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Sborník prací Přírodovědecké fakulty University Palackého v Olomouci. Matematika, Vol. 14 (1974), No. 1, 83--91

Persistent URL: http://dml.cz/dmlcz/120035

1974 — ACTA UNIVERSITATIS PALACKIANAE OLOMUCENSIS
FACULTAS RERUM NATURALIUM — TOM 45

Katedra algebry a geometrie přírodovědecké fakulty University Palackého v Olomouci: Vedoucí katedry: Prof. RNDr. Josef Šimek

A GENERALIZATION OF CONFIGURATIONS IN THE SENSE OF GLADKIJ

by JOSEF ZEDNÍK

(Received June 26th, 1973)

In [1] Gladkij defined configurations of order n for every language and every natural number n. In this paper, the author defines the notion of a configuration of a language whose order is an arbitrary ordinal number and studies some properties of these configurations. Furthermore, the author defines the class of bounded [2] and the class of finitely characterizable languages and studies some connections between these classes.

We now introduce basic notions. If V is a set, we denote by V^* the free monoid on V, i.e. the set of all finite sequences of elements of the set V including the empty sequence Λ, this set being provided with a binary operation of concatenation. The elements of V^* are called strings. We identify one-member sequences with elements of V; it follows $V \subseteq V^*$. If $x = x_1 x_2 \ldots x_n \in V^*$, where n is a natural number and $x_i \in V$ for $i = 1, 2, \ldots, n$, we put $|x| = n$; further we put $|\Lambda| = 0$. For $x, y \in V^*$ we put $x \preceq y$ if there exist such strings $u, v \in V^*$ that $y = uxv$. For $P \subseteq V^* \times V^*$; $x, y \in V^*$ we write $y \to x(P)$ instead of $(y, x) \in P$ and put $y \Rightarrow x(P)$ if there exist such strings $u, v, s, t \in V^*$ that $y = usv$, $x = utv$, and $s \to t(P)$. Finally we put, for $x, y \in V^*$, $y \overset{*}{\Rightarrow} x(P)$ if there exists an integer $n \geqq 0$ and some strings $t_0, t_1, \ldots, t_n \in V^*$ such that $y = t_0$, $t_n = x$, and $t_{i-1} \Rightarrow t_i(P)$ for $i = 1, 2, \ldots, n$. The sequence $(t_i)_{i=0}^{n}$ is called a y-derivation of x of length n in P.

An ordered pair (V, L) where V is a set and $L \subseteq V^*$ is called a language.

1. Generalization of configurations in the sense of Gladkij

1.1. Definition. Let (V, L) be a language, $M \subset L$ a set, and $x, y \in V^*$. We put $y > x(M)$ if $uyx \in L - M$ implies $uxv \in L$ for each $u, v \in V^*$. For $M = \phi$ write $y > x$ instead of $y > x(\phi)$.

1.2. Lemma. Let (V, L) be a language, $M \subseteq N \subset L$ sets. Then the relation $> (M)$ is a subrelation of (the relation) $> (N)$.

Proof. Let $x, y \in V^*$ be strings, $M \subseteq N \subset L$ sets, and $y > x(M)$. Then for each $u, v \in V^*$ the condition $uyv \in L - N$ implies $uyv \in L - M$ and it follows $uxv \in L$. Thus $y > x(N)$.

1.3. Definition. Let (V, L) be a language, α an ordinal number. We put $E_\alpha = \phi[\bar{E}_\alpha = \phi]$ for $\alpha = 0$; let $\alpha > 0$ be an ordinal number and suppose that E_ι $[\bar{E}_\iota]$ has been defined for each $\iota < \alpha$. We put $C_\iota = \{x \in V^*;\ \exists\, y \in V^*: (y, x) \in E_\iota\}$ $[\bar{C}_\iota = \{x \in V^*;\ \exists\, y \in V^*: (y, x) \in \bar{E}_\iota\}]$ for each ordinal number $\iota < \alpha$. We put $Q_\alpha(x) = \{q \in L;\ \exists\, u, v, z, z_1, z_2, a, b \in V^*: q = uxv,\ z \in \bigcup_{\iota < \alpha} C_\iota,\ z_1 \neq \Lambda \neq z_2,$ and $z = z_1 z_2$, $((u = az_1$ and $z_2 b = xv)$ or $(ux = az_1$ and $z_2 b = v))\}$ $[\bar{Q}_\alpha(x) = \{q \in L;\ \exists\, u, v, z, z_1, z_2, a, b \in V^*;\ q = uxv,\ z \in \bigcup_{\iota < \alpha} \bar{C}_\iota,\ z_1 \neq \Lambda \neq z_2,\ z = z_1 z_2,$ and $((u = az_1$ and $z_2 b = xv)$ or $(ux = az_1$ and $z_2 b = v))\}]$. We put $E_\alpha = \{(y, x) \in V^* \times V^*;\ |y| < |x|,\ y > x,$ and $x > y(Q_\alpha(x))\}$ $[\bar{E}_\alpha = \{(y, x) \in V^* \times V^*;\ 1 = |y| < |x|,\ y > x,$ and $x > y(\bar{Q}_\alpha(x))\}]$. Finally, we put $C_\alpha = \{x \in V^*;\ \exists y \in V^*: (y, x) \in E_\alpha\}$ $[\bar{C}_\alpha = \{x \in V^*;\ \exists\, y \in V^*: (y, x) \in \bar{E}_\alpha\}]$. For $x, y \in V^*$ we say that x is a [strong] configuration of order α of the language (V, L), y its resultant if $(y, x) \in E_\alpha[(y, x) \in \bar{E}_\alpha]$. The elements of the set $C_\alpha[\bar{C}_\alpha]$ are called [strong] configurations of order α of the language (V, L).

