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ABSTRACTS OF Ph.D. THESES IN MATHEMATICS
defended recently at Charles University, Prague

TEACHING OF FINANCIAL AND INSURANCE MATHEMATICS AT SECONDARY AND HIGHER PROFESSIONAL SCHOOLS — COINSURANCE IN BONUS-MALUS SYSTEMS IN AUTOMOBILE INSURANCE

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(January 30, 2006; supervisor D. Vorlíčková)

The thesis consists of two parts. The first part is divided into two chapters and is devoted to the teaching of financial and insurance mathematics at secondary and higher professional schools.

There are thirteen lectures on financial mathematics in chapter one and thirteen lectures on insurance mathematics in chapter two. A seminar in Excel has been also made to each lecture. Those seminars together with a sort of repetition of the parts of secondary mathematics, which are required at financial and insurance mathematics, are recorded on the CD enclosed.

Chapter one deals with simple and compound interest and their combination, various sorts of interest rates; further it concentrates on the equation of value and the yield on a transaction, various types of annuities and finally on the valuation of securities.

The substantial part of the chapter two is devoted to the calculations of premiums in life insurance. Only the last three lectures are devoted to general (non-life) insurance, claim reserving and (the very last one) to bonus-malus systems (BMS) in particular.

The second part of the thesis continues in dealing with BMS. It is known that BMS lead to the individualization of premiums. One of the ways how to express the extent of the individualization of premiums quantitatively is proposed in chapter four.

In the prevailing BMS the level of the premium depends on the number of claims and is independent of the amounts of these claims. A system that takes into account both the number of claims and partially their amounts is suggested in chapter five.

To solve a quite common problem which is connected with BMS in practice, the so called hunger for bonus, carriers might offer a deductible coinsurance to their clients. Chapter six is devoted to the calculation of the optimal strategy of the choice of the deductible coinsurance from the policyholders' point of view. The positive influence of the possibility of the deductible coinsurance and the use of the optimal strategy on the elasticity of the BMS is also shown in this chapter.

The expounded problems are illustrated numerically with the BMS used in some countries in the last chapter of the thesis.

SCIENTIFIC WORK OF BOHUMIL BYDŽOVSKÝ

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(March 6, 2006; supervisor L. Boček)

Bohumil Bydžovský was a famous Czech mathematician of the 20th century. The dissertation thesis describes his extensive and large-scale scientific work. It is divided into seven chapters, in which it is possible to find the main areas of his mathematical research. Professor Bydžovský is an author of 111 mathematical articles and textbooks for secondary schools and universities.

Bydžovský's first works discuss rational curves in general and especially those of third degree. Masterly applications of the theory of elliptic functions in geometry brought Academician Bydžovský world acclaim in the first half of his life.

The second topic, which we can find in Bydžovský's works, is the theory of plane and space collineations and their groups and consequently algebraic correspondences of higher degree, especially Cremona transformations and their groups.

Only two Bydžovský's works are not related to his major research topic of algebraic geometry and deals with differential geometry, especially investigates geodesic curves in general on surface of revolution of second degree.

Professor Bohumil Bydžovský became reputable also with contribution in the theory of plane configurations, especially configurations of the type $(12_4, 16_3)$.

ULTRAFILTERS AND SMALL SETS

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(April 28, 2006; supervisor P. Simon)

There have been several attempts to connect ultrafilters with families of "small" sets. One of them was made by James Baumgartner who introduced $\mathscr{I}$-ultrafilters, the key notion for the thesis: Let $\mathscr{I}$ be a family of subsets of a set X such that $\mathscr{I}$ contains all singletons and is closed under subsets. Given a free ultrafilter $\mathscr{U}$ on ω, we say that $\mathscr{U}$ is an $\mathscr{I}$-ultrafilter if for every $F : \omega \to X$ there is $A \in \mathscr{U}$ such that $F[A] \in \mathscr{I}$.

In the thesis $\mathscr{I}$-ultrafilters are studied in the setting $X = \omega$ and $\mathscr{I}$ is an ideal on ω or another family of "small" subsets of natural numbers that contains finite sets and is closed under subsets. Several collections of "small" subsets are introduced in the first chapter and later on used to define corresponding classes of

$\mathscr{I}$-ultrafilters. Among others we consider as $\mathscr{I}$ the family of sets with asymptotic density zero, the summable ideal or the family of thin sets or (SC)-sets.

