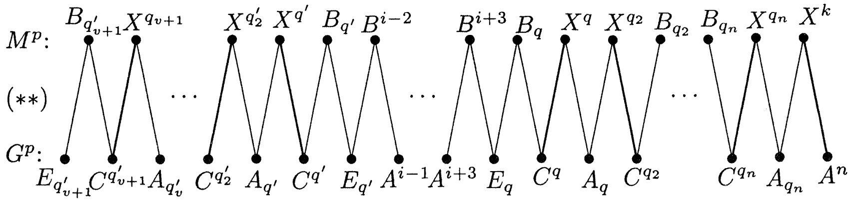


Figure 6

According to Theorem 9 there does not exist $B \in M^p$ such that $E_{q'_{v+1}}\ I^p\ B$ and $B \neq B_{q'_{v+1}}$, thus $E_{q'_{v+1}} = A^0$. Figure 6 shows the graph of $\mathcal{J}^p$ emphasizing the substructure $\mathcal{J}_1^p$. By assumption, $A^i \cap A^{i+1} = A^{i+1} \cap A^{i+2}$; from Theorem 6 we have $C^{q_j} \cap A_{q_j} = A_{q_j} \cap C^{q_{j+1}}$ if $q_1 := q$ and $j \in \{1, \ldots, u\}$. At the same time, $C^{q'_j} \cap A_{q'_j} = A_{q'_j} \cap C^{q'_{j+1}}$ if $q'_1 := q'$ and $j \in \{1, \ldots, v\}$. Furthermore, $B^{i+2} \cap B^{i+3} = B^{i+3} \cap B_q = \{b, m_{i+1}\} \cup (Q^i - \{n_k, n_q\})$ and $B^{i-1} \cap B^{i-2} = B^{i-2} \cap B_{q'} = \{b, m'_i\} \cup (Q^i - \{n_k, n_{q'}\})$. It follows from Theorem 9 that $B_{q_j} \cap Y_{q_j} = Y_{q_j} \cap B_{q_{j+1}}$ for $j \in \{2, \ldots, u-1\}$ and $B_{q'_j} \cap Y_{q'_j} = Y_{q'_j} \cap B_{q'_{j+1}}$ for $j \in \{2, \ldots, v-1\}$. If $p = 2q + 1$, then $n = 5q + 3$.

2. Assume that $u \neq k$. Then, by Proposition 7, also $u' \neq k$ where $B^{i+2} = \{b, m_{i+1}\} \cup (Q^i - \{n_k\})$, $B^{i-1} = \{b, m'_i\} \cup (Q^i - \{n_k\})$, $A^{i+3} = \{d, a_{i+2}\} \cup (R^i - \{g_u\})$, $A^{i-1} = \{e, a'_i\} \cup (R^i - \{g_{u'}\})$. Since $\mathcal{J}$ is of type (p, n) there exist norming mappings $\alpha_1 : A^{i+3} \to B^{i+2}$, $\alpha_2 : A^{i-1} \to B^{i-1}$.

Proposition 10 *The following statements hold:*

(i) $d \not{I}\ n_u, n_k$; $d \not{I}\ m'_i \Leftrightarrow A^{i+3}\ I^p\ B_u$, $d\ I\ m'_i \Leftrightarrow A^{i-1} = \{d, a'_i\} \cup (R^i - \{g_u\})$,

(ii) $e \not{I}\ n_{u'}, n_k$; $e \not{I}\ m_{i+1} \Leftrightarrow A^{i-1}\ I^p\ B_{u'}$, $e\ I\ m_{i+1} \Leftrightarrow A^{i+3} = \{e, a_{i+2}\} \cup (R^i - \{g_{u'}\})$.

Proof (i) It follows from $u \neq k$ that $\alpha_1(d) = n_u$, thus $d \not I n_u$, $\alpha_1(a_{i+2}) = m_{i+1}$, $\alpha_1(g_k) = b$ and $\alpha_1(g_r) = n_r$ for $r \neq k, u$. If $d\ I\ n_k$, then $A^{i+3}\ I^p\ B^{i+1}$ which is a contradiction and hence $d \not I n_k$.

If $d \not I m_i'$, then $A^{i+3}\ I^p\ B_u$. Conversely, $A^{i+3}\ I^p\ B_u$ implies the existence of just one norming mapping $\alpha : A^{i+3} \to B_u$ with $\alpha(d) = m_i'$ and thus $d \not I m_i'$. If $d\ I\ m_i'$, then $A'\ I^p\ B^{i-1}$ where $A' = \{d, a_i'\} \cup (R^i - \{g_u\})$. Thus $A' = A^{i-1}$ because $A' \neq A^i$. Let $A^{i-1} = \{d, a_i'\} \cup (R^i - \{g_u\})$. Then just one norming mapping $\alpha : A^{i-1} \to B^{i-1}$ exists with $\alpha(d) = n_u$ and $\alpha(a_i') = m_i'$. This yields $d\ I\ m_i'$.

(ii) The proof is similar to (i). □

Proposition 11 *The following equivalences hold:*

$$u = u' \iff d\ I\ m_i' \iff e\ I\ m_{i+1} \iff d = e.$$

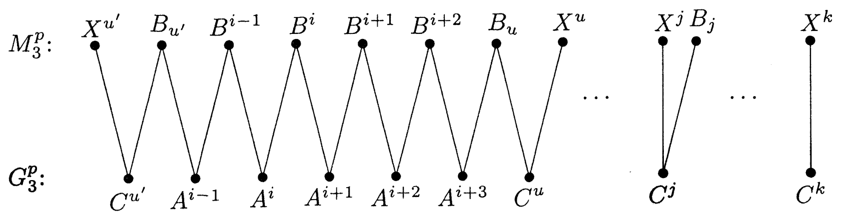
Proof Let us suppose that $u = u'$. Moreover, assume that $d \not I m_i'$. Then $A^{i+3}\ I^p\ B_u$ by Proposition 10. If $e \not I m_{i+1}$, then $A^{i-1}\ I^p\ B_u$ which contradicts $C^u\ I^p\ B_u$. If $e\ I\ m_{i+1}$, then $A^{i+3} = \{e, a_{i+2}\} \cup (R^i - \{g_u\}) = \{d, a_{i+2}\} \cup (R^i - \{g_u\})$ which yields $e = d$; thus $e \not I m_{i+1}$, a contradiction. Hence $d\ I\ m_i'$.