1.4. Lemma. $E_\alpha \subseteq E_\beta[\bar{E}_\alpha \subseteq \bar{E}_\beta]$ holds for an arbitrary language (V, L) and for arbitrary ordinal numbers $\alpha \subseteq \beta$.

Proof. For $\alpha = 0$ and for an ordinal number β we have $E_0 = \phi \subseteq E_\beta$. Let $0 < \alpha \leqq \beta$. Then $\bigcup_{\iota < \alpha} E_\iota \subseteq \bigcup_{\iota < \beta} E_\iota$ and thus $\bigcup_{\iota < \alpha} C_\iota \subseteq \bigcup_{\iota < \beta} C_\iota$. It follows $Q_\alpha(x) \subseteq Q_\beta(x)$ for an arbitrary string $x \in V^*$. If $(y, x) \in E_\alpha$ then $|y| < |x|, y > x$, and $x > y(Q_\alpha(x))$. It follows $x > y(Q_\beta(x))$, by 1.2. Thus $(y, x) \in E_\beta$ holds.

1.5. Corollary. $C_\alpha \subseteqq C_\beta[\bar{C}_\alpha \subseteq \bar{C}_\beta]$ holds for an arbitrary language (V, L) and for arbitrary ordinal numbers $\alpha \leqq \beta$.

1.6. Lemma. Let (V, L) be a language and α an arbitrary ordinal number. Then $\bar{E}_\alpha \subseteq E_\alpha$.

Proof. For an arbitrary ordinal number α, we denote by $V(\alpha)$ the following condition: $\bar{E}_\alpha \subseteqq E_\alpha$. Then $V(0)$ is satisfied. Let $\gamma > 0$ be an arbitrary ordinal number and suppose that $V(\delta)$ is satisfied for each $\delta < \gamma$. If $(y, x) \in \bar{E}_\gamma$ then $1 = |y| < |x|$,

$y > x$, and $x > y(\bar{Q}_\gamma(x))$. By $V(\delta)$, $\bar{C}_\delta \subseteqq C_\delta$ for $\delta < \gamma$. It follows $\bar{Q}_\gamma(x) \subseteqq Q_\gamma(x)$ for each $x \in V^*$. Hence $x > y(Q_\gamma(x))$, by 1.2 and $V(\gamma)$ is satisfied. By induction follows that $V(\alpha)$ is satisfied for each ordinal number α.

1.7. Corollary. Let (V, L) be a language and α an arbitrary ordinal number. Then $\bar{C}_\alpha \subseteqq C_\alpha$.

It is also true, by 1.4. and 1.5:

1.8. Corollary. Let (V, L) be a language and α an arbitrary ordinal number. Then $\bigcup_{\iota < \alpha} E_\iota \subseteq E_\alpha [\bigcup_{\iota < \alpha} \bar{E}_\iota \subseteq \bar{E}_\alpha]$, $\bigcup_{\iota < \alpha} C_\iota \subseteq C_\alpha [\bigcup_{\iota < \alpha} \bar{C}_\iota \subseteq \bar{C}_\alpha]$.

1.9. Lemma. Let (V, L) be a language, α, β arbitrary ordinal numbers with the properties $\alpha \geqq 1$, $\beta \geqq 1$, $\alpha \neq \beta$. Then $(E_\alpha - \bigcup_{\iota < \alpha} E_\iota) \cap (E_\beta - \bigcup_{\iota < \beta} E_\iota) = \phi$ $[(\bar{E}_\alpha - \bigcup_{\iota < \alpha} \bar{E}_\iota) \cap (\bar{E}_\beta - \bigcup_{\iota < \beta} \bar{E}_\iota) = \phi]$.

