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TIME CONDITIONAL PROPOSITIONS

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In this paper I want to perform such a formalization of propositions the truth of which depends on time, so that deductions obtained in extended predicate calculus of the first order are due to intuitions about time-truth relations. Works on problems of logic of time (chronologic logic etc.) do not deal with this question. The selection of synthetic works dealing with implementation of time into logical systems is referred to in bibliography.

1. Introduction

Let's suppose that time acquires values from the set of rational numbers Q and is the entity of specific type. Then let's consider discrete linear time (according terminology e.g. [1]) so as it corresponds to common human dimensions and let's take no account of some possible properties of time, which physics or philosophy admit.

Let's have constructable function which can transform conventional time into the set Q and vice versa. In [3] this function is called dating procedure. Specially, we require from this function to deliminate in set Q interval corresponding with time predicate "now". Generally, it will be an interval, but in dependence on context, i.e. in this case of exactness it can be only an element of the set Q. Actual specification of predicate "now" is necessary for the interpretation of so called pseudodata (acc. [3]) such for instance "today", "yesterday", "this year" etc.

2. Presumptions of formalization

Let the propositions, analysed here, state about the world outside time and refer - in the sense of validity of the statement - to a certain time set (subset of Q). Let's call the first component of such propositions proper assertion and the second time assertion. Evidently, each of propositions can be transformed to this form. In an extreme case Q itself will be the proper time set (regardless dating procedure).

We are not able to formalize propositions, the truth of which depends on time, only on basis of the propositional logic, because the aparatus of propositional logic is not sufficient for adequate formalization of time assertion. The authors dealing with this problems either write time dependent propositions in metalanguage (only for demonstration case) or they used so called operator of time realization of proposition. The second way, though more adequate in inconvenient in the respect that formalization of proper assertions is limited to means of propositional logic (otherwise the logic of time would be build as the second order theory).

Let's consider the proper assertion to be nonanalysed propositions. We will not deal with its structure, so generality will not be touched, we do not demand any restrictions. Let's use the time assertion in the sense of the predicate logic, i.e. let's deliminate the class of time individual variables and the class of time predicates for which suitable symbols must be defined. In addition, if we introduce explicitely type differentiation, the above mentioned exception dealing with proper assertions seems to be groundless.

3. Formalization

Let some assertion A (proper assertion) be valid for the time set T. Let this fact be denoted by A(T). We establish the definition of this expression by the way of the mentioned formalization means so that the natural demand is respected in the way that negation of such an expression must differentiate two cases. If the statement A(T) is non-valid it can mean a) the proper assertion is invalid at all or b) it is not accepted for the time set given.

Let's define it like this:

$$A(T) = \text{df } \forall t[T(t) \longrightarrow A] \ , \qquad \text{(D1)}$$

where t is a time individual variable, T(t) is a propositional function which limits the time set T (this quasiambiguity, according to me, contributes to better readability), the implication is material.

This definition can be postulated alternatively (in accordance with the principles of predicate calculus) as follows:

$$A(T) = \text{df } \exists t T(t) \longrightarrow A \ , \qquad \text{(D2)}$$

the adequacy of which is intuitively less evident.

Expression A must not be parametrized by time, i.e. to content free time variable which is in accordance with presumptions mentioned above. With this exception A can be any wellformed formula of predicate calculus. If the structure of expression A is more complicated, we will put it into square brackets.

The expression

$$\exists t T(t) \wedge \neg A$$

is according both definitions the negation of the time conditional proposition A(T), i.e.

$$\neg A(T) \leftrightarrow \exists t T(t) \wedge \neg A ,$$

which corresponds with the mentioned demands on falce of A(T).

4. Consequences of formalization

Intuitions of time-logical relations support even provable formulas consequent from the definition. They are above all theorems for different proper assertions on the common time set:

$$A(T) \wedge B(T) \leftrightarrow [A \wedge B](T) \qquad (T1)$$

$$A(T) \vee B(T) \leftrightarrow [A \vee B](T) \qquad (T2)$$

$$[A \rightarrow B](T) \rightarrow [A(T) \rightarrow B(T)] \qquad (T3)$$

$$[A \rightarrow B](T) \rightarrow [A \rightarrow B(T)] \qquad (T4)$$

Formula T1 characterizes the conjuction and T2 the disjunction with the same time assertion. These formulas demand no comment. Theorems T3 and T4 reflect a more complicated character of the implication in a time context. Their meaning is more obvious from the equivalent formulas, which arise from T3 and T4 after the change of premises

$$A(T) \rightarrow [[A \rightarrow B](T) \rightarrow B(T)] \qquad (T3')$$

$$A \rightarrow [[A \rightarrow B](T) \rightarrow B(T)] , \qquad (T4')$$

when they express two possible variations of elimination of the time dependent implication and show possibilities to formulate the rule modus ponens for the time conditional propositions.