Consider $d\ I\ m_i'$. It means that $A^{i-1} = \{d, a_i'\} \cup (R^i - \{g_u\}) = \{e, a_i'\} \cup (R^i - \{g_{u'}\})$. Since $d, e \notin R^i$ we have $d = e$ and $u = u'$. From $\alpha_1(d) = n_u$ we obtain $d\ I\ m_{i+1}$ and thus $e\ I\ m_{i+1}$. Similarly, $e\ I\ m_{i+1}$ yields $d = e$, $u = u'$ and $d\ I\ m_i'$.

If $d = e$, then $\alpha_1(d) = n_u$, $\alpha_1(e) = n_{u'}$. Hence $n_u = n_{u'}$ and $u = u'$. □

a) Let us assume that $u \neq u'$. Then $d \neq e$, $d \not I n_u, n_k, m_i'$, $A^{i+3}\ I^p\ B_u$ and $e \not I n_{u'}, n_k, m_{i+1}$, $A^{i-1}\ I^p\ B_{u'}$.

Let $\mathcal{J}_3 = (G_3, M_3, I_3)$ be a substructure of $\mathcal{J}$ with $G_3 = G_1 \cup \{d, e\}$ and $M_3 = M_1 \cup \{b\}$. The graph of $\mathcal{J}_3^p$ has the following form:



See incidence structures $\mathcal{J}_3$, $\mathcal{J}_3^p$ for $p = 5$, $k = 2$, $u = 1$, $u' = 3$ at Enclosure 14.

Theorem 11 *If* $A\ I^p\ B_j$, $A \neq C^j$, *then also* $A\ I^p\ B_{j'}$ *for* $j' \neq k, u, u', j$.

Proof Let $A\ I^p\ B_j$ and $A \neq C^j$. Then $A \subseteq G - G_3$ and there exists $a \in A$, $a \in G - G_3$. Now, from $|A \cap C^j| = p - 1$ and $C^j = \{a_i', a_{i+2}\} \cup (R^i - \{g_j\})$ we obtain $A = \{a, a_i'\} \cup (R^i - \{g_j\})$ or $A = \{a, a_{i+2}\} \cup (R^i - \{g_j\})$ or $A =$

$\{a, a'_i, a_{i+1}\} \cup (R^i - \{g_j, g_{j'}\})$. Since $A\ I^p\ B_j$ there exists a norming mapping $\alpha : A \to B_j$.

(i) First assume that $A = \{a, a'_i\} \cup (R^i - \{g_j\})$. Then $\alpha(a) = m_{i+1}$, $\alpha(a'_i) = m'_i$ and $\alpha(g_r) = n_r$ for $r \neq k, j$. If $a\ I\ n_k$, then $A\ I^p\ X^j$ which is a contradiction. Thus $a \not{I}\ n_k$. If $a\ I\ n_j$, then $A'\ I^p\ B^i$ where $A' = \{a, a'_i\} \cup (R^i - \{g_k\})$. This is a contradiction. In the case $a \not{I}\ n_j$ we get $A\ I^p\ B^{i-1}$ which also contradicts our assumption.

(ii) If $A = \{a, a_{i+2}\} \cup (R^i - \{g_j\})$, then one can get a contradiction similarly to (i).

(iii) According to (i), (ii) we have $A = \{a, a'_i, a_{i+2}\} \cup (R^i - \{g_j, g_{j'}\})$. Let $j' = k$. Then $\alpha(a) = b$, $\alpha(a_{i+2}) = m_{i+1}$, $\alpha(a'_i) = m'_i$ and $\alpha(g_r) = n_r$ for $r \neq k, j$. If $a\ I\ n_j$, then $A'\ I^p\ B^{i+2}$ for $A' = \{a, a_{i+2}\} \cup (R^i - \{g_k\})$ which is a contradiction. In the case $a \not{I}\ n_j$ we obtain $A\ I^p\ X^k$. It follows from Theorems 6, 7 that $A\ I^p\ X^{\varphi(k)}$ and because of $B_j \neq X^{\varphi(k)}$ we have a contradiction.

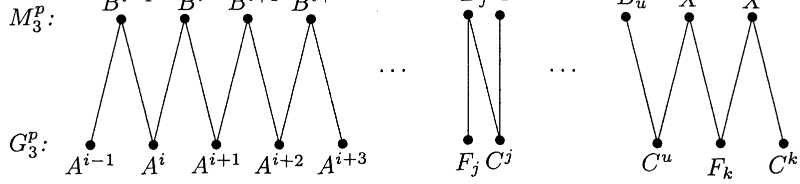
Let $j' = u$. Then $\alpha(a) = n_u$, $\alpha(a'_i) = m'_i$, $\alpha(a_{i+2}) = m_{i+2}$, $\alpha(g_k) = b$ and $\alpha(g_r) = n_r$ for $r \neq u, k, j$. If $a \not{I}\ n_j$, then $A\ I^p\ B_u$ which is a contradiction. In the case $a\ I\ n_j$ we obtain $A'\ I^p\ B^{i+2}$ where $A' = \{a, a_{i+2}\} \cup (R^i - \{g_u\})$. This is a contradiction again. We proceed in a similar way in the case $j' = u'$. Hence $j' \neq k, u, u', j$.

We have $\alpha(a) = n_{j'}$, $\alpha(a'_i) = m'_i$, $\alpha(a_{i+2}) = m_{i+1}$, $\alpha(g_u) = b$ and $\alpha(g_r) = n_r$ for $r \neq k, j, j'$. If $a\ I\ n_k$, then $A\ I^p\ X^j$ which is a contradiction. Thus $a \not{I}\ n_k$. If $a\ I\ n_j$, then $A'\ I^p\ B^{i+2}$ where $A' = \{a, a_{i+2}\} \cup (R^i - \{g_{j'}\})$. This is a contradiction again. Thus $a \not{I}\ n_j$ an in this case $A\ I^p\ B_{j'}$. □

Let us put $L = \{1, \dots, p-1\}$ and $L' = L - \{k, u, u'\}$. To every $j \in L'$ there exists $A \in G^p$ such that $A\ I^p\ B_j$. Then, by Theorem 11, there exists $j' \in L'$, $j' \neq j$ such that $A\ I^p\ B_{j'}$. In this way we get a mapping $\xi : L' \to L'$ which is involutory. However, this contradicts the fact that the positive integer $|L'|$ is odd. Hence, an incidence structure of type (p, n) satisfying the requirements 2, a) does not exist.

b) Let us assume that $u = u'$. Then, by Propositions 10, 11, we have $d = e$, thus $A^{i+3} = \{d, a_{i+2}\} \cup (R^i - \{g_u\})$, $A^{i-1} = \{d, a'_i\} \cup (R^i - \{g_u\})$ and $d \not{I}\ n_u, n_k$, $d\ I\ m'_i, m_{i+1}$.