Proof. E.g., suppose that $\alpha < \beta$. If there exists $(y, x) \in (E_\alpha - \bigcup_{\iota < \alpha} E_\iota) \cap (E_\beta - \bigcup_{\iota < \beta} E_\iota)$ then $(y, x) \in E_\alpha - \bigcup_{\iota < \alpha} E_\iota$ and $(y, x) \in E_\beta - \bigcup_{\iota < \beta} E_\iota = E_\beta - (\bigcup_{\iota < \alpha} E_\iota \cup \bigcup_{\alpha \leqq \iota < \beta} E_\iota)$. It follows $(y, x) \in E_\alpha$ and $(y, x) \notin E_\alpha$ which is a contradiction.

1.10. Corollary. Let (V, L) be a language, α, β arbitrary ordinal numbers, with the properties $\alpha \geqq 1$, $\beta \geqq 1$, and $\alpha \neq \beta$. Then $(C_\alpha - \bigcup_{\iota < \beta} C_\iota) \cap (C_\beta - \bigcup_{\iota < \beta} C_\iota) = \phi$ $[(\bar{C}_\alpha - \bigcup_{\iota < \alpha} \bar{C}_\iota) \cap (\bar{C}_\beta - \bigcup_{\iota < \beta} \bar{C}_\iota) = \phi]$.

1.11. Lemma. If (V, L) is a language then there exists an ordinal number $\alpha > 0$ $[\bar{\alpha} > 0]$ such that $E_\alpha - \bigcup_{\iota < \alpha} E_\iota = \phi$ $[\bar{E}_{\bar{\alpha}} - \bigcup_{\iota < \alpha} \bar{E}_\iota = \phi]$.

Proof. If $V = \phi$ then $V^* = \{\Lambda\}$ and $L = \phi$ or $L = \{\Lambda\}$. Clearly $E_1 = \phi = E_1 - E_0$. Thus $\alpha = 1$ for each language (ϕ, L). Suppose $V \neq \phi$ and $E_\alpha - \bigcup_{\iota < \alpha} E_\iota \neq \phi$ for each ordinal number α, $1 \leqq \alpha < \nu$ where ν is an ordinal number with the property card $\nu >$ card $(V^* \times V^*)$. It follows $E_\nu = \bigcup_{1 \leqq \iota \leqq \nu} (E_\iota - \bigcup_{1 \leqq \lambda < \nu} E_\lambda)$ and the sets $E_\iota - \bigcup_{1 \leqq \lambda < \nu} E_\lambda$ are pairwise disjoint, by 1.9. It follows card $(V^* \times V^*) < \sum_{1 \leqq \iota \leqq \nu}$ card $(E_\iota - \bigcup_{1 \leqq \lambda < \nu} E_\lambda) =$ card $E_\nu \leqq$ card $V^* \times V^*$ which is a contradiction.

1.12. Corollary. If (V, L) is a language then there exists an ordinal number $\alpha[\bar{\alpha}]$ such that $C_\alpha - \bigcup_{\iota < \alpha} C_\iota = \phi$ $[\bar{C}_{\bar{\alpha}} - \bigcup_{\iota < \alpha} \bar{C}_\iota = \phi]$.

1.13. Lemma. Let (V, L) be a language, $\alpha \geqq 1[\bar{\alpha} \geqq 1]$ an ordinal number with the property $E_\alpha - \bigcup_{\iota < \alpha} E_\iota = \phi$ $[\bar{E}_\alpha - \bigcup_{\iota < \bar{\alpha}} \bar{E}_\iota = \phi]$. Then $E_\beta - \bigcup_{\iota < \alpha} E_\iota = \phi$ $[\bar{E}_\beta - \bigcup_{\iota < \bar{\alpha}} \bar{E}_\iota = \phi]$ for each ordinal number $\beta \geqq \alpha$ $[\beta \geqq \bar{\alpha}]$.

