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ANALOG SOLUTION OF A GIVEN PROBLEM IN THE BACK TIME

KAREL BENEŠ

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The work continues the work [1], there are further methods of analog solution mentioned.

Let us have, for example, the differential equation

$$y' + y = 2e^t \quad , \quad y(0) = 1 \quad , \quad t \in \langle -1,1 \rangle \quad , \tag{1}$$

the function $y = e^t$ is its solution. (Further we will not consider overstepping of the machine unit of single variables on the output of calculating units.) At first let us solve the equation (1) in the interval $t \in \langle 0,1 \rangle$. The program diagram for solution of the equation (1) is in the fig.1. The equation

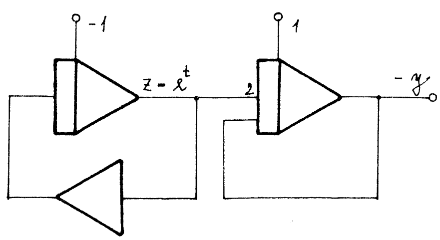


Fig. 1

is programmed in the form $y' = 2e^t - y$. If we have to solve the same equation in the interval $t \in \langle 0;-1 \rangle$, then, for a change, it is $y(0) = 1$. We generate the function $z = e^t$ (right side of the equation (1)) by the solution of the differential equation $z' - z = 0$ with the initial condition $z(0) = 1$. We do not presuppose the change of sign of integrators with composition of the program diagram. The program diagram for the solution of the equation (1) in the interval $t \in \langle 0;-1 \rangle$ is in the fig.2. If we apply real integrators which ones change sign,

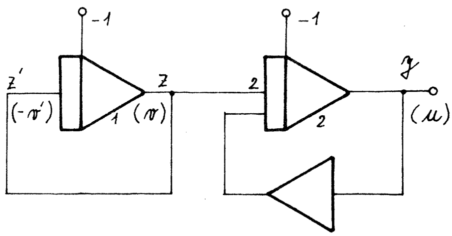


Fig. 2

then the integrator 1 solves the equation $-v' = v$, i.e. $v' + v = 0$ with the initial condition $v(0) = 1$, the function $v = e^{-t}$ is its solution. The integrator 2 solves the equation $-u' = 2v - u = 2e^{-t} - u$, i.e. $u' - u = -2e^{-t}$, $u(0) = 1$, the function $u = e^{-t}$ is its solution. This solution is solution of the equation (1) with inversly oriented time axis. (See the fig.3.)

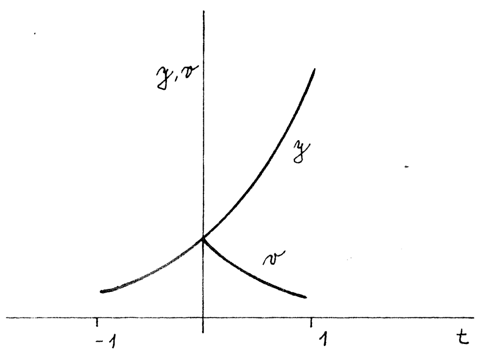


Fig. 3

Further let us have the equation

$$y'' + y' + y = 3e^{t} , \quad y(0) = 1 , \quad y'(0) = 1 ,$$

$$t \in \langle -1,1 \rangle \qquad (2)$$

the function $y = e^{t}$ is its solution. At first we solve the equation (2) in the interval $t \in \langle 0;1 \rangle$. The program diagram for this case is in the fig.4. If we solve the equation (2) in

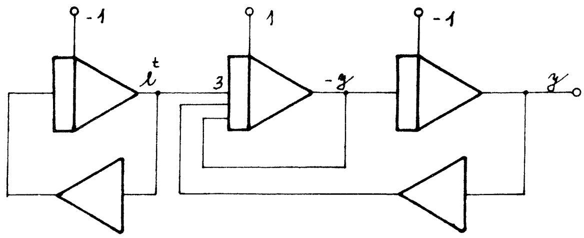


Fig. 4

the interval $t \in \langle 0;-1 \rangle$, then we proceed with generation of the right side of the equation (2) like in the proceeding case, we do not consider the change of signs of output values of integrators with composition of the program diagram. The program diagram for this case is in the fig.5. If we for a