Chapter two provides basic facts on $\mathscr{I}$-ultrafilters, e.g. that there are no $\mathscr{I}$-ultrafilters if the ideal $\mathscr{I}$ is not tall. For tall ideals then a proof is given that the existence of corresponding $\mathscr{I}$-ultrafilters is consistent with ZFC. Special attention receive $\mathscr{I}$-ultrafilters for particular families $\mathscr{I}$ from chapter one and their relationships to other well-known classes of ultrafilters — selective ultrafilters, P-points, Q-points or rapid ultrafilters. For instance, a hereditarily rapid ultrafilter that is not a Q-point is constructed.

In chapter three sums and product of $\mathscr{I}$-ultrafilters are investigated. It is proved that $\mathscr{I}$-ultrafilters are closed under $\mathscr{I}$-sums if the ideal $\mathscr{I}$ is a P-ideal and that $\mathscr{I}$-ultrafilters need not be closed even under products if $\mathscr{I}$ is not a P-ideal.

In the last chapter the notion of $\mathscr{I}$-ultrafilter is weakened in such a way that only finite-to-one or one-to-one functions are considered in the definition of an $\mathscr{I}$-ultrafilter. This approach generalizes Gryzlov's 0-points defined in the following way: a free ultrafilter $\mathscr{U}$ on ω is called a 0-point if for every one-to-one function $f : \omega \to \omega$ there exists $U \in \mathscr{U}$ such that $f[U]$ has asymptotic density zero. Gryzlov constructed such ultrafilters in ZFC and his result is strengthened in the last part of the thesis. It provides a ZFC construction of a summable ultrafilter, i.e. an ultrafilter which is defined by replacing the density zero ideal in the definition of a 0-point by the (strictly smaller) summable ideal.

DIFFERENTIAL EQUATIONS AND THEIR APPLICATIONS IN ECONOMICS

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(April 28, 2006; supervisor J. Stará)

The main objective of the work was to create a mathematical text, well readable for users, aimed at the application of the differential equations in economics. Necessary parts of knowledge of ordinary differential equations (ODE) theory and optimal control theory are summarised in the text as well. Focusing on the correctness of mathematical approach, the mathematical models are studied and the results are interpreted in the original context.

The text presents derived mathematical models of various economic problems, where ODE are used. It was presented and proved in the work that it could be useful to use ODE while studying the time development of economic quantities and that with the help of a suitable interpretation of the solution it is possible to verify the well-known pieces of knowledge of the economic theory, or even to obtain new ones. The suitability of including ODE into training of economics and management students was proved there too.

THEOREMS OF THE ALTERNATIVE AND LINEAR PROGRAMMING IN INFINITE-DIMENSIONAL SPACES

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The content of the thesis is divided into three chapters. In the first one, we study theorems of the alternative, like Farkas' Lemma, and linear programming with finite number of linear constraints in the abstract setting of (possibly) infinite-dimensional vector spaces W and V over a linearly ordered field F where the space V is also linearly ordered. In the second chapter, we study the infinite case of the mentioned topics: we study theorems of the alternative and linear programming with (possibly) infinite number of linear constraints in the setting of an infinite-dimensional real vector space X. In the third chapter, which supplements the first one, we establish a basic theory of convex polyhedra, formulate the simplex algorithm in infinite-dimensional spaces, and we also introduce the concept of integer linear programming in a real infinite-dimensional space. Most of the results of the thesis can be found in [2] and [1].

References

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SOME PROBLEMS OF THE PRIVATE HEALTH INSURANCE

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In the first part of the thesis a method of the life tables (LT) combination is described. Examples are shown using the combination of the LT of the specific part of the population (non-smokers or non-drinkers of alcohol) and LT of general population. In the second part some differences between men and women are shown. Some of the examples are: numbers of average illnesses days, different kind of diseases, different average age, etc. The result was that there are differences between the genders. On 13/12/2004 the EU Directive No 2004/113/ES "Equal treatment between women and men in the access to and supply of goods and services" was issued. In Article 5 of this directive there is written that the gender

(sex) cannot be used as the parameter for the premium nor annuity calculation. The use of this parameter was considered as a discrimination. The impact of this directive is calculated in the remaining parts of the thesis.