Proof. For each $\beta \geqq \alpha$, denote by $V(\beta)$ the following condition: $E_\beta - \bigcup_{\iota < \alpha} E_\iota = \phi$. Then $V(\alpha)$ is satisfied. Let γ be an ordinal number with the property $\gamma > \alpha$. We suppose that $V(\delta)$ is true for each ordinal number $\delta(\alpha \leqq \delta < \gamma)$. It follows $E_\delta = \bigcup_{\iota < \alpha} E_\iota$ for each $\delta(\alpha \leqq \delta < \gamma)$. Thus $\bigcup_{\iota < \gamma} E_\iota = \bigcup_{\iota < \alpha} E_\iota \cup \bigcup_{\alpha \leqq \delta < \gamma} E_\delta = \bigcup_{\iota < \alpha} E_\iota \cup \bigcup_{\alpha \leqq \delta < \gamma} (\bigcup_{\iota < \alpha} E_\iota) = \bigcup_{\iota < \alpha} E_\iota$. If $(y, x) \in E_\gamma$ then $|y| < |x|$, $y > x$, and $x > y(Q_\gamma(x))$. The equality $\bigcup_{\iota < \alpha} E_\iota = \bigcup_{\iota < \alpha} E_\iota$ implies $\bigcup_{\iota < \gamma} C_\iota = \bigcup_{\iota < \alpha} C_\iota$ and therefore $Q_\gamma(x) = Q_\alpha(x)$ for each $x \in V^*$. Thus, $x > y(Q_\alpha(x))$ and $(y, x) \in E_\alpha$. We have proved $E_\gamma \subseteqq E_\alpha$. Since $\alpha < \gamma$ we have $E_\alpha \subseteqq E_\gamma$, by 1.4. Therefore $E_\gamma = E_\alpha$. We have now $E_\gamma - \bigcup_{\iota < \alpha} E_\iota = E_\alpha - \bigcup_{\iota < \alpha} E_\iota = \phi$ which is $V(\gamma)$.

1.14. Lemma. For each language (V, L) there is smallest ordinal number $\varrho \geqq 1$ $[\bar{\varrho} \geqq 1]$ with the property $E_\varrho = \bigcup_{\iota < \varrho} E_\iota$ $[\bar{E}_{\bar{\varrho}} = \bigcup_{\iota < \bar{\varrho}} \bar{E}_\iota]$.

Proof. Let $m = \text{card } V^* \times V^*$, $n > m$ be cardinal number, and ν an ordinal number with the property card $\nu = n$. We put $M = \{\alpha \text{ an ordinal number}; 1 \leqq \alpha \leqq \nu$ and $E_\alpha - \bigcup_{\iota < \alpha} E_\iota = \phi\}$. By 1.11, the set M is a nonempty set of ordinal numbers and, therefore, the set M has the smallest element ϱ. Then $E_\varrho - \bigcup_{\iota < \varrho} E_\iota = \phi$, thus $E_\varrho = \bigcup_{\iota < \varrho} E_\iota$ and ϱ is the smallest ordinal number with this property.

1.15. Lemma. Let (V, L) be a language, $\varrho[\bar{\varrho}]$ the number defined in 1.14. Then $C_\varrho = \bigcup_{\iota < \varrho} C_\iota [\bar{C}_{\bar{\varrho}} = \bigcup_{\iota < \bar{\varrho}} \bar{C}_\iota]$.

1.16. Definition. Let (V, L) be a language, $\varrho \geqq 1$ $[\bar{\varrho} \geqq 1]$ the smallest ordinal number with the property $E_\varrho = \bigcup_{\iota < \varrho} E_\iota [\bar{E}_{\bar{\varrho}} = \bigcup_{\iota < \bar{\varrho}} \bar{E}_\iota]$. We put $E = E_\varrho [\bar{E} = \bar{E}_{\bar{\varrho}}]$, $C = C_\varrho [\bar{C} = \bar{C}_{\bar{\varrho}}]$, $B = L - V^*CV^*$ $[\bar{B} = L - V^*\bar{C}V^*]$.

It follows by 1.6, 1.8, and 1.16:

1.17. Theorem. Let (V, L) be a language. Then $\bar{E} \subseteqq E$, $\bar{C} \subseteqq C$.

It follows by 1.17 and 1.16:

1.18. Theorem. $B \subseteqq \bar{B}$ holds for each language.

1.19. Example. Let $V = \{a\}$, $L = \{a^n;\ n$ an integer, $n \geqq 2\}$. We find the sets $E, \bar{E}, C, \bar{C}, B, \bar{B}$.

The language (V, L) has the following property: $x \in L$ iff $x \in V^*$ and $2 \leqq |x|$. Let $0 \leqq s \leqq r$ be arbitrary integers. Then $a^s > a^r$. It follows that for all $u, v \in V^*$ the condition $ua^sv \in L$ implies $2 \leqq |ua^sv| = |u| + s + |v| \leqq |u| + r + |v| = |ua^rv|$, thus $ua^rv \in L$. Especially, $\Lambda > a^r$ for each integer $r \geqq 0$ and $a > a^r$ for each integer $r \geqq 1$.

