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MATHEMATICAL DESCRIPTION OF PROGRAM DIAGRAM

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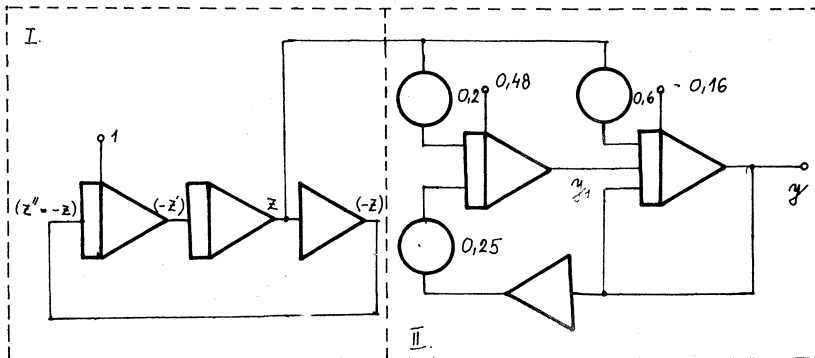
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Introduction

In practise there is sometimes a case to determine the equation to definite program diagram which is solved by this program diagram. In the case of analog program diagram we start from reality that each mathematical operation is simulated on the base of valid physical laws by partial unit. Mathematical relations for the dependence of output value belonging to count unit on the input quality are known. We obtain the system of equations describing the given program diagram, this system of equations is transposed to the equation for the variable which we are interested in. It is convenient to divide the program diagram (until as it is evident from the program diagram) into several independent parts as it is clear from following Figures.

Theoretical and application section

E.g. the program diagram in the Fig.1 is able to be divided into two parts and we are interested in the variable y (the output value of the integrator 2). We denote the output variables of calculate units and complete the mathematical description of single units. We obtain the system of equations which is then transposed to the equations for such the variable we are interested in. It is possible to determine easily the output values of the elements making multiplying by fixed coefficient, inverters and integrators with one input. If we denote e.g. in the Fig.1 (sec.I) the output value of the integrator 2 by the letter z , we shall determine from this value both the output values and the input ones of the remaining units.



The program diagram in the Fig.1 is described by the following system of equations (we suppose the change of signs in inputs of initial conditions, too):

Section II:

$$y = - \int_0^t (0,6 z + y + y_1) dt + 0,16$$

$$y_1 = - \int_0^t (0,2 z - 0,25 y) dt - 0,48 ,$$

i.e.

$$y' = -0,6 z - y - y_1 \quad (1)$$

$$y_1' = -0,2 z + 0,25 y .$$

The system of equations (1) is transposed to the equations of the second order for wanted equation of y

$$y'' = -0,6 z' - y' - y_1' , \quad (2)$$

by supplying for y_1' from the 2nd equations of the system (1) we obtain

$$y'' = -0,6 z' - y' + 0,2 z - 0,25 y ,$$

i.e.

$$y'' + y' + 0,25 y = -0,6 z' + 0,2 z . \quad (2a)$$

The section I of program diagram is described by the equation

$$z'' + z = 0 , z(0) = 0 , z'(0) = 1 , \quad (3)$$

which solution is function $z = \sin t$. The equation (2a) may be rewrite to the form

$$y'' + y' + 0,25 y = -0,6 \cos t + 0,2 \sin t . \quad (4)$$

The initial conditions are determined from the program diagram and from the first equation of the system (1), i.e.