The unisex LT were calculated — independence on the gender of the people alive and dead was taken into account. The Czech Statistical Office methodology for the creation of the LT was used (reconstructed on the basis of some diploma thesis). Then the impact of using these LT on the financial results of the insurance company was calculated. For the comparison the Fair Value methodology was used. Basic idea was: the company uses the unisex LT for the premium and reserve calculation but the persons in the portfolio are dying with the probabilities relevant to their gender. So, dependently on the percentage of the men and women in the portfolio, there are more or less persons dead or alive than expected. The calculation was made for the three basic types of the insurance — endowment insurance, risk term insurance and annuity insurance.

Finally, the recommendation for the Fair Value calculation was presented — in case the unisex LT are used, it is also necessary to take into account the percentage of the men and women in the company's portfolio.

ALGORITHMS FOR MINKOWSKI PYTHAGOREAN HODOGRAPH CURVES

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(June 23, 2006; supervisor A. Karger)

As observed by Choi et al. (1999), curves in Minkowski space $\mathbb{R}^{2,1}$ are very well suited to describe the medial axis transform (MAT) of a planar domain, and Minkowski Pythagorean hodograph (MPH) curves correspond to domains, where both the boundaries and their offsets are rational curves (Moon, 1999).

In this thesis, we focus mainly on MPH cubics. We show that any MPH cubic (including the case of light-like tangents) is a cubic helix in Minkowski space. With the help of tangent indicatrices of MPH curves, we derive a complete classification of planar and spatial MPH cubics.

Based on these results, we give a thorough discussion of G^1 Hermite interpolation by MPH cubics, focusing on solvability and approximation order. Among other results, it is shown that any space-like C^∞ curve without isolated Minkowski inflections can be approximately converted into a G^1 spline curve composed of MPH cubics with the approximation order being equal to four. The theoretical results are illustrated by several examples. In addition, we show how the curvature of a curve in Minkowski space is related to the boundaries of the associated planar domain.

UTILITY FUNCTIONS IN PORTFOLIO OPTIMIZATION

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(September 11, 2006; supervisor J. Dupačová)

This thesis analyzes portfolio selection problems with respect to choice of utility functions. Chapter 2 deals with the stability of optimal portfolio and its expected utility. Firstly, for the class of twice-differentiable, strictly increasing utility functions, the stability of optimal investment strategy due to changes in Arrow-Pratt absolute risk aversion measure is discussed. Secondly, applying the theory of variational analysis, the limit set of optimal portfolios is analyzed.

Chapter 3 develops characterizations of multiperiod risk premium. The construction of multiperiod risk premium is based on the preference indifference between accepting a multiperiod game and rejecting this game. The possibility of accepting the game only in some time periods is included.

Chapter 4 describes SSD rules concerning the portfolio selection problem. Using the relationship between CVaR and SSD for discrete probability distributions of returns, necessary and sufficient conditions for efficient portfolios relative to all possible portfolios created from a set of assets are derived. We develop a SSD portfolio efficiency measure which is consistent with second-order stochastic dominance. Moreover, we explore the convexity of this measure. We adopt these results for testing second-order stochastic efficiency of mean-VaR optimal portfolios.

Finally, Chapter 5 develops a test for FSD efficiency of a given portfolio of choice alternatives relative to all possible portfolios. We introduce the class of representative utility functions for FSD portfolio efficiency tests. We provide a linear programming criterion for implementing the test. To identify the input for the linear programming problem, we suggest mixed-integer linear programming or subsampling techniques.

M–ESTIMATION IN NONLINEAR REGRESSION FOR LONGITUDAL DATA

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We consider the nonlinear longitudinal regression model $Z_i^j = m(\theta_0, \mathbb{X}_i(T_i^j)) + \varepsilon_i^j$, $i = 1, \dots n$, $j = 1, \dots N_t^i$, where Z_i^j is the j-th measurement on the i-th subject at random time T_i^j in the interval of observation $[0, t]$, $m(\theta_0, .)$ is a known regression function depending on an unknown d-dimensional

parameter θ_0, $\mathbb{X}_i(T_i^j)$ a p-dimensional covariate process of subject i at time T_i^j and ε_i^j is a noise. In this thesis we introduce the assumptions which guarantee the consistency and asymptotic normality of smooth M-estimator of unknown parameter θ_0 which is defined as a minimizer of $\frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{N_t^i}\rho\big(Z_i^j - m(\theta, \mathbb{X}_i(T_i^j))\big)$, where $\rho(.)$ is a real continuous function. Some theoretical results are numerically demonstrated by a short simulation study.