Let r, s be arbitrary integers with $r \geqq 0$, $s \geqq 2$. Then $a^r > a^s$. It follows that for each $u, v \in V^*$ the condition $2 \leqq s \leqq |u| + |v| + s = |ua^sv|$ is satisfied, which implies $ua^sv \in L$.

If $r \geqq 1$ is an arbitrary integer then $a^r > \Lambda$ does not hold. E.g., for $u = a$, $v = \Lambda$ we have $|ua^rv| = r + 1$, thus $ua^rv \in L$, and $|u \Lambda v| = 1$, thus $u \Lambda v \notin L$.

Similarly if $r \geqq 2$ is an integer then $a^r > a$ does not hold. E.g., for $u = v = \Lambda$ we have $|ua^rv| = r \geqq 2$, thus $ua^rv \in L$, and $|uav| = 1$, thus $uav \notin L$.

We have proved $a^s > a^r$ and $a^r > a^s$ for arbitrary integers $r, s \geqq 2$. It follows $E_1 = \{(a^r, a^s);\ r, s \text{ integers } 2 \leqq r < s\}$, $C_1 = \{a^s;\ s \geqq 3 \text{ integer}\}$.

Further, we prove Λ, a, $a^2 \notin C_2$. If $\Lambda \in C_2$ then there exists $y \in V^*$ such that $|y| < |\Lambda| = 0$ which is impossible. Therefore $\Lambda \notin C_2$. If $a \in C_2$ then there exists $y \in V^*$ such that $|y| < |a| = 1$ and it follows $y = \Lambda$. Further, we have $a > \Lambda$ $(Q_1(a))$, where $Q_1(a) = \{a^3, a^4, a^5, \ldots\}$. Thus for each $u, v \in V^*$, the condition $uav \in \in L - Q_1(a) = \{a^2\}$ implies $u \Lambda v \in L$, but, e.g. for $u = a$, $v = \Lambda$, the last implication does not hold. Thus $(\Lambda, a) \notin E_2$ and $a \notin C_2$. If $a^2 \in C_2$ then there exists $y \in V^*$ such that $|y| < |a^2| = 2$ which implies $y = \Lambda$ or $y = a$. We have $Q_1(a^2) = \{a^3, a^4, a^5, \ldots\}$, $L - Q_1(a^2) = \{a^2\}$. If $(\Lambda, a^2) \in E_2$ or $(a, a^2) \in E_2$, then $a^2 > \Lambda\ (Q_1(a^2))$ or $a^2 > a(Q_1(a^2))$. Thus for each $u, v \in V^*$ the condition $ua^2v \in L - Q_1(a^2) = \{a^2\}$ implies $u \Lambda v \in L$ or $uav \in L$, but, e.g. for $u = v = \Lambda$, the last implication does not holds. Thus (Λ, a^2), $(a, a^2) \notin E_2$ and $a_2 \notin C_2$. Therefore $E_2 = E_1$, $C_2 = C_1$, $E = E_2 = \{(a^r, a^s);\ r, s \text{ integers, } 2 \leqq r < s\}$, $C = C_2 = \{a^s;\ \text{s integer}, s \geqq 3\}$, $B = \{a^2\}$. The condition $(y, x) \in E$ implies $|y| \geqq 2$ and it follows $\bar{E} = \phi$. Thus $\bar{C} = \phi$ and $\bar{B} = L$.

2. Bounded languages

Let (V, L) be a language in the following text.

2.1. *Definition.* Let R be a subrelation of the relation $>$ in (V, L) with the property $(y, x) \in R$ implies $|y| \leqq |x|$. Then R is called a sufficient set for (V, L).

2.2. *Definition.* Let R be a sufficient set for (V, L) and $(y, x) \in R$ implies $|y| = 1$. Then R is called a strongly sufficient for (V, L).

Clearly the following two lemmas hold:

2.3. *Lemma.* Let R be a strongly sufficient set for (V, L). Then R is a sufficient for (V, L).

2.4. *Lemma.* Let R be a [strongly] sufficient set for (V, L), $S \subseteqq R$ a set. Then S is a [strongly] sufficient set for (V, L).

2.5. *Definition.* Let R be a sufficient set for (V, L). For each $s \in L$ we put $s \in B_R$ iff for each $t \in L$ the condition $t \overset{*}{\Rightarrow} s(R)$ implies $|t| = |s|$.

2.6. Lemma. Let R be a sufficient set for (V, L). Then, for each $z \in L$, there exists $s \in B_R$ such that $s \overset{*}{\Rightarrow} z(R)$.