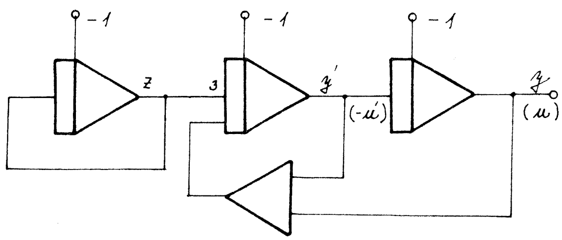


Fig. 5

change consider the real integrators, which ones change sign of output quantity, then the program diagram in the fig.5 is described by the equation $u'' = u' - u + 3e^{-t}$, $u(0) = 1$, $u'(0) = -1$, i.e.

$$u'' - u' + u = 3e^{-t} \tag{3}$$

the function $u = e^{-t}$ with mentioned initial conditions is its solution. This solution is for a change solution of the equation (2) with an inversely oriented time axis.

Let us still mention the case of the differential equation

$$y'' - 3y' + 2y = 0 \ , \quad y(0) = 0 \ , \quad y'(0) = 1 \ ,$$
$$t \in \langle -1;1 \rangle \tag{4}$$

the function $e^{t}\sin t$ is its solution. The program diagram for solution of the equation (4) in the interval $t \in \langle 0,1 \rangle$ is in the fig.6. The program diagram for solution (4) in the interval

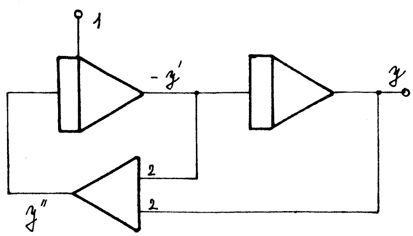


Fig. 6

$t \in \langle 0;-1 \rangle$ is in the fig.7. The equation is programmed in the

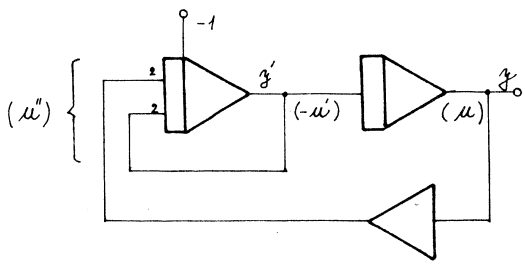


Fig. 7

form $y'' = 2y' - 2y$, we do not consider change of signs of output values of integrators. If we for a change consider the real integrators, which one change sign of output quantity, then the program diagram in the fig.7 is described by the equation

$$u'' + 2u' + 2u = 0 \ , \quad u(0) = 0 \ , \quad u'(0) = -1 \ , \qquad (5)$$

the function $u = -e^{-t}\sin t = e^{-t}\sin(-t)$ is its solution, which one is solution of the equation (4) with an inversely oriented time axis.

We also can execute the integration with respect to dt in the way we put inverter (analog summer) in front of every integrator, respective we omit inverter (analog summer). We can replot the program diagram in the fig.1 to the form in the fig.8. We get the program diagram in the fig.2 by omission of

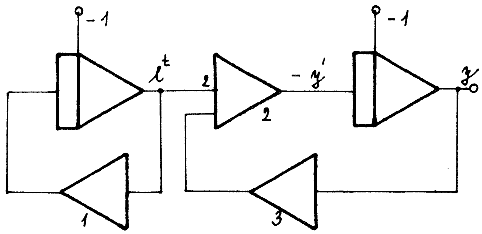


Fig. 8

the inverter 1 and the analog summer 2. We can replot the program diagram in the fig.4 to the form in the fig.9 (we pro-

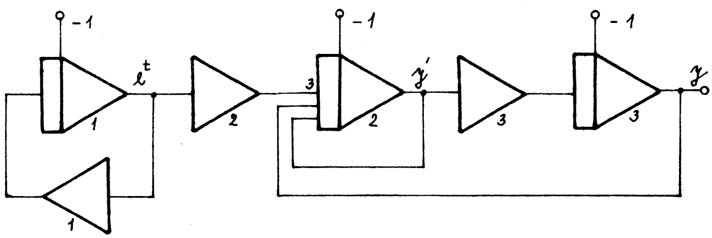


Fig. 9

gram the equation (2) in the form $- y'' = -3e^{t} + y' + y$). We get the program diagram in the fig.5 by omission of the inverters 1, 2 and 3 and by addition of inverters (analof summers) into the lower inputs of the integrator 2. We also get the program diagram for solution of the equation (2) in the back time directly from the program diagram in the fig.4 for solution in direct time by addition of the inverters (analog summers respectively), by omission inverters from the inputs of the integrators respectively, how it is clear from the fig. 10. The program diagram is described by the equation $u'' - u' +$