Proposition 12 *If $\mathcal{J}_3 = (G_3, M_3, I_3)$ is a substructure of $\mathcal{J}$ with $G_3 = G_1 \cup \{d\}$ and $M_3 = M_1 \cup \{b\}$, then a graph of $\mathcal{J}_3^p$ has a form*

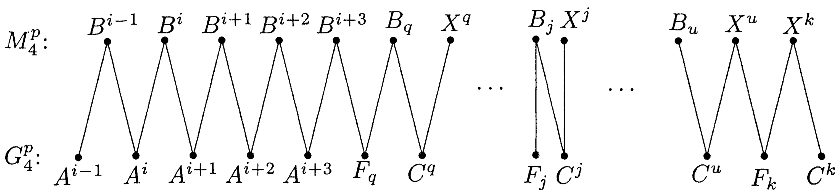


where $F_j = \{d, a'_i, a_{i+2}\} \cup (R^i - \{g_u, g_j\})$ for $j \neq u$.

Enclosure 15 shows the incidence structures $\mathcal{J}_3$ and $\mathcal{J}_3^5$ for $k = 2$ and $u = 1$.

Since the incidence structure $\mathcal{J}$ is of type (p, n) there exists either a set $B^{i+3} \in M^p$ where $A^{i+3}\ I^p\ B^{i+3}$ or a set $B^{i-2} \in M^p$ where $A^{i-1}\ I^p\ B^{i-2}$.

Proposition 13 *Let a set* $B^{i+3} \in M^p$ *exist. Then* $B^{i+3} = \{x, b, m_{i+1}\} \cup (Q^i - \{n_k, n_q\})$ *where* $x \notin M_3$. *If* $\mathcal{J}_4 = (G_4, M_4, I_4)$ *is a substructure of* $\mathcal{J}$ *with* $G_4 = G_1 \cup \{d\}$ *and* $M_4 = M_1 \cup \{b, x\}$, *then a graph of* $\mathcal{J}_4^p$ *has a form*



Proof Since $B^{i+2} = \{b, m_{i+1}\} \cup (Q^i - \{n_k\})$ and $|B^{i+2} \cap B^{i+3}| = p - 1$ we have $B^{i+3} = \{x, m_{i+1}\} \cup (Q^i - \{n_k\})$ or $B^{i+3} = \{x, b\} \cup (Q^i - \{n_k\})$ or $B^{i+3} = \{x, b, m_{i+1}\} \cup (Q^i - \{n_k, n_q\})$ where $x \notin M_3$. There exists a norming mapping $\alpha : A^{i+3} \to B^{i+3}$ because $A^{i+3}\ I^p\ B^{i+3}$.

a) First suppose that $B^{i+3} = \{x, m_{i+1}\} \cup (Q^i - \{n_k\})$. Then $\alpha(d) = n_u$, $\alpha(a_{i+2}) = m_{i+1}$, $\alpha(g_k) = x$ and $\alpha(g_r) = n_r$ for $r \neq u, k$. Let $g_u\ I\ x$. Then $A^{i+2}\ I^p\ B^{i+3}$ which is a contradiction. Hence $g_u \not{I}\ x$. If $a_i'\ I\ x$, then $C^u\ I^p\ B'$ where $B' = \{x, m_i', m_{i+1}\} \cup (Q^i - \{n_k, n_u\})$ which is a contradiction. If $a_i' \not{I}\ x$, then $F_k\ I^p\ B'$ where $B' = \{x, m_{i+1}\} \cup (Q^i - \{n_k\})$. This is also a contradiction.

b) Let $B^{i+3} = \{x, b\} \cup (Q^i - \{n_k\})$. Then $\alpha(d) = n_u$, $\alpha(a_{i+2}) = x$, $\alpha(g_k) = b$ and $\alpha(g_r) = n_r$ for $r \neq u, k$. Let $g_u\ I\ x$. Then $A^{i+2}\ I^p\ B'$ where $B' = \{x\} \cup Q^i$. This is a contradiction. Hence $g_u \not{I}\ x$. If $a_i'\ I\ x$, then $A^i\ I^p\ B'$ where $B' = \{x, m_i'\} \cup (R^i - \{n_u\})$ which is a contradiction. Finally, if $a_i' \not{I}\ x$, then $A^{i-1}\ I^p\ B^{i+3}$ which is also a contradiction.

c) Now it is clear that $B^{i+3} = \{x, b, m_{i+1}\} \cup (Q^i - \{n_k, n_q\})$. First assume that $q = u$. Then $\alpha(d) = x$, $\alpha(a_{i+2}) = m_{i+1}$, $\alpha(g_k) = b$ and $\alpha(g_r) = n_r$ for $r \neq u, k$. Let $g_u\ I\ x$. Then $A^i\ I^p\ B'$ where $B' = \{x\} \cup Q^i$. This is a contradiction. Hence $g_u \not{I}\ x$.

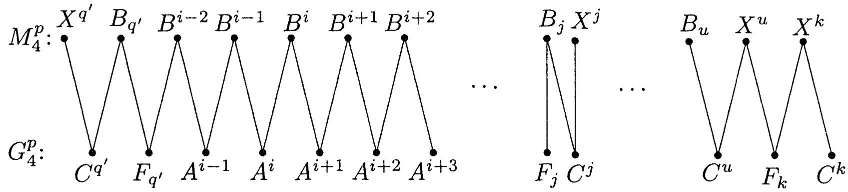
Let $a_i'\ I\ x$. Then for an arbitrary $j \neq u$ we have $C^j\ I^p\ B'$ where $B' = \{x, m_i', m_{i+1}\} \cup (Q^i - \{n_k, n_u\})$ which is a contradiction. If $a_i' \not{I}\ x$, then $C^u\ I^p\ B'$ where $B' = \{x, m_{i+1}\} \cup (Q^i - \{n_u\}$ which is also a contradiction. Thus $q \neq u$. Then $\alpha(d) = n_u$, $\alpha(a_{i+2}) = m_{i+1}$, $\alpha(g_k) = b$, $\alpha(g_q) = x$ and $\alpha(g_r) = n_r$ for $r \neq k, u, q$. Let $g_u\ I\ x$. Then $A^{i+1}\ I^p\ B^{i+3}$ which is a contradiction.