Proof. There exists $s \in L$ such that $s \overset{*}{\Rightarrow} z(R)$; e.g., $s = z$. We choose a string s such that $s \overset{*}{\Rightarrow} z(R)$ and $|s|$ is minimal. Then $s \in B_R$.

2.7. Definition. Let R be a sufficient set for (V, L).

(1) If $s, t \in V^*$, $s \Rightarrow t(R)$, then we put $|(s, t)|_R = \min \{|q|; (p, q) \in R, s \Rightarrow t(\{(p, q)\})$.

(2) If $s, t \in V^*$ and $(t_i)_{i=0}^p$ is an s-derivation of t in R, then we put $\|(t_i)_{i=0}^p\|_R = 0$ for $p = 0$ and $\|(t_i)_{i=0}^p\|_R = \max \{|(t_{i-1}, t_i)|_R; i = 1, 2, \dots p\}$, otherwise. The integer $\|(t_i)_{i=0}^p\|_R$ is called the norm of the s-derivation $(t_i)_{i=0}^p$ of t in R.

(3) If $s, t \in V^*$, $s \overset{*}{\Rightarrow} t(R)$, then we put $\|(s, t)\|_R = \min \{\|(t_i)_{i=0}^p\|_R; (t_i)_{i=0}^p$ is an s-derivation of t in $R\}$. The integer $\|(s, t)\|_R$ is called the norm of the ordered pair (s, t).

(4) If $t \in L$, then we put $\|t\|_R^L = \min \{\|(s, t)\|_R; s \in B_R, s \overset{*}{\Rightarrow} t(R)\}$. The integer $\|t\|_R^L$ is called the norm of the element $t \in L$ in R.

2.8. Definition. Let R be a sufficient set for (V, L). We put $Z_R = \{(y, x) \in R; \exists z \in L: |x| \leqq \|z\|_R^L\}$.

2.9. Definition. Let R be a sufficient set for (V, L). Then (V, L) is called (1) R-bounded if the sets V, B_R, Z_R are finite (2) R-hyperbounded if the sests V, B_R, R are finite.

The following lemma is a corollary of theorem 3.11 of [2]:

2.10. Lemma. Let $R \subseteqq S$ be [strongly] sufficient sests for (V, L). Each [strongly] R-bounded language (V, L) is [strongly] S-bounded.

2.11. Theorem. (1) The set $E[\bar{E}]$ is [strongly] sufficient for each (V, L). (2) Each $\bar{E}$-bounded language (V, L) is E-bounded. (3) $B_E = B$, $B_{\bar{E}} = \bar{B}$. (4) There exists an E-bounded language (V, L) which is not $\bar{E}$-bounded.

Proof. (1) There exists the smallest ordinal number $\varrho \geqq 1$ with the property $E_\varrho = \bigcup_{\iota < \varrho} E_\iota$, by 1.14. There is $E = E_\varrho$ and E_ϱ is a subrelation of the relation $>$ in (V, L), by 1.16 and 1.3. Thus E is a sufficient set for (V, L). By 1.17 we have $\bar{E} \subseteqq E$ and thus $\bar{E}$ is a sufficient set for (V, L), by 2.4. At the same time each pair $(y, x) \in \bar{E}$ has the property $|y| = 1$. Thus $\bar{E}$ is a strongly sufficient set for (V, L).

(2) $\bar{E} \subseteqq E$, by 1.17. It follows that each $\bar{E}$-bounded language (V, L) is E-bounded by 2.10.

(3) If $s \in B$, then $s \in V^*$ and there exist no strings $x, y \in V^*$ such that $x \prec s$ and $(y, x) \in E$. Thus the condition $t \overset{*}{\Rightarrow} s(E)$ implies $|t| = |s|$ for each $t \in \overline{L}$. It is $s \in B_E$. Hence $B \subseteqq B_E$. If $s \in B$, then $s \in L$ and for each $t \in L$ the condition $t \overset{*}{\Rightarrow} s(E)$ implies $|t| = |s|$. It follows that there exist no strings $x, y \in V^*$ such that $x \preceq s$ and $(y, x) \in E$. Thus $s \in L$ and $s \notin V^*CV^*$. Hence $s \in B$ and $B_E \subseteqq B$. We have proveed $B_E = B$. Similarly as above we can prove the second equality.