Another group of provable formulas expresses the relations for the conjunction and the disjunction of the proper assertion on the different time sets:

$$A(T_1) \wedge A(T_2) \leftrightarrow A(T_1 \cup T_2) \qquad (T5)$$

$$A(T_1 \cap T_2) \rightarrow A(T_1) \vee A(T_2) \qquad (T6)$$

Symbols $\cap$ and $\cup$ denote here an intersection and a union of the time sets respectively. Immediate results are in this case poorer and any simple characteristic of the implication in this respect is not accepted, with the only exception that T_1 is in the set implication relation with T_2:

$$A(T_1' \cup T_2) \rightarrow [A(T_1) \rightarrow A(T_2)] \quad , \qquad (T7)$$

symbol $'$ denotes complement of set T_1 to Q.

In the case of different proper assertions on the different time sets there is only one formula provable:

$$A(T_1) \wedge B(T_2) \leftrightarrow [A \vee B](T_1 \cup T_2) \qquad (T8)$$

The following provable formulas are valid for the mixed expressions consisting of the time conditional propositions and from the propositions independent of time:

$$A(T) \wedge B \rightarrow [A \wedge B](T) \qquad (T9)$$

$$A(T) \vee B \leftrightarrow [A \vee B](T) \qquad (T10)$$

$$[A(T) \rightarrow B] \rightarrow [A \rightarrow B](T) \qquad (T11)$$

$$[A \rightarrow B(T)] \leftrightarrow [A \rightarrow B](T) \qquad (T12)$$

The usage of the negation at the formulation of the time dependent propositions has a special place. The negation of the time conditional proposition $\neg A(T)$ was referred to above. If in the time conditional proposition the proper assertion is negated, i.e. $[\neg A](T)$, we obtain a proposition of a different quality than $\neg A(T)$. The relation between these propositions is

$$\neg A(T) \longrightarrow [\neg A](T) \qquad (T13)$$

and this implication cannot be conversed. Nevertheless it corresponds with the wanted intentions, because the proposition $\neg A(T)$ has a different meaning than the proposition $[\neg A](T)$.

5. Extending the system of the predicate calculus

We insert the theory of the time conditional propositions into a suitable system of the predicate logic of the first order so that we extend this system with the definition of time conditional proposition (D1 or D2), we complete the set of axioms with axioms characteristic for the time depending propositions and we add into the set of rules of inference the rules the presumptions of which are the time dependent propositions.

In this sense the set of axioms can be extended only by one formula

$$A \longrightarrow A(T) \ . \qquad (AT)$$

We add two variants of modus ponens for the time dependent propositions to the rules of the inference of the given system:

(TMP1) If the formulas $[A \longrightarrow B](T)$ and $A(T)$ are deducible, then the formula $B(T)$ is deducible, as well.

(TMP2) If the formulas $[A \longrightarrow B](T)$ and A are deducible, then the formula $B(T)$ is deducible, as well.

6. The semantics of an extended system

Let's suppose the semantics of the original system of the predicate logic to be introduced in the usual way, i.e.

we take suitable universum U and interpret the formulas in the standard way above it. We use for the interpretation of the extended system a set $\mathcal{T}$ (subset of Q). Next, we restrict a 0-ary function that gives the value from $\mathcal{T}$, for which the predicate "now" is true and the relation of ordering " $\leq$ " on the set $\mathcal{T}$. The formulas with the time parameter are then interpreted above the Cartesian product $U^n \times \mathcal{T}$, where $n \geq 0$ is a number of non-time variables in the interpreted formula.

As to the verbal interpretation of the time conditional propositions, we must point out an important fact that has not been explicitely stated yet. We suppose that all time conditional propositions are asserted in the time "now". The time conditional propositions, which are asserted in any time given explicitely, are not admissible, as it is evident from conditions of definitions D1 and D2. Such possibility - permitting an iteration of time condition, would lead to undesirable results as to the time localization of the proper assertions.

7. Applications of the extended system

The extension of the system of the predicate calculus is realized by simple means and it is founded on the classical logical principles, which seems to be the greatest advantage for possible applications.

In the formal system of the predicate logic, which was modified in the mentioned way, the time operators of the logic of time, the chronologic logic etc. can be implemented and we can add proper definitions, axioms and rules of inference. In this way we obtain formal systems, which are comparable with the systems described in the references.

The applications of the presented system are immediately possible in the database systems respecting the time factor and in the systems of artificial intelligence. It can be applied in the deontic logic and in the normative systems, as well.

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ČASOVĚ PODMÍNĚNÉ VÝROKY

Souhrn

V článku je aplikován pojem časově podmíněného výroku a je zavedena formální definice na bázi predikátového počtu 1.řádu. Dále jsou zde uvedeny teorémy, které bezprostředně vyplývají z této formalizace a charakterizují základní logické spojky v časově podmíněných výrocích. Kromě toho je navrženo rozšíření systému klasického predikátového počtu o časový kontext, jsou diskutovány sémantické otázky a možnosti aplikace.

ВРЕМЕННО УСЛОВНЫЕ ВЫСКАЗЫВАНИЯ

Резюме

В статье развертывано понятие временно условного высказывания и проведено формальное определение на базе предикатного исчисления первого порядка. Далее здесь приведены теоремы, которые непосредственно вытекают из этой формализации и

характеризуют элементарные логические связки в временно условных высказываниях. Кроме того предложено расширение системы классического предикатного исчисления временным контекстом, дискутированы семантические вопросы и возможности применения.

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