ON PORTMANTEAU TESTS OF RANDOMNESS

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This work deals with simple portmanteau tests of randomness that are based on autocorrelation-like coefficients, computed in time domain and do not require any simulations. They are extensively reviewed in Chapter 1. Those based on ranks appear the most promising and they are therefore discussed in the rest of the thesis. Chapter 2 summarizes some common features of most Monte Carlo studies conducted in this work, including the null hypothesis, alternatives, software and realization details.

Chapter 3 introduces weighted serial modifications of the well known sign and turning point test statistics to any positive lag and investigates their asymptotic properties as well as finite sample moment characteristics. Furthermore, their orthonormal versions are proposed and shown to be advantageous in the case of short time series and also in Chapter 4 where they are employed in a newly developed methodology for assessing random number generators. Chapter 5 extends the theory regarding Kendall's rank autocorrelations by computing their exact variances at higher lags. This makes it possible to construct correctly sized portmanteau tests based on them. Computationally less demanding weighted Kendall's rank autocorrelations are introduced in Chapter 6 and their finite sample and asymptotic properties are investigated there. Chapter 7 deals with certain (signed-)rank autocorrelations of scores and shows their surprisingly good performance in testing against conditional heteroscedasticity.

Chapter 8 criticises any portmanteau statistic based on a (possibly weighted) sum of certain common serial correlation coefficients and shows both theoretically and empirically that its finite sample and asymptotic distribution may differ significantly even for quite long time series. In Chapter 9, a new and unifying view of portmanteau testing is proposed, original portmanteau statistics are introduced and their dominance over common benchmarks is illustrated in many important cases. Chapter 10 establishes joint uncorrelatedness and asymptotic independence of various types of rank autocorrelation coefficients and deals with the problem how to combine them optimally in a single portmanteau statistic, which

is shown to be potentially advantageous. Finally, some results are summarized, their application is discussed and further extensions are suggested.

CONTINUOUS POPULATION MODELS FOR SINGLE SPECIES

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The thesis deals with the applications of the theory of differential equations in biology, especially in population ecology. There are shown and derived models, that describe the population growth of various species under different conditions. The models are then discussed and its results are used as a prediction of the behaviour of real populations.

The thesis consists of three parts. In the first one some elements of the theory of autonomous systems of differential equations are presented, such as existence and uniqueness of solutions, continuous dependence of solutions on initial conditions and parameters, equilibria end their stability, phase portraits of one-dimensional systems and bifurcations. In the work the qualitative approach known in the field of dynamical systems is used, the qualitative properties of the autonomous systems are deduced from the corresponding properties of the vector fields. The main output is the phase-portrait, from which it is possible to get all the information about the dynamics of the investigated systems.

The core of the thesis lies in its second part. Here the models of population growth are derived, starting from the easiest one describing the exponential growth, continuing with the models where the growth is kept in check by density-dependent and finally ending with models with parameters, where the dependence of the behaviour of the model on parameters is investigated. At the end the model for the situation in which one species (the predator) preys on the other species (the pray) is studied in detail. It is the spruce-budworm model, which enables to gain understanding of the dynamics of the tree and insect interaction, and from it we decide what measures can be effectively taken to protect wood industry from great economic damages.

In the third part the short history of the theory of differential equations is presented with the emphasis on the major trends in its evolution and on its most known creators and contributors.

SEQUENTIAL CHANGE-POINT ANALYSIS

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The work concerns sequential procedures for detection of changes in location and linear regression models. Particularly, several classes of monitoring schemes to (sequentially) detect a structural change in a linear model after training period of size m are proposed and their statistical properties studied.

The thesis extends the results by Horváth, Hušková, Kokoszka and Steinebach (2004) and Aue, Horváth and Hušková (2006) to L_1 procedures that are not sensitive w.r.t. outliers but sensitive w.r.t. changes in parameters. Procedures based on weighted CUSUM's of residuals with a wide spectrum of boundary functions are studied. They are designed so that the tests have a small probability of false alarm and asymptotic power 1, as $m \to \infty$. Related estimators of the change point are also studied.