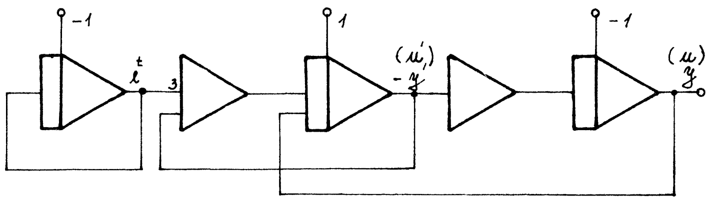


Fig. 10

$+ u = 3e^{-t}$, $u(0) = 1$, $u'(0) = -1$. The equation is programmed in the form $-u'' = -3e^{-t} - u' + u$, the function $u = e^{-t}$ is its solution, i.e. solution of the equation (2) with an inversely oriented time axis. We take away the back solution and derivations of the back solution from the outputs of identical calculating units like solution and its derivation in direct solution.

If we arrange the program diagram in the fig.6 for the solution in the back time by the mentioned process, we get the program diagram in the fig.11. The program diagram is des-

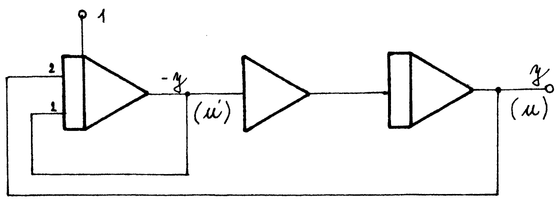


Fig. 11

cribed by the equation $-u'' = 2u' + 2u$ with initial conditions $u(0) = 0$, $u'(0) = 1$, the function $u = e^t \sin(-t)$ is its solution, how it was mentioned already. (Compare with the fig. 7.) If we want to watch course of y" in the back time, too, then the program diagram, which one solves a problem in the back time, has the form in accordance with the fig.12 on the basis of the fig.6.

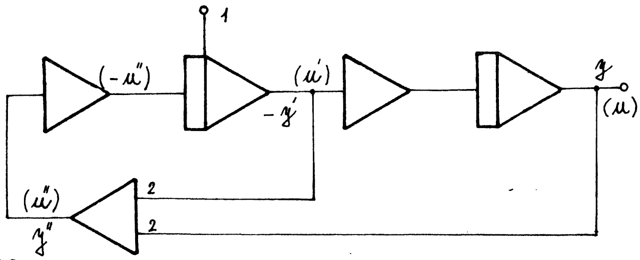


Fig. 12

The mentioned methods of solution of a problem in the back time (unconsidering of a change of sign of output value of the integrator, addition (taking-away) of inverter to input of integrator) one method is a modification of the second method, is possible to apply as well on next type of differential equations.

For example, non-linear differential equation

$$y' + y^2 = 1 + t^2 \quad , \quad y(0) = 0 \quad , \quad t \in \langle -1,1 \rangle \qquad (6)$$

$y = t$ is its solution. The program diagram for solution in the interval $t \in \langle 0;1 \rangle$ is in the fig.13, the equation is pro-

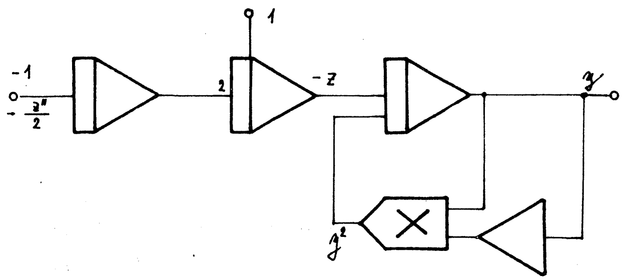


Fig. 13

grammed in the form $-y' = y^2 - (1 + t^2)$, the right side of the equation (6) is generated by solution of the equation $z'' = 2$ with the initial conditions $z(0) = 1$, $z'(0) = 0$. The program diagram for solution of the equation in the interval $t \in \langle 0,-1 \rangle$ is in the fig.14, we do not consider the change of