$$y(0) = 0,16$$

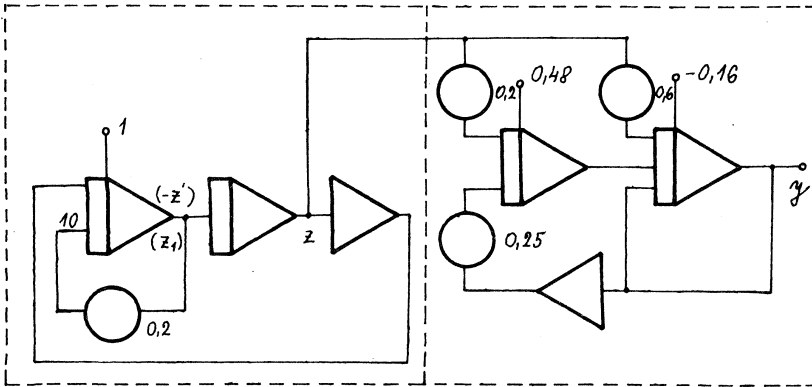
$$y'(0) = -0,6 z(0) - y(0) - y_1(0) =$$

$$= -0,16 + 0,48 = 0,32 .$$

The section II of the program diagram on the Fig.2 is described again the equation (2a), but the function of z is different. This function is described by the equation (in according to program diagram)

$$z'' + 2z' + z = 0, \quad z(0) = 0, \quad z'(0) = 1, \quad (5)$$

We can also proceed by general way in determination of equations of function z that the program diagram is described by the system of equations (see determination of equation for y in the Fig.1) and this we transpose to equation of higher order.



It holds

$$z = - \int_0^t z_1 dt$$

$$z_1 = - \int_0^t (2 z_1 - z) dt,$$

i.e.

$$z' = - z_1, \quad z_1' = z - 2 z_1. \quad (6)$$

The system (5) is transposed to equation of the second order for z , i.e.

$$z'' + 2z' + z = 0, \quad z(0) = 0, \quad z'(0) = 1. \quad (7)$$

The equations (5) and (6) are the same. The characteristics equation to the equation (7) has the form $\lambda^2 + 2\lambda + 1 = 0$ and it has the double root $\lambda_{1,2} = -1$.

Then the solution has the form

$$z = (c_1 + t c_2) e^{-t}, \quad (8)$$

Further it holds

$$z' = c_2 e^{-t} - e^{-t} (c_1 + t c_2). \quad (8a)$$

We will determine the coefficients c_1 and c_2 from initial conditions, it is $c_1 = 0$, $c_2 = 1$, then the solution of the equation (7) is according to (8)

$$z = t e^{-t}. \quad (9)$$

The program diagram in the Fig.2 is described by the equation (see (2a))

$$y'' + y' + 0,25 y = -0,6 e^{-t} + 0,6 t e^{-t} + 0,2 t e^{-t},$$

i.e.

$$y'' + y' + 0,25 y = 0,8 t e^{-t} - 0,6 e^{-t}, \quad (10)$$

$$y(0) = 0,16, \quad y'(0) = -0,16 + 0,48 = 0,32.$$

There is more complicated contigence of the program diagram in the Fig.3, when we are interested in the run of the output value of summer.

According to the Fig.3 it holds

$$y = -0,25 z - y_2$$

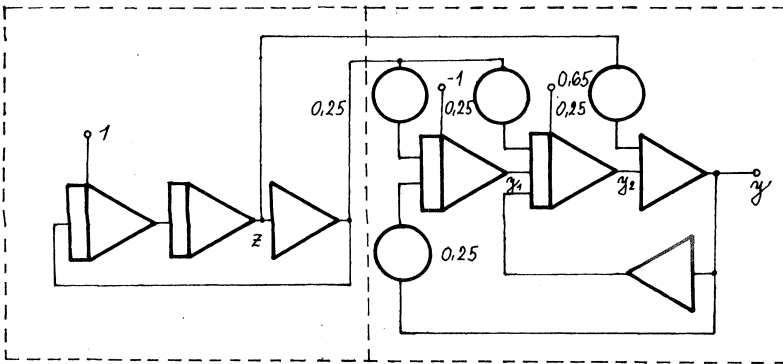
$$y_2 = -\int_0^t (-0,25 z + y_1 - y) dt - 0,65$$

$$y_1 = - \int_0^t (-0,25 z + 0,25 y) dt + 1, \quad (11)$$

$$y_2' = 0,25 z - y_1 + y$$

$$y_1' = 0,25 z - 0,25 y$$

$$y_2'' = 0,25 z' - y_1' + y' = 0,25 z' - 0,25 z + 0,25 y + y' .$$



From the first equation of the system (11) it holds

$$\begin{aligned} y'' &= -0,25 z'' - y_2'' = - \\ &= -0,25 z'' - 0,25 z' + 0,25 z - 0,25 y - y' \end{aligned}$$

i.e.