Theoretical results are accompanied by a simulation study.

SECOND ORDER PROPERTIES OF SOME M-ESTIMATORS AND R-ESTIMATORS

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Throughout the whole thesis we consider the linear model

$$Y_i = \beta_1 x_{i1} + \ldots + \beta_p x_{ip} + e_i = \boldsymbol{\beta}^T \boldsymbol{x}_i + e_i, \qquad i = 1, \ldots, n,$$

where $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_p)^T$ is a vector of unknown parameters, $\boldsymbol{x}_i = (x_{i1}, \ldots, x_{ip})^{\mathsf{T}}$, for $i = 1, \ldots, n$, are known constants, and $e_1, \ldots, e_n$ are independent, identically distributed random variables with a cumulative distribution function F.

The problem of estimation of the regression parameter $\boldsymbol{\beta}$ is a very old but still very active area of statistics and many different estimators have been developed. In the robust statistics two of the main families of estimators are M-estimators and R-estimators. In this work we study the second order asymptotic properties of M-estimators (based on a continuous or a step function) and R-estimators based on Wilcoxon scores. The existing results usually deal with the case of a location parameter or a simple linear regression (without an intercept). We generalize these results to the case of multi-dimensional regression parameters. We cover studentized M-estimators as well.

The first part of the work is of technical character and derive some technical results about asymptotic behavior of M-processes and R-processes which may be of independent interest. In the second part of the work we utilize these auxiliary results to derive a two-term von Mises expansion for an M-estimator based on a 'smooth' function ψ and for an R-estimator based on Wilcoxon scores. Further we propose an alternative way of constructing a confidence interval for a single regression parameter and we investigate its properties. We compare this alternative

procedure with the 'traditional' (Wald type) approach. Finally, we are dealing with a sequential problem of a confidence interval of a bounded width.

FULL EMBEDDINGS AND THEIR MODIFICATIONS

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The thesis summarizes my contributions to the theory of representations in categories.

The first and the second chapter enhances the collection of known alg-universal categories, i.e. categories which contain an equivalent copy of every variety of universal algebras. The first chapter concerns set functors, i.e. endofunctors of the category of all sets and mappings. We prove that the category of finitary set functors and natural transformations is alg-universal and present an example of a rigid proper class of accessible set functors. In the next chapter, we show that the category of varieties and interpretations (or, in other words, clones and clone homomorphisms) is alg-universal as well.

The final chapter deals with the theory of functor slices and baskets of concrete categories. The five-member collection of baskets introduced by J. Sichler and V. Trnková is enlarged in the following way: For every ordinal α we introduce a new basket $\mathbb{E}_\alpha$. We prove that every essentially algebraic category of height α is a slice of $\mathbb{E}_\alpha$, characterize small slices of $\mathbb{E}_\alpha$ and generalize known results about slices of the algebraic basket $\mathbb{A}$.

LADISLAV SVANTE RIEGER (1916–1963)

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This thesis is dedicated to the prominent Czech mathematician Ladislav Svante Rieger. His scientific work can be divided into three main mathematical areas: algebra, mathematical logic, and axiomatic set theory. Despite being appreciated worldwide, L.S. Rieger especially contributed to the mathematical research in our countries. He is regarded as the founder of mathematical logic and axiomatic set theory in Czechoslovakia. The aim of the thesis is to give a full account of Rieger's personality and his achievements in the individual mathematical areas.

In the first — biographic — part, a detailed description of L.S. Rieger's personal and professional life is given. This section also provides information on prominent members of his family.

The main — mathematical — part deals with Rieger's scientific work. His publications are divided into four thematic sections. Attention is paid to the principal results and their evaluation in the context of the world mathematics. References to Rieger's followers are a valuable contribution to his mathematical legacy. Each chapter of this part is concluded by a brief historical overview of the related mathematical area with an emphasis on the Czechoslovak science.

The final part of the thesis consists of factual appendices (the list of Rieger's publications, reviews in the journal Mathematical Reviews, and the outline of his teaching activities). The illustrated appendix contains copies of documents from Rieger's estate.