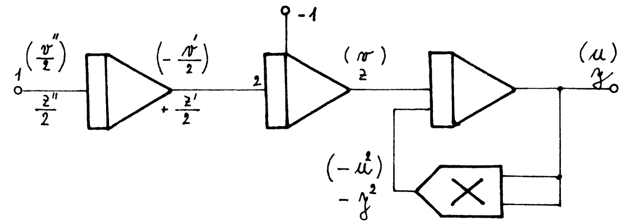


Fig. 14

signs of output values of integrators during the programming. The equation is programmed in the form $y' = 1 + t^2 - y^2$. The program diagram is described by the equation $-u' = v - u^2 =$ $= 1 + t^2 - u^2$, i.e. $-u' + u^2 = 1 + t^2$, $u(0) = 0$, the function $u = -t$ is its solution.

If we apply the method of addition (taking-away) of inverter to every input of integrator, then we get the program diagram in the fig.15. The program diagram is described again

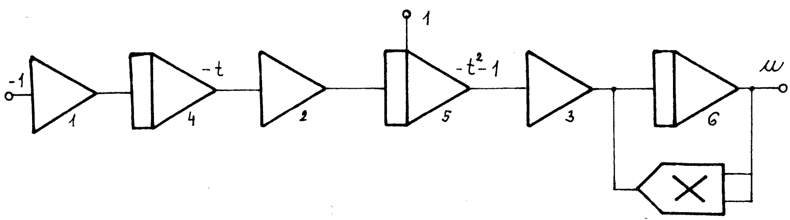


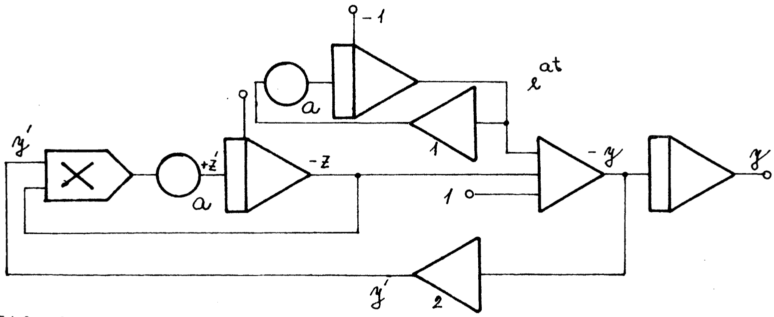
Fig. 15

by differential equation $-u' + u^2 = 1 + t^2$ with the initial condition $u(0) = 0$, the function $u = -t$ is its solution, too. We can optimalize the program diagram in the fig.15 by omission of the inverters 1 and 2, further we get the program diagram in the fig.14 by omission of the inverter 3 and by a change of sign of input machine unit on the beginning of the integral string and by a change of sign at the initial value at the integrator 5.

In the similar way we proceed as well at more complicated non-linear differential equations, like for example at the equation

$$y' + e^{ay} = 1 + e^{at} , \quad y(0) = 0 , \quad t \in \langle -1;1 \rangle \tag{7}$$

The function y = t is solution of the equation (7). The program diagram for solution of the equation (7) in the interval $t \in \langle 0,1 \rangle$ is in the fig.16. The function $z = e^{ay}$ is generated



Fif. 16

by solution of the equation $z' = ay'.e^{ay} - ay'.z$ with the initial condition $z(0) = e^{ay(0)} = 1$. The program diagram for solution of the equation (7) in the interval $t \in \langle 0;-1 \rangle$ is in the fig.17, we do not consider a change of signs of integrators. The function $v = e^{at}$ is generated by solution of the equation $v' - av = 0$, $v(0) = 1$. In accordance with the fig.17 it holds $r' = au'.r$, $r(0) = 1$, i.e. $r = e^{au}$, $u(0) = 0$. Further it holds $u' = -1 + r - e^{-at} = -1 + e^{au} - e^{-at}$, i.e. the program diagram in the fig.17 is desc ribed by the equation

$$u' - e^{au} = -1 - e^{-at} , \quad u(0) = 0 \tag{7a}$$

the function u = -t is its solution.