Hence $g_u \not{I}\ x$. If $a_i'\ I\ x$, then $C^u\ I^p\ B'$ where $B' = \{x, m_i', m_{i+1}\} \cup (Q^i - \{g_u, g_q\})$ which is also a contradiction. Hence $a_i' \not{I}\ x$. However, then $F_q\ I^p\ B^{i+3}$ and $\mathcal{J}_4^p$ has a graph presented in the proposition. □

Enclosure 16 shows $\mathcal{J}_4$ and $\mathcal{J}_4^5$ for $k = 2, u = 1, q = 3$.

In a similar way we can prove the following proposition:

Proposition 14 *If there exists a set $B^{i-2} \in M^p$, then $B^{i-2} = \{y, b, m'_i\} \cup (Q^i - \{n_k, n_{q'}\})$ where $y \notin M_3$. If $\mathcal{J}_4 = (G_4, M_4, I_4)$ is a substructure of $\mathcal{J}$ with $G_4 = G_1 \cup \{d\}$ and $M_4 = M_3 \cup \{b, y\}$, then the graph of $\mathcal{J}_4^p$ has a form*



Remark 5 If both sets B^{i+3}, B^{i-2} exist, then $q \neq q'$. Indeed, in the contrary case we have $F_q\ I^p\ B^{i+3}, B^{i-2}$ which is a contradiction.

Theorem 12 *Let us put $L = \{1, \ldots, p-1\}$ and $L' = L - \{k, u, q, q'\}$. If $F_j\ I^p\ B$ for $j \in L'$ where $B \neq B_j$, then also $F_{j'}\ I^p\ B$ for $j' \in L', j' \neq j$.*

Proof Let $F_j\ I^p\ B$, $j \in L'$ and $B \neq B_j$. If $\mathcal{J}_5 = (G_5, M_5, I_5)$ is a substructure of $\mathcal{J}$ with $G_5 = G_1 \cup \{d\}$ and $M_5 = M_1 \cup \{b, x, y\}$ (x, y are from Propositions 13, 14), then $B \in M_5^p$ and there exists $z \in B$, $z \in M - M_5$. With respect to $|B \cap B_j| = p-1$ we have $B = \{z, m'_i, m_{i+1}\} \cup (Q^i - \{n_k, n_j\})$ or $B = \{z, b, m'_i\} \cup (Q^i - \{n_k, n_j\})$ or $B = \{z, b, m_{i+1}\} \cup (Q^i - \{n_k, n_j\})$ or $B = \{z, b, m'_i, m_{i+1}\} \cup (Q^i - \{n_k, n_j, n_{j'}\})$. Moreover, there exists a norming mapping $\alpha : F_j \to B$.

Assume that $B = \{z, m'_i, m_{i+1}\} \cup (Q^i - \{n_k, n_j\})$. Then $\alpha(d) = n_u$, $\alpha(a'_i) = m'_i$, $\alpha(a_{i+2}) = m_{i+1}$, $\alpha(g_k) = z$ and $\alpha(g_r) = n_r$ for $r \neq k, u, j$. If $g_u\ I\ z$, then $C^j\ I^p\ B$ which is a contradiction. Thus $g_u \not{I}\ z$. Let $g_j\ I\ z$. This yields $C^u\ I^p\ B'$ where $B' = \{z, m'_i, m_{i+1}\} \cup (Q^i - \{n_k, n_u\})$. This is a contradiction. From $g_j \not{I}\ z$ we obtain $F_k\ I^p\ B$ which is a contradiction again. In a similar way we can prove that also the two following cases are impossible.

Hence, $B = \{z, b, m'_i, m_{i+1}\} \cup (Q^i - \{n_k, n_j, n_{j'}\})$. Assume that $j' = u$. Then $\alpha(d) = z$, $\alpha(a_{i+2}) = m_{i+1}$, $\alpha(a'_i) = m'_i$, $\alpha(g_k) = b$ and $\alpha(g_r) = n_r$ for $r \neq k, u, j$. If $g_j \not{I}\ z$, then $C^u\ I^p\ B$ which is a contradiction. If $g_j\ I\ z$, then $F_k\ I^p\ B'$ where $B' = \{z, m'_i, m_{i+1}\} \cup (Q^i - \{n_u, n_k\})$. This is a contradiction again. Thus $j' \neq u$.

Assume that $j = q$. Then $\alpha(d) = n_u$, $\alpha(a_{i+2}) = m_{i+1}$, $\alpha(a'_i) = m'_i$, $\alpha(g_k) = b$, $\alpha(g_q) = z$ and $\alpha(g_r) = n_r$ for $r \neq u, j, k, q$. If $g_j \not{I}\ z$, then $F_q\ I^p\ B$ which is a contradiction. If $g_j\ I\ z$, then $F_k\ I^p\ B'$ where $B' = \{z, m'_i, m_{i+1}\} \cup (Q^i - \{n_q, n_k\})$. This is also a contradiction. Therefore, $j' \neq q$. Similarly we prove that $j' \neq q'$. We have obtained that $j' \in L'$ and $\alpha(d) = n_u$, $\alpha(a'_i) = m'_i$, $\alpha(a_{i+2}) = m_{i+1}$, $\alpha(g_k) = b$, $\alpha(g_{j'}) = z$ and $\alpha(g_r) = n_r$ for $r \neq u, j, k, j'$. If $g_u\ I\ z$, then $C^j\ I^p\ B$ which is a contradiction. Thus $g_u \not{I}\ z$. If $g_j\ I\ z$, then $C^u\ I^p\ B'$ where $B' = \{z, b, m'_i, m_{i+1}\} \cup (Q^i - \{n_k, n_u, n_{j'}\})$. This is also a contradiction. Hence $g_j \not{I}\ z$ which yields $F_{j'}\ I^p\ B$. □

Remark 6 Enclosure 17 shows the situation described in Theorem 12 for $p = 7$, $k = 2$, $q = 3$, $q' = 4$, $j = 6$, $j' = 5$. In Theorem 12 there is supposed that both sets B^{i+3}, B^{i-2} exist. However, this theorem also holds if one of those sets does not exist. Then $L' = L - \{k, u, q\}$ resp. $L' = L - \{k, u, q'\}$.