(4) Let (V, L) be the language from example 1.19. Then $V = \{a\}$, $L = \{a^n;$ n integer, $n \geqq 2\}$, $E = \{(a^r, a^s);$ r, s integers, $2 \leqq r < s\}$, $C = \{a^s;$ s integer, $s \geqq 3\}$, $B = \{a^2\}$, $\bar{E} = \bar{C} = \phi$, $\bar{B} = L$. Find the set Z_R. Let $m \geqq 3$ be an arbitrary integer. The longest a^2-derivation of the string a^m in E is $a^2 \Rightarrow a^3 \Rightarrow \ldots \Rightarrow$ $\Rightarrow a^m$. Its length is $m - 2$ and this derivation is a derivation in the one-element subset $\{(a^2, a^3)\} \subseteqq E$. For each $i = 2, 3, \ldots, m - 1$ there is $|(a^i, a^{i+1})|_E = 3$ and thus the integer 3 is the norm of the derivation $a^2 \Rightarrow a^3 \Rightarrow \ldots \Rightarrow a^m$. The other a^2-derivations of the string a^m have the lengths $p = 1, 2, \ldots, m - 3$. Let $(t_i)_{i=0}^p$ be one of these derivations. Then there exist integers i, j, k such that $0 < i \leqq p$, $2 \leqq j <$ $< j + 2 \leqq k$, $t_{i-1} = a^j \Rightarrow a^k = t_i$, the condition $t_{i-1} \Rightarrow t_i(\{(a^2, a^3)\})$ does not hold, and the condition $t_{i-1} \Rightarrow t_i(\{(a^2, a^{2+(k-j)})\})$ holds. Thus $|(t_{i-1}, t_i)|_E = 2 + (k - j) \geqq$ $\geqq 4$. Hence $\|(a^2, a^m)\|_E = 3 = \| a^m \|_E^L$. It follows $Z_E = \{(a^2, a^3)\}$. We have proved that the sets V, B, Z_E are finite and thus, the language (V, L) is E-bounded. But the set $\bar{B} = L$ is not finite and thus the language (V, L) is not $\bar{E}$-bounded.

3. Finitely characterizable languages

3.1. Definition. Let (V, L) be a language, α an arbitrary ordinal number, $C_\alpha[\bar{C}_\alpha]$ the set of all [strong] configurations of order α of (V, L). We put $P_\alpha = \{x \in C_\alpha;$ the conditions $x' \in C_\alpha$, $x' \preceq x$ imply $x = x'\}$ $[\bar{P}_\alpha = \{x \in \bar{C}_\alpha;$ the conditions $x \in \bar{C}_\alpha$, $x' \preceq x$ imply $x = x'\}]$. An element of P_α $[\bar{P}_\alpha]$ is called a [strong] simple configuration of order α of (V, L).

3.2. Definition. Let (V, L) be a language. We put $v = \{x \in V^*;\ \exists\, u, v \in V^*: uxv \in$ $\in L\}$. An element of v is called necessary.

3.3. Definition. Let (V, L) be a language. We put $D_\alpha^v = E_\alpha \cap (v \times P_\alpha)$ $[\bar{D}_\alpha^v =$ $= \bar{E}_\alpha^v \cap (v \times \bar{P}_\alpha)]$. For $x \in V^*$ we put $x \in P_\alpha^v[x \in \bar{P}_\alpha^v]$ iff there exists $y \in V^*$ such that $(y, x) \in D_\alpha^v[(y, x) \in \bar{D}_\alpha^v]$; we say that x is a [strong] simple configuration of order α with a necessary resultant y of (V, L).

3.4. Definition. Let (V, L) be a language, $\varrho[\bar{\varrho}]$ the smallest ordinal number with the property $E_\varrho = \bigcup_{\iota < \varrho} E_\iota$ $[\bar{E}_{\bar{\varrho}} = \bigcup_{\iota < \bar{\varrho}} \bar{E}_\iota]$. We put $D^v = \bigcup_{\iota \leqq \varrho} D_\iota^v$ $[\bar{D}^v = \bigcup_{\iota \leqq \bar{\varrho}} \bar{D}_\iota^v]$.

3.5. Lemma. $D^v \subseteqq E$ $[\bar{D}^v \subseteqq \bar{E}]$ for each language (V, L).

Proof. If ϱ is the smallest ordinal number with the property $E_\varrho = \bigcup_{\iota < \varrho} E_\iota$, then $D^v =$

$$= \bigcup_{\iota \leqq \varrho} D_\iota^v = D_\varrho^v \cap \bigcup_{\iota < \varrho} D_\iota^v \subseteqq E_\varrho \cap \bigcup_{\iota < \varrho} E_\iota = E_\varrho = E.$$

3.6. Definition. Let (V, L) be a language. We say that (V, L) is [strongly] finitely characterizable if (V, L) is $[\bar{D}^v]$ D^v-hyperbounded.