ON EXISTENCE AND REGULARITY OF SOLUTIONS TO PERTURBED SYSTEMS OF STOKES TYPE

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The dissertation thesis is devoted to the existence and regularity of weak solutions to systems of stationary equations suggested by problems of fluid mechanics. Chapter 2 concentrates on the main subject of the theses, i.e. partial regularity of solutions to generalized stationary Navier-Stokes equations which describe motion of a fluid whose viscosity depends on shear and on pressure. Chapter 1 contains some preliminary results about systems which represent a linearization of problems studied in Chapter 2.

ANALYSIS OF THE DISCONTINUOUS GALERKIN METHOD FOR ELLIPTIC PROBLEMS

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In the present thesis Discontinuous Galerkin formulations for a scalar linear elliptic partial differential equation of the second order with various boundary conditions are studied. Two variants of Discontinuous Galerkin formulations are considered. These are the *symmetric* and *nonsymmetric interior penalty Galerkin* schemes, see, e.g., [1]. Well-known results concerning the existence and uniqueness of the solution and a priori error estimates are recapitulated and extended.

Original output of this thesis is the study of the effect of numerical integration used in the definition of the discrete problem. Our result states that if certain properties of quadrature formulae are satisfied then there exists a unique solution of the problem. Moreover, if sufficiently precise quadrature formulae are used then the order of convergence is not decreased.

Suitable modification of Discontinuous Galerkin schemes which allows the so called *semiregular* triangulations is presented. This part extends [3].

In the final part the Poisson equation is equipped with a nonlinear Newton boundary condition. This problem was studied in [2] and in a sequence of following papers. The Discontinuous Galerkin formulation of such a problem is considered. Existence and uniqueness of the solution is obtained with the aid of theory of monotone operators and error estimate with respect to mesh-dependent error functional is derived.

Computer implementation of studied schemes is also discussed and few illustrative numerical examples are presented to assess theoretical results.

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SEVERAL DIRAC OPERATORS IN PARABOLIC GEOMETRY

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(November 7, 2006; supervisor V. Souček)

In this thesis, we show the existence of a sequence of differential operators starting with the Dirac operator in k Clifford variables, $D = (D_1, \dots, D_k)$, where $D_i = \sum_j e_j \cdot \partial_{ij} : C^\infty((\mathbb{R}^n)^k, \mathbb{S}) \to C^\infty((\mathbb{R}^n)^k, \mathbb{S})$ ($\mathbb{S}$ is the spinor module). This operator is the Cauchy-Riemann operator for $n = 2$ and its resolution is the Dolbeault complex. For higher n, the resolution of D is not known in general. While this problem was treated many times in the language of Clifford analysis and some partial results are known, we give a description of this operator in Parabolic geometry, which is a special type of Cartan geometry modeled on G/P, where P is a Parabolic subgroup of G. We construct sequences of invariant differential operators starting with the Dirac operator in several variables and assume that these sequences coincide in some cases with the resolution. We describe the structure of these sequences precisely in case the dimension n is odd and give a conjecture that these sequences have similar structure for n even, $k \leq n/2$ (the s.c. *stable range*). We also give some information about these sequences in case n even, $k > n/2$. In the last chapter, explicit formulas for the operators are derived for the case $k = 2$.

EXPLICIT DESCRIPTION OF SUPPORTING AND SEPARATING HYPERPLANES OF TWO CONVEX POLYHEDRAL SETS DEPENDING ON PARAMETERS

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Separation of convex sets is widely used in many branches of mathematics. Often in practice input data are known only approximately. We combine these two principles — separation and uncertainty — together. Uncertainty is treated both with the help of parametrization and interval analysis. In the parametrization part of this work we study three main cases: parameters are situated either in the right-hand side of inequalities, or in one row or in one column of the constraint matrix. For each case we are concerned with the basic separation properties (existence, description, stability etc.) of two convex polyhedral sets with parameters. We define so called solution set (a set of parameters for which the given convex polyhedral sets are strongly separable) and stability sets (a set of parameters for which separability of the convex polyhedral sets has the same characteristics). We provide a lot of examples, which were carried out on a computer. Interval analysis deals with real intervals instead of real numbers. We propose a way how to check whether given convex polyhedral sets are separable for some or for all realizations of the interval data. Some of the proposed problems can be checked efficiently, while the others are NP-hard.