First let us assume that both sets B^{i+3}, B^{i-2} exist and consider the sets L, L' from Theorem 12 where $|L'| = p - 5$. If $F_j \in G^p$ for $j \in L'$, then there exists a set $Y_j \in M^p$, $Y_j \neq B_j$, such that $F_j \ I^p \ Y_j$. Moreover, by Theorem 12, $F_{j'} \ I^p \ Y_j$ for a certain $j' \in L'$ distinct from j. If we put $\xi(j) = j'$ for an arbitrary $j \in L'$, then ξ is involutory mapping of the set L'. For every $j \in L'$ let us consider a substructure $\mathcal{J}_j = (G_j, M_j, I_j)$ of $\mathcal{J}$ where $G_j = \{C^j, F_j, F_{j'}, C^{j'}\}$, $M_j = \{X^j, B_j, Y_j, B_{j'}, X^{j'}\}$. It is obvious that $\mathcal{J}_j = \mathcal{J}_{\xi(j)}$.

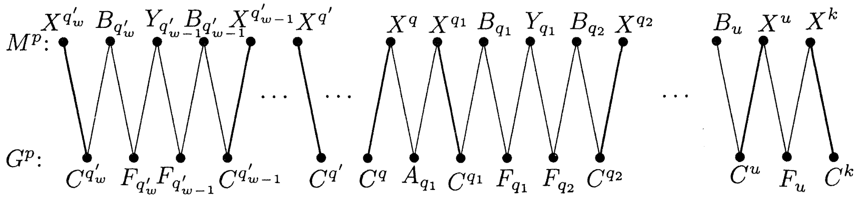


Figure 7

Let us consider an involutory mapping φ of the set L described in Theorem 7. Then $\varphi(u) = k$ and φ induces an involutory mapping of a set $L'' = L - \{u, k\}$ where $|L''| = p - 3$.

Let $\mathcal{J}_5 = (G_5, M_5, I_5)$ be a substructure of $\mathcal{J}$ where $G_5 = G_1 \cup \{d\}$ and $M_5 = M_1 \cup \{b, x, y\}$. We assign a graph of this structure by means of Propositions 13, 14. If we put $\varphi(q) = q_1$, then $q_1 \in L'$ and there exists a set $A_{q_1} \in G^p$ such that $A_{q_1} \ I^p \ X^q, X^{q_1}$ (see Figure 7). Furthermore, let $\xi(q_1) = \xi\varphi(q) = q_2$ where $q_2 \in L''$. Consider a substructure $\mathcal{J}_{q_1}$ where $X^{q_2} \in M_{q_1}$; let us put $\varphi(q_2) = \varphi\xi\varphi(q) = q_3$ and $\xi(q_3) = q_4$, consider $\mathcal{J}_{q_3}$ where $X^{q_4} \in M_{q_3}$ etc.

Similarly, let $\varphi(q') = q'_1$, $\xi\varphi(q') = \xi(q'_1) = q'_2$ etc. There exist positive integers v, w where

$$\underbrace{\xi\varphi \ldots \xi\varphi}_{v}(q) = q_v \quad \text{and} \quad \underbrace{\xi\varphi \ldots \xi\varphi}_{w}(q') = q'_w$$

such that $v + w = p - 5$ (Figure 7). With respect to $|L''| = p - 3$ we obtain $\varphi(q_v) = q_w$. Hence, there exists a set $A \in G^p$ such that $A \ I^p \ X^{q_v}, X^{q_w}$ which is a contradiction. Thus, any incidence structure described above does not exist.

Let us suppose that B^{i+2} exists and B^{i-1} does not, i. e. $i = 1$. Then there also exists an involutory mapping ξ of the set $L' = L - \{k, u, q\}$ which contradicts the fact that L' has an odd number of elements. Similarly we obtain a contradiction if B^{i-1} exists and B^{i+2} does not.

Thus, any incidence structure of type (p, n) satisfying the requirements 2, b) does not exist.

Main Theorem *Let* $\mathcal{J} = (G, M, I)$ *be an incidence structure of type* (p, n) *where* $p, n > 2$. *Let* $R^i = R^{i+1}$ *for a certain* i, $0 \leq i \leq n - 2$ *and* $a'_i \not I \, m'_i$. *Then* p *is odd, thus* $p = 2q + 1$ *and a graph of the incidence structure* $\mathcal{J}^p$ *is either* $(*)$ *from [5] where* $n = 3q + 2$ *or* $(**)$ *where* $n = 5q + 3$.

References

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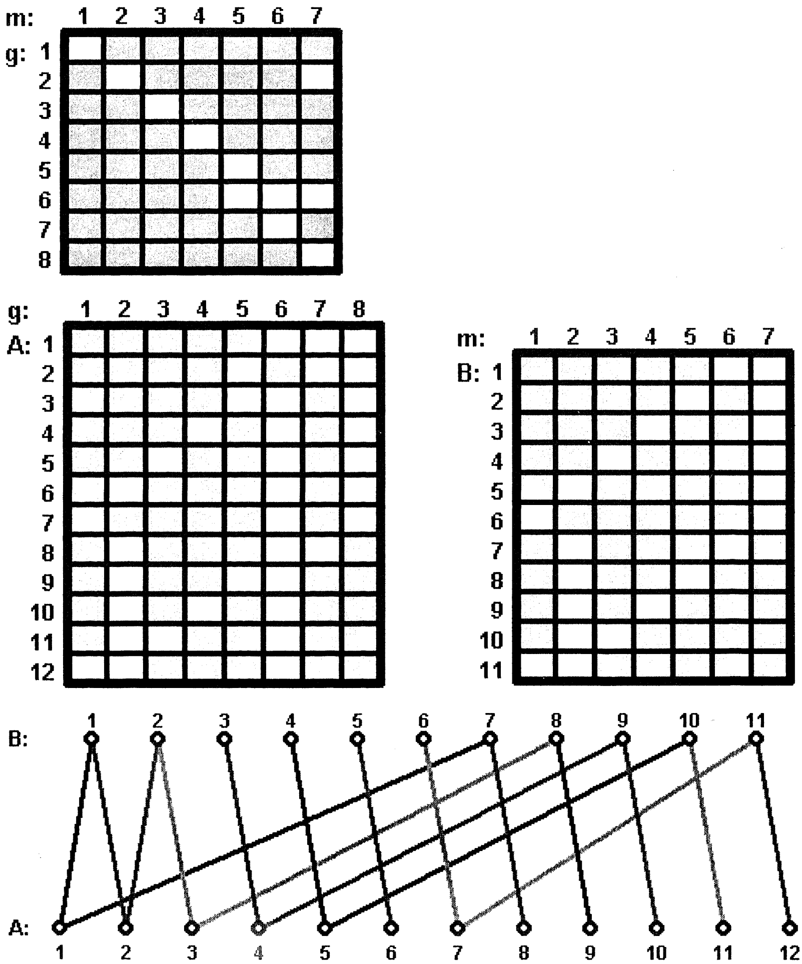
[3] Machala, F.: *Incidence structures of independent sets.* Acta Univ. Palacki. Olomuc., Fac. rer. nat., Math. **38** (1999), 113–118.