$$y'' + y' + 0,25 y = -0,25 z'' - 0,25 z' + 0,25 z, \quad (12)$$

where according to (3) is $z = \sin t$. Then the equation (12) has the form

$$y'' + y' + 0,25 y = 0,5 \sin t - 0,25 \cos t. \quad (13)$$

For initial conditions $y(0)$ and $y'(0)$ it holds

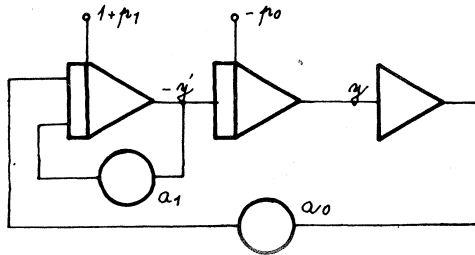
$$y(0) = -0,25 z(0) - y_2(0) = 0,65$$

$$\begin{aligned}
 y'(0) &= -0,25 z'(0) - y_2'(0) = \\
 &= -0,25 - (0,25 z(0) - y_1(0) + y(0)) = \\
 &= -0,25 + 1 - 0,65 = 0,1 .
 \end{aligned}$$

There are dynamic systems excited by Dirac's function occur often in practice. These systems respective program diagrams of these systems can be described in several ways. In the Fig. 4 there is the program diagram for the solution of the equation

$$y'' + a_1 y' + a_0 y = 0, \quad y(0) = p_0, \quad (14)$$

$$y'(0) = 1 + p_1,$$



in the Fig.5 there is the program diagram for the solution of the equation

$$y_1'' + a_1 y_1' + a_0 y_1 = \delta(t), \quad (15)$$

$$y_1(0) = p_0, \quad y_1'(0) = p_1,$$

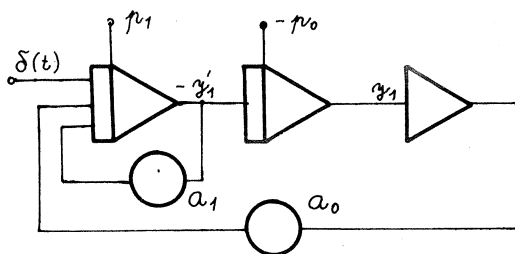
where $\delta(t)$ is Dirac's function. Laplace's image of the equation (14) has the form

$$s^2 Y(s) - s p_0 - 1 - p_1 + a_1 (s Y(s) - p_0) + a_0 Y(s) = 0,$$

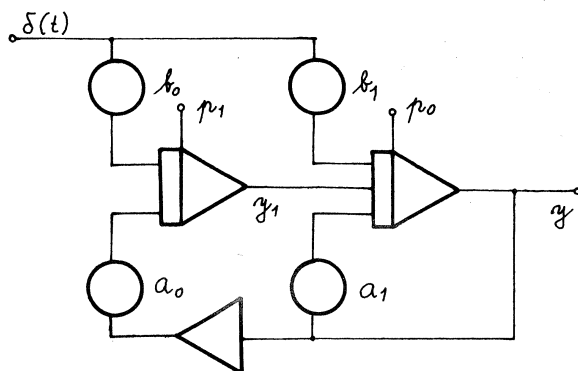
Laplace's image of the equation (15) has the form

$$s^2 Y_1(s) - s p_0 - p_1 + a_1 (s Y_1(s) - p_0) + a_0 Y_1(s) = 1.$$

The imagines (14) and (15) are except of describing of seeked functions identical and the equations have identical solutions. The program diagrams in the Fig.4 and 5 are equivalent and the corresponding physical systems are equivalent, too.