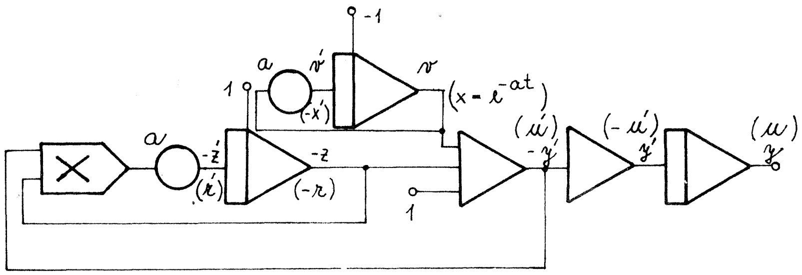


Fig. 17

If we apply the modificated method with addition, taking--away respectively, of inverters on inputs of integrators and if we get out from the origin program diagram in the fig.16, we get the program diagram in the fig.18. The inverters 1 and

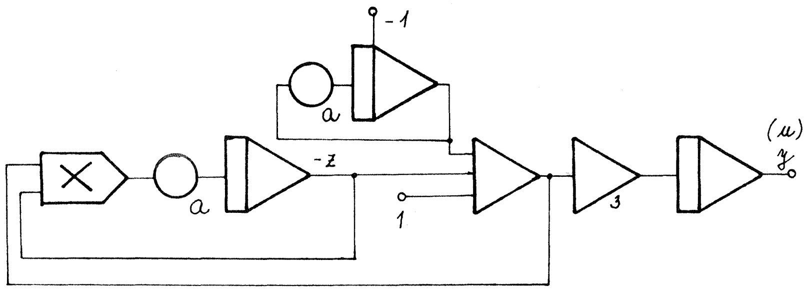


Fig. 18

2 are omited, the inverter 3 is addead. The program diagram in the fig.18 is described by the equation (7a), too.

Comment: How it was said, it is not (from reason of easier orientation in the program diagrams) considered a taking-away of machine unit on outputs of some calculating units. This reality can be easy removed during the solution of a given problem on computer by amplitude transformation or by normalization. The adjusted program diagram from the fig.4 is in the fig.19. It holds for output values u_v of all calculating

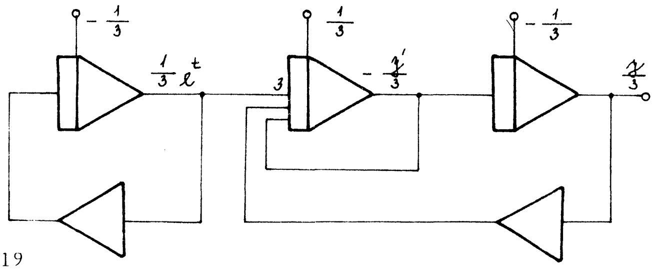


Fig. 19

units $|u_v| < 1$ on the considered interval of solution $t \in \langle 0;1 \rangle$. The adjusted program diagram in accordance with the fig.8 is in the fig.20.

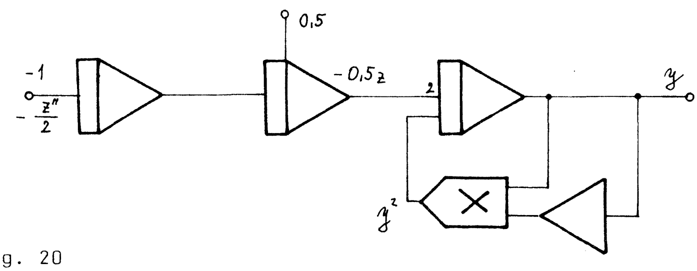


Fig. 20

SOUHRN

ANALOGOVÉ ŘEŠENÍ DANÉ ÚLOHY VE ZPĚTNÉM ČASE

KAREL BENEŠ

V práci jsou uvedeny úpravy analogových programových schémat pro řešení daných úloh (diferenciálních rovnic) ve zpětném čase, tj. úloh s opačně orientovanou časovou osou (osou nezávisle proměnné).

РЕЗЮМЕ

АНАЛОГОВОЕ РЕШЕНИЕ ДАННОГО ЗАДАНИЯ В ОБРАТНОМ ВРЕМЕНИ

К. БЕНЕШ

В работе показаны обработки аналоговых программных карт для решений данных задач /дифференциальных уравнений/ в обратном времени, это значит заданий с временной осью /ось с независимой переменной/ ориентированной в обратную сторону.

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