[4] Machala, F.: *Incidence structures of type (p,n).* Czech. Math. Journal (to appear).

[5] Machala, F.: *Special incidence structures of type* (p, n). Acta Univ. Palacki. Olomuc., Fac. rer. nat., Math.**39** (2000), 123–134.

Enclosures



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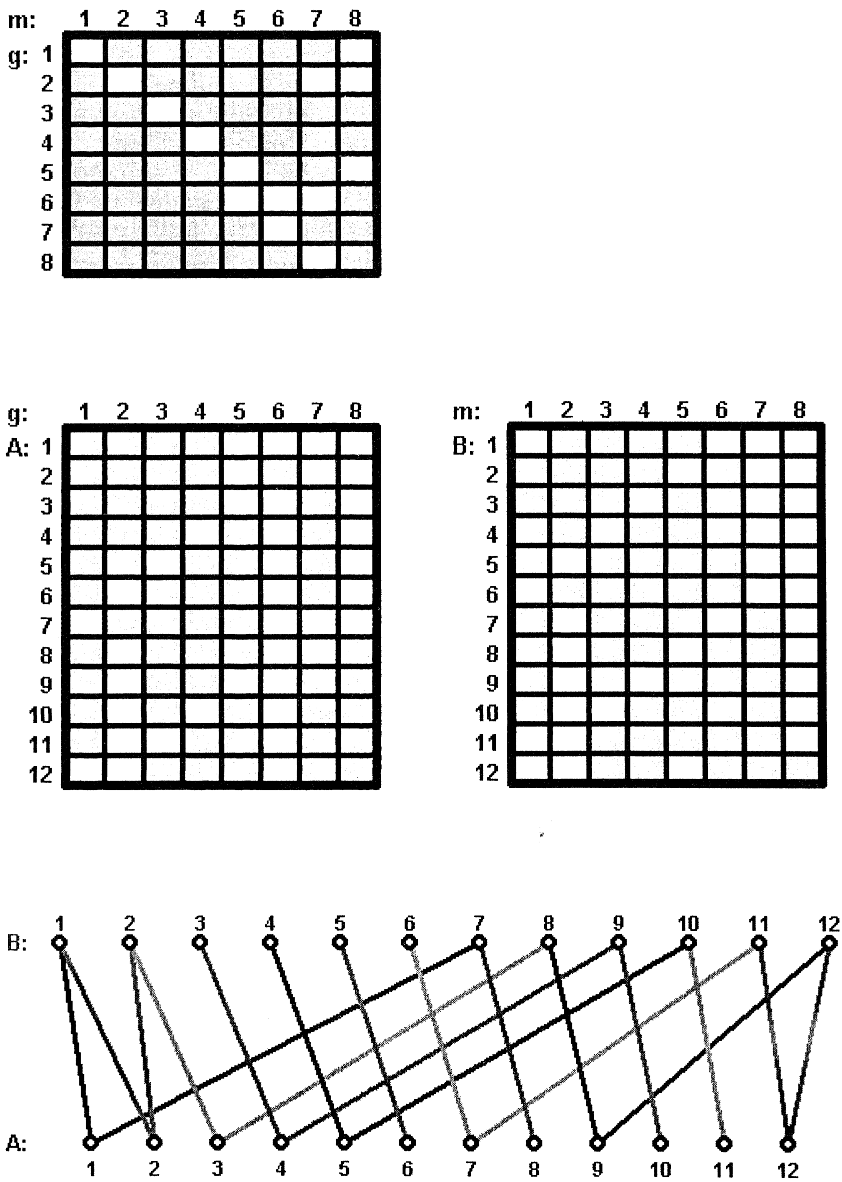

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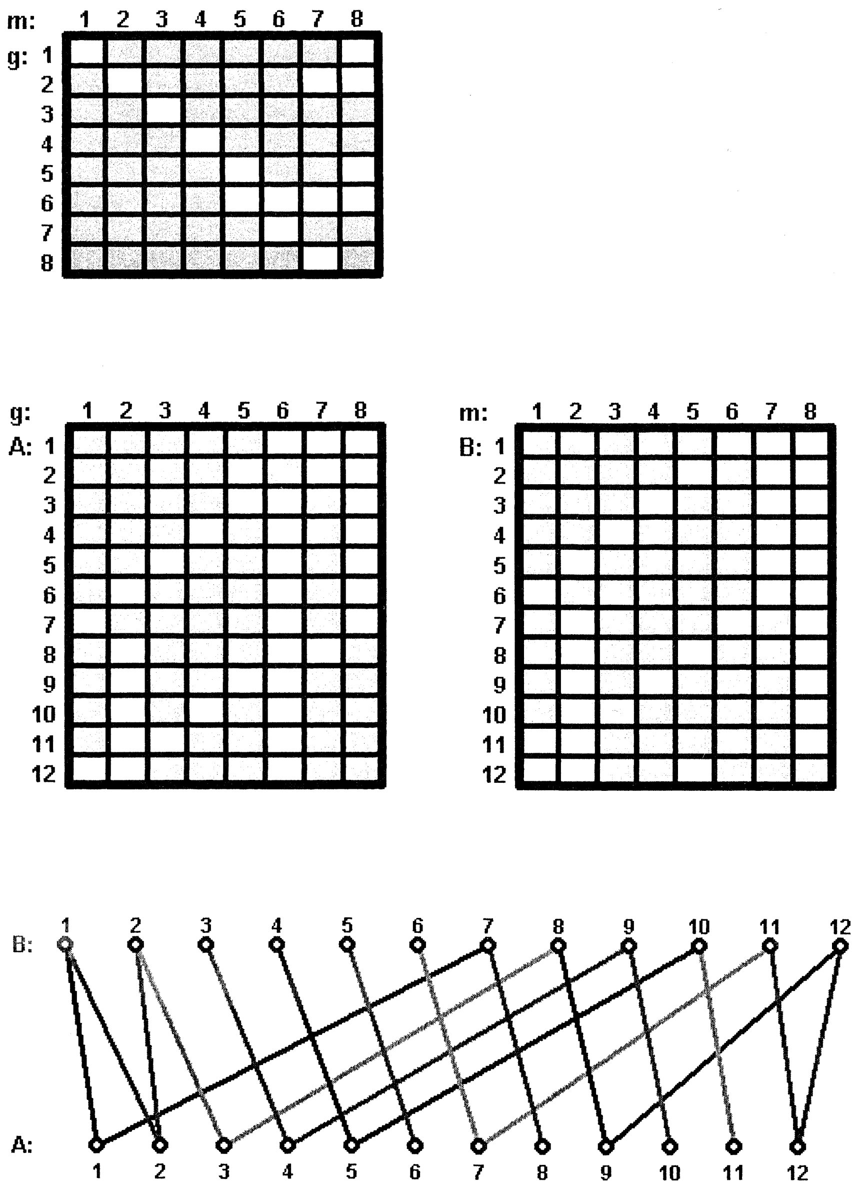