VALUATION TECHNIQUES OF LIFE INSURANCE LIABILITIES

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(November 10, 2006; supervisor T. Cipra)

The intention of this work is to contribute to the current actuarial discussion about valuation of life liabilities with a summary of current most frequent valuation methodologies. It started with the most traditional one (Statutory valuation approach), gone through the more developed ones (deterministic Embedded Value approach and deterministic estimation of the Fair Value) to the most recent one — stochastic Fair Value approach via interest rates simulations. It is intended to give a more detailed overview of the methodologies not only in a way of a general description but in a way of specific mathematical formulas and numerical examples as well in order to observe their mutual relations and similarities, their positives and negatives. The real process of the stochastic liability fair value calculation under the interest rate simulations is showed at the second part of the work. The procedure explaining all important issues, steps and results are

presented.

STATIONARY DISTRIBUTION OF TIME SERIES

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(November 10, 2006; supervisor J. Anděl)

The thesis deals with methods of computation of stationary distribution in various models of time series. The goal is to find a stationary density of the series given the distribution of innovations.

In the first part we study linear processes, the AR(1) process in particular. Even in this relatively simple situation the exact solution to our problem is known only in very few special cases. We describe and explore two algorithms, both originally designed for AR(1) model, which yield approximations of the stationary density. The first one, proposed by Anděl and Hrach (2000), is based on the approximation of the solution to some integral equation. We proved that under some rather mild conditions on the density of the noise sequence it generates a sequence of densities which converges uniformly and exponentially fast to the desired stationary density. We generalized this method to processes of higher order (AR(2)) and dimension (VAR(1)). The second approach was proposed by Haiman (1998). It investigates the behaviour of partial sums $\varepsilon_1+\rho\varepsilon_2+\ldots+\rho^n\varepsilon_{n+1}$ as n tends to infinity. The properties of this algorithm were originally derived under very strong assumptions. We relaxed these assumptions significantly and again we showed that its output converges uniformly and exponentially fast. This was done by studying properties of convolution of densities. Finally, we extended this method to causal linear processes and ARMA processes.

In the second part we deal with nonlinear models of time series. First we describe a method of approximation of stationary density for a general nonlinear autoregression of first order. Then we focus on the model of absolute autoregression AAR(1). For several distributions of innovations (normal, Cauchy, discrete uniform, Laplace) we derived exact form of stationary density and other characteristics of the stationary distribution. We also proposed a procedure for approximation of the stationary density when its exact form is not known. In the final part, we review the results of Loges (2004) who computed stationary density in a threshold TAR(1) model driven by Laplace innovations.

KINEMATIC GEOMETRY IN N-DIMENSIONAL EUCLIDEAN SPACE

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Plane and space curves have been described frequently in the literature. In the thesis, some known curves are mentioned as well. In particular, a list of the plane curves is presented that are created kinematically. We do not give attention to their characteristic properties, but we focus especially on possible ways of curves' origin. In other words, curves are investigated as trajectories of points during space motion. The study of the space motion is the main point of the thesis.

The mathematical branch which deals with the study of space motion is called kinematic geometry. A lot has been written about plane and space kinematic geometry. The space motion in the n-dimensional Euclidean space and its characteristic properties have been investigated much more intensively since the second half of last century, also due to the development of modern technology.

The thesis should serve as the short and well-arranged overview of kinematic geometry in the n-dimensional Euclidean space. It surely does not deal with all the parts of kinematic geometry in the n-dimensional Euclidean space but only with the study of properties and relations of the space motion in E_n. An overview of the rules for the computation of the values of the invariants of the space motion in E_n is presented; these rules can be denoted as a generalization of the so called Euler's formulas.

Further, we restrict our attention to the four-dimensional Euclidean space, where the situation of a space motion is not known enough as it is in the Euclidean spaces E_2 or E_3.

In E_4 we study two special examples of the space motion — the screw motion with the trajectories lying on the unit sphere and the 2D-Darboux motion. For the former one we also use the term "screw motion in the elliptical space E_3". We have computed the particular values of the invariants of the 2D-Darboux motion in E_4 and drawn the trajectories of the screw motion with the trajectories lying on the unit sphere and of the 2D-Darboux motion in E_4.