3.7. Remark. Clearly if (V, L) is a language, $\bar{\varrho}$ from 1.14 is a natural number for (V, L), and each $x \in V$ is necessary, then (V, L) is finitely characterizable (in our sense) iff (V, L) is finitely characterizable in the sense of Gladkij [1].

3.8. Lemma. There exists an E-bounded language (V, L) which is not finitely characterizable.

3.9. Remark. Novotný has proved that for $V = \{a, b, c, d\}$, $L_1 = \{ba^{2k} ca^{2^{2n}-k}b;$ $n = 0, 1, \ldots; k = 1, 2, \ldots, 2^{2n}\}$, $L_2 = \{ba^{2^{2n+1}-k} da^{2k} b;\ n = 0, 1, \ldots;\ k = 1, 2, \ldots, 2^{2n+1}\}$, $L = L_1 \cup L_2$ the language (V, L) satisfies the conditions of lemma 3.8. Put $R = \{(aca, a^3c), (ada, da^3), (bda, ba^2c), (acb, da^2b)\}$. He has proved that: (1) $R \subseteq E$, thus R is a finite sufficient set for (V, L) (2) $B_R \subseteq \{x \in L; |x| \leq 4\}$, thus B_R is a finite set. Hence the language (V, L) is R-bounded (even R-hyperbounded) and it is also E-bounded, by (1) and 2.10. Novotný has proved that the set D^v is not finite and thus the language (V, L) is not finitely characterizable at the same time.

3.10. Theorem. If a language is finite characterizable then it is E-bounded.

Proof. If a language (V, L) is finitely characterizable, then (V, L) is D^v-hyperbounded and thus also D^v-bounded. It follows that the language (V, L) is E-bounded, by 3.5 and 2.10.

3.11. Remark. The converse implication to 3.10 is false by 3.8.

3.12. Corollary. If a language is strongly finitely characterizable then it is $\bar{E}$-bounded.

3.13. Problem. Does there exist an $\bar{E}$-bounded language such that it is not strongly finitely characterizable?

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Souhrn

ZOBECNĚNÍ KONFIGURACÍ VE SMYSLU GLADKÉHO

JOSEF ZEDNÍK

Tato práce se zabývá zobecněním konfigurací Gladkého [1], a to ve dvou směrech: 1. Připouští se, aby výsledek konfigurace měl jakoukoliv délku. 2. Připouští se, aby řádem konfigurace bylo jakékoliv ordinální číslo. Konfigurace Gladkého jsou v našich

úvahách zahrnuty, předpokládá-li se, že délka výsledku je rovna 1, že se připustí jen konfigurace konečného řádu a že v daném jazyce není nepotřebných prvků. Rozlišujeme konfigurace od silných konfigurací, při nichž má výsledek délku rovnou 1. Dvojice skládající se z výsledku a z konfigurace se považuje za pravidlo. Jazyk se nazývá (silně) omezený, existuje-li proň gramatika s pravidly, která se skládají z výsledků a ze (silných) konfigurací. V práci se dokazuje, že každý silně omezený jazyk je omezený. Pak se definují prosté konfigurace jako takové, které neobsahují konfigurace téhož řádu mezi svými vlastními podřetězy. Jazyk se nazývá (silně) konečně charakterizovatelný, jestliže jeho vět bez (silných) konfigurací je jen konečný počet. Ukazuje se, že (silně) konečně charakterizovatelný jazyk je (silně) omezený.

Резюме

ОБОБЩЕНИЕ КОНФИГУРАЦИЙ ГЛАДКОГО

ИОСИФ ЗЕДНИК

Настоящая работа занимается обобщением конфигураций Гладкого [1] именно в двух направлениях: 1. Допускается, чтобы результат конфигурации имел любую длину. 2. Допускается, чтобы рангом конфигурации было любое ординальное число. Конфигурации Гладкого включены в наших исследованиях, если мы предполагаем что длина результатов ровна 1, что мы допускаем только конфигурации конечного ранга и что в даном языке несуществует ненужных элементов. Мы отличаем конфигурации от сильных конфигураций, у которых длина результата ровна 1.

Упорядоченная пара состающая из результата и конфигурации считается правилом. Язык называется (сильно) ограниченный, когда существует грамматика, правила которой составлены из результатов и из (сильных) конфигураций. В работе доказывается, что каждый сильно ограниченный язык является ограниченным. После того определяются простые конфигурации, это такие, которые незавключают конфигураций того же самого ранга в своих собственных правильных подфразах.

Язык называется (сильно) конечно характеризуемым, если он обладает только конечным числом фраз без (сильных) конфигураций и если он имеет тоже только конечное число простых конфигураций всех рангов. Показывается, что (сильно) конечно характеризуемый язык является (сильно) ограниченным.