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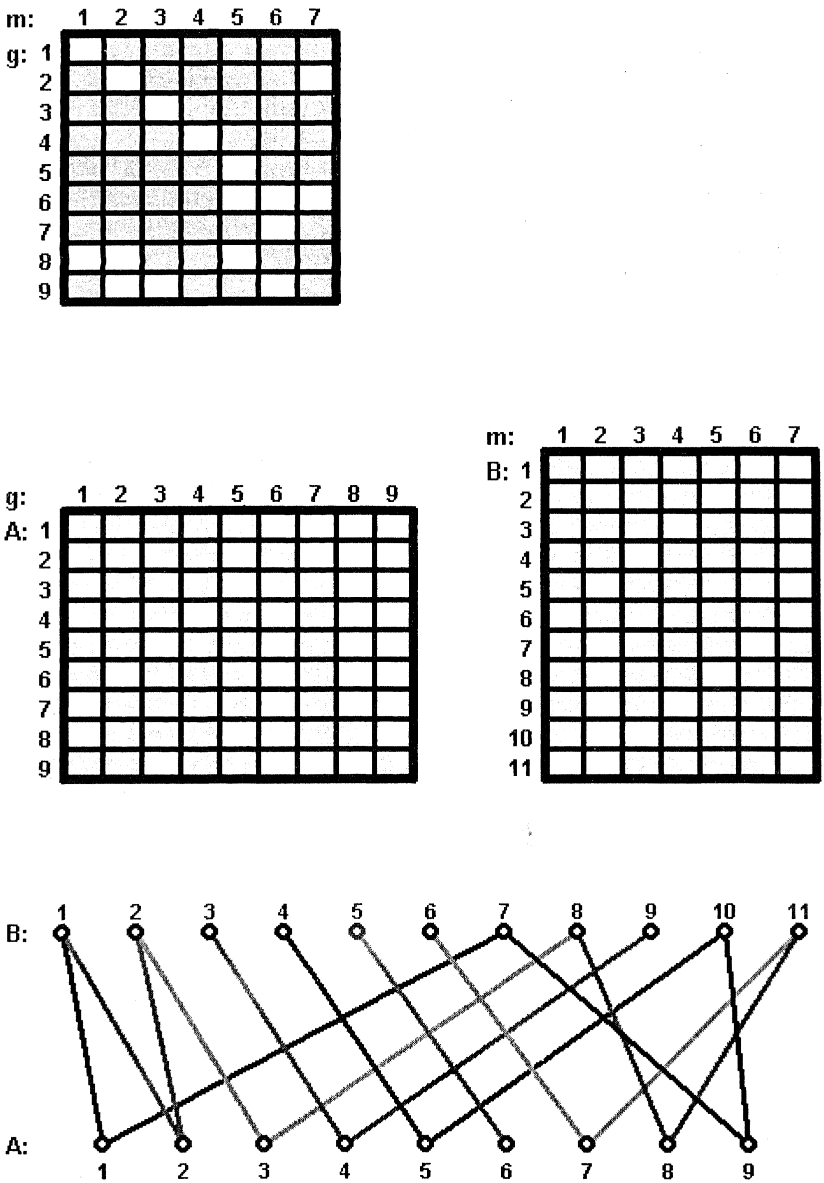

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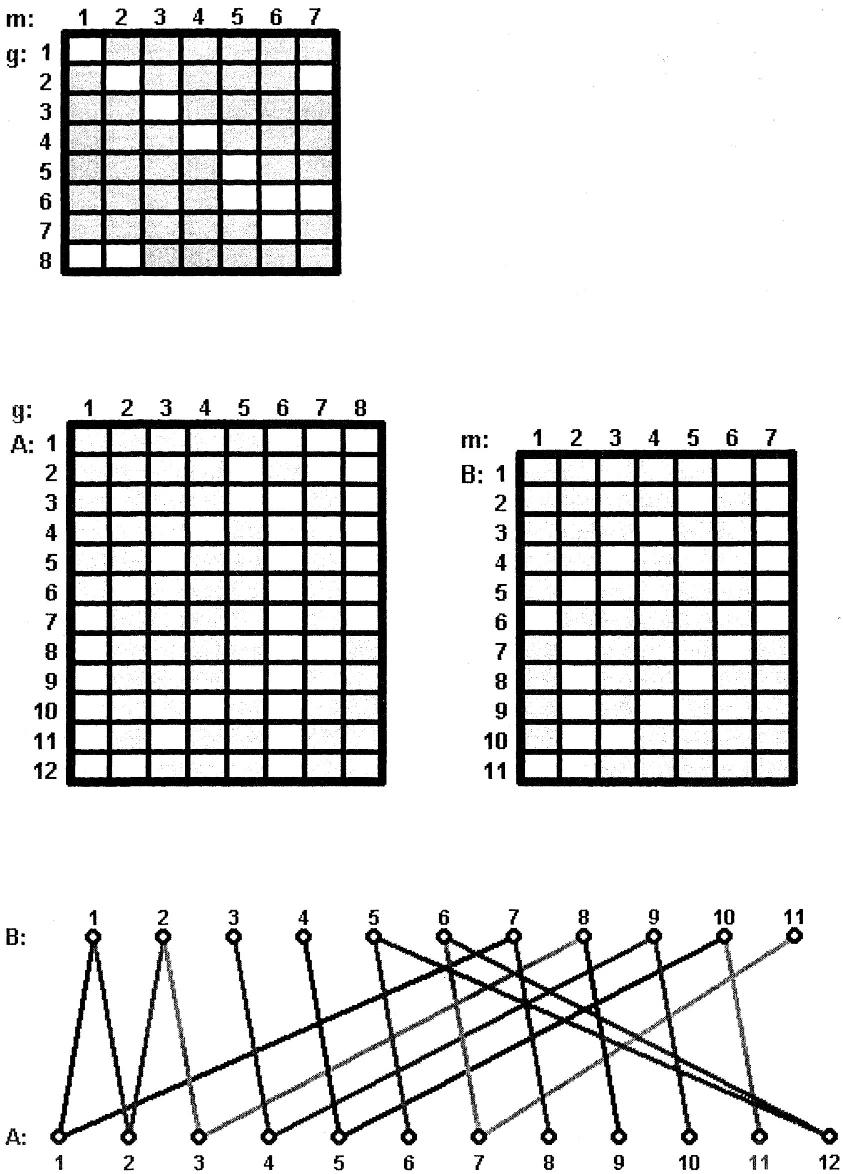