In a similar way the program diagram in the Fig.6 is described by the equations (compare with the Fig.1)



$$\begin{aligned}
 y_1 &= - \int_0^t (b_0 \delta(t) - a_0 y) dt - p_1 = \int_0^t a_0 y dt - b_0 - p_1 \\
 y &= - \int_0^t (b_1 \delta(t) + y_1 + a_1 y) dt - p_0 = \\
 &= - \int_0^t (y_1 + a_1 y) dt - b_1 - p_0 ,
 \end{aligned} \tag{16}$$

i.e.

$$y_1' = a_0 y, \quad y' = -y_1 - a_1 y \quad (17)$$

We will transposed the system (17) to equation of the second order for function y ,

i.e.

$$y'' + a_1 y' + a_0 y = 0 \quad (18)$$

with initial conditions according (17)

$$y(0) = -b_1 - p_0, \quad y'(0) = b_0 + p_1 + a_1(b_1 + p_0).$$

After the initiation of the solution (arrival Dirac's impulse) initial values of the integrator change about value $-b_0$ respective $-b_1$.

From the system (16) we can proceed also as follows:

$$y_1' = -b_0 \delta(t) + a_0 y, \quad (19)$$

$$y' = -b_1 \delta(t) - y_1 - a_1 y.$$

We will transposed the system (19) to equation of the second order for y ,

$$y'' = -b_1 \delta'(t) + b_0 \delta(t) - a_0 y - a_1 y',$$

i.e.

$$y'' + a_1 y' + a_0 y = -b_1 \delta'(t) + b_0 \delta(t) \quad (20)$$

with initial conditions $y(0) = -p_0$, $y'(0) = p_1 + a_1 p_0$.
(At function $\delta(t)$ we take her limit from the right).

Laplace's imagine of equation (18) has the form

$$s^2 Y(s) + s b_1 + s p_0 - b_0 - p_1 - a_1 (b_1 + p_0) + \\ + a_1 s Y(s) + a_1 b_1 + a_1 p_0 + a_0 Y(s) = 0,$$

i.e.

$$s^2 Y(s) + a_1 s Y(s) + a_0 Y(s) = -s(b_1 - p_0) + b_0 + p_1 \cdot \quad (21)$$

Laplace's image of the equation (20) has the form

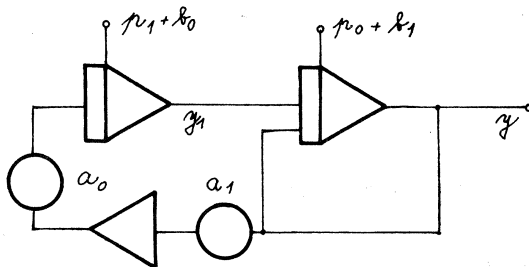
$$s^2 Y(s) + s p_0 - p_1 - a_1 p_0 + a_1 s Y(s) + a_1 p_0 + a_0 Y(s) = -s b_1 + b_0 ,$$

i.e.

$$s^2 Y(s) + a_1 s Y(s) + a_0 Y(s) = -s(b_1 - p_0) + b_0 + p_1 \cdot \quad (22)$$

The equations (21) and (22) are identical, the mentioned processes of derivation of the description of the program are equivalent.

The program diagram in the Fig.7 is equivalent with the program diagram in the Fig.6, which is described by the system (17) respective by the equation (18) with the initial conditions $y(0) = -p_0 - b_1$, $y'(0) = b_0 + p_1 + a_1 (b_1 + p_0)$.



Summary

In the work there is described the method of determination of differential equation solved according to given program diagram.

Souhrn

MATEMATICKÝ POPIS PROGRAMOVÉHO SCHÉMA

V práci je popsána metoda určení diferenciální rovnice řešené dle daného programového schéma.

Р е з ю м е

МАТЕМАТИЧЕСКОЕ ОПИСАНИЕ ПРОГРАМНОЙ СХЕМЫ

В работе описан метод извлечения дифференциального уравнения для данной программной схемы.

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