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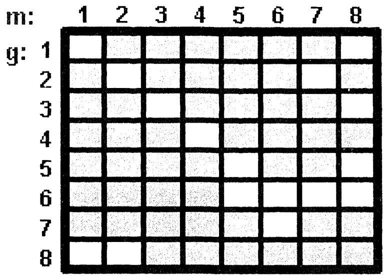

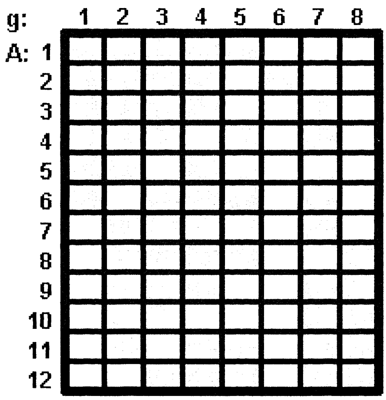

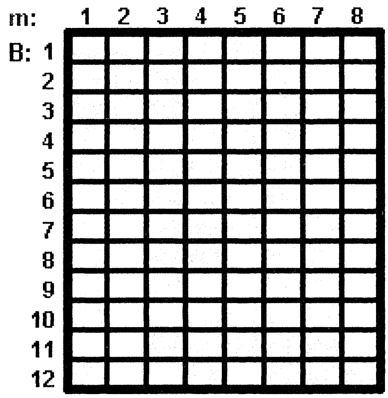

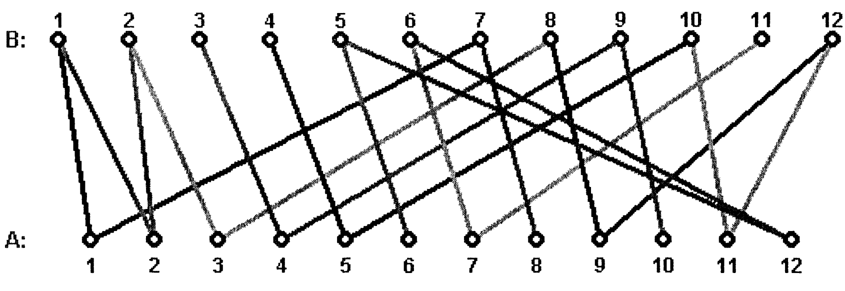

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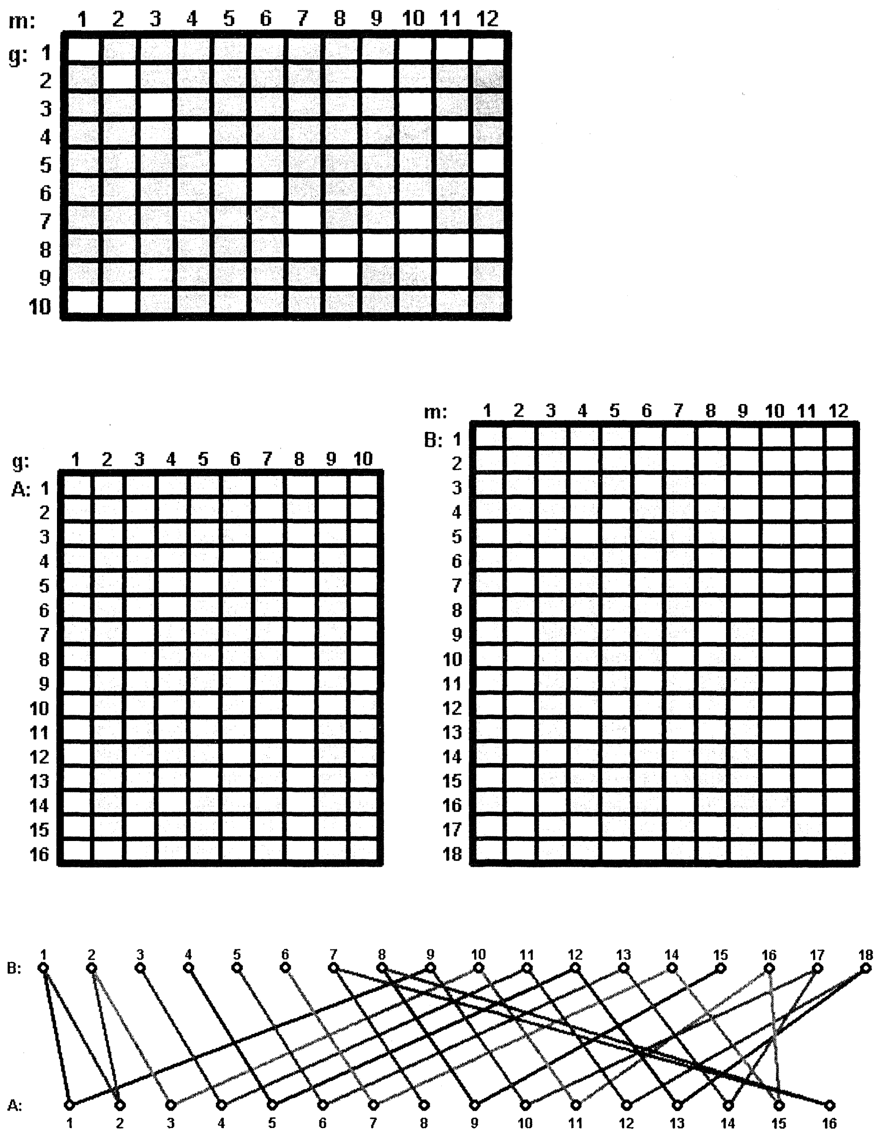

Enclosure 17