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Dimensional compactness in biequivalence vector spaces

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Abstract. The notion of dimensionally compact class in a biequivalence vector space is introduced. Similarly as the notion of compactness with respect to a π-equivalence reflects our nonability to grasp any infinite set under sharp distinction of its elements, the notion of dimensional compactness is related to the fact that we are not able to measure out any infinite set of independent parameters. A fairly natural Galois connection between equivalences on an infinite set s and classes of set functions $s \to Q$ is investigated. Finally, a direct connection between compactness of a π-equivalence $R \subseteq s^2$ and dimensional compactness of the class $\mathbf{C}[R]$ of all continuous set functions from $\langle s, R \rangle$ to $\langle Q, \doteq \rangle$ is established.

Keywords: alternative set theory, biequivalence vector space, π-equivalence, continuous function, set uniform equivalence, compact, dimensionally compact

Classification: Primary 46S20, 46S10, 46E25; Secondary 03E70, 03H05

1. Introduction.

The present paper is a contribution to the topic of biequivalence vector spaces (BVS) initiated in [Sm–Z 1991] as a counterpart to classical topological vector spaces within the framework of alternative set theory (AST). Most of the results stated below were announced in [Z 1989]. Besides of [Sm–Z 1991], the reader is assumed to be acquainted with the AST in an extent covered by [V 1979] and with some basic notions and facts concerning biequivalences and continuous functions which can be found in [G–Z 1985]. Throughout the paper the terminology and notation of [Sm–Z 1991], the latter subject to some minor selfexplanatory changes, will be used.

The central position in the paper is due to the notion of a dimensionally compact class in a BVS and to its connection to the notion of indiscernibility equivalence.

To explain what goes on, let us recall that the AST-counterpart of a classical topological space can frequently be represented as a pair $\langle s, R \rangle$ where s is a set and R is an indiscernibility equivalence, i.e., a compact π-equivalence on s. The notion of compactness is motivated by the empirical fact that one cannot grasp perfectly any infinite set in its totality within distinction among all of its elements; whenever an infinite set is observed, some couple of its elements occurs indiscernible. Thus an equivalence $R \subseteq s^2$ is said to be compact on a class $X \subseteq s$ if there is no infinite set $u \subseteq X$ of pairwise discernible elements; R is called compact if it is compact on s.

Similarly, the notion of dimensional compactness reflects the property of real measurement that we are not able to measure out an actually infinite set of really independent parameters; whenever an infinite number of measurements is carried out, some dependence between their values occurs. This leads us to the following definition.

Let $\langle W, M, G\rangle$ be a BVS. A class $A \subseteq W$ will be called *dimensionally compact* (in $\langle W, M, G\rangle$) if each independent set $w \subseteq A$ is finite. (For the notion of independence in a BVS see [Sm–Z 1991, Section 8]).

Now, assume that various numerical parameters or characteristics of objects forming an infinite set s are measured. Such parameters can be considered as set functions $f : s \to Q$. Given a class of such parameters $A \subseteq Q^s$, one can naturally call two objects $x, y \in s$ A-indiscernible if $f(x) \doteq f(y)$ for all $f \in A$. Obviously

$$\mathbf{T}[A] = \{\langle x, y\rangle \in s^2;\ (\forall\, f \in A)(f(x) \doteq f(y))\}$$

is an equivalence on s for each $A \subseteq Q^s$. Moreover, $\mathbf{T}[A]$ is the coarsest one among the equivalences R on s such that each $f \in A$ is "continuous" with respect to R in the sense that $\langle x, y\rangle \in R$ implies $f(x) \doteq f(y)$ for all $x, y \in s$.

On the other hand, if R is an equivalence on s, then we can consider the class

$$\mathbf{C}[R] = \{f \in Q^s;\ (\forall\, x, y \in s)(\langle x, y\rangle \in R \ \Rightarrow\ f(x) \doteq f(y))\}$$

of all functions continuous with respect to R, i.e. the class of all continuously varying numerical characteristics of objects from s.

Intuitively it is clear that there should be some connection between compactness of the equivalence R and dimensional compactness of the class $\mathbf{C}[R] \subseteq Q^s$, provided a suitable BVS structure on Q^s is introduced. We will show that this is really the case.

Of course, on the Sd-class Q^s the structure of a vector space over the field Q of all rationals is defined componentwise. Sometimes we will also utilize the fact that Q^s even is a linear algebra over Q under componentwise multiplication. In most cases the relevant BVS structure on Q^s will be of the form $\langle Q^s, IQ^s, BQ^s\rangle$. This convenient restriction is justified by our present subject, because, as shown in [G–Z 1985], all "reasonable" π-equivalences which can be introduced on Q^s coincide with the π-equivalence $\doteq^s$, given by

$$f \doteq^s g \ \Leftrightarrow\ (\forall\, x \in s)(f(x) \doteq g(x)),$$

when restricted to classes of the form $\mathbf{C}[R]$.

2. Compactness and dimensional compactness.

Throughout the paper s denotes a fixed set; to avoid trivialities we assume that s is infinite. Let us start with the following four lemmas which will be needed in further proofs.

Lemma 1 ([M 1979]). *Let R be a π-equivalence on s. Then there is a set metric $h : s^2 \to Q$ such that for all $x, y \in s$ we have*

$$h(x, y) \le 1 \qquad \textit{and} \qquad \langle x, y\rangle \in R \ \Leftrightarrow\ h(x, y) \doteq 0.$$

Lemma 2 ([V 1979a]). *Let $\{R_i;\ i \in I\}$ be a codable system of indiscernibility equivalences on s. Then $\bigcap_{i\in I} R_i$ is an indiscernibility equivalence if and only if there is a countable class $J \subseteq I$ such that $\bigcap_{i\in I} R_i = \bigcap_{i\in J} R_i$.*

Lemma 3 ([G–Z 1985]). *Let R be an indiscernibility equivalence on s. Then for every infinite set $w \subseteq \mathbf{C}[R] \cap BQ^s$ there exist functions $f, g \in w$ such that $f \neq g$ and $f(x) \doteq g(x)$ for each $x \in s$.*

Lemma 4. *Let R be a σ-equivalence on s. Then R is compact on s if and only if R is a set and the factor set s/R is finite.*

Proof: Consider the class

$$C = \{n \in N;\ (\exists\, u \subseteq s)(|u| < n\ \ \&\ \ R \cap u^2 \subseteq \mathrm{Id})\}.$$

Then C obviously is a π-cut on N (see [K–Z 1988]). If R is compact, then $C \subseteq FN$. As FN is not a π-class, we have $C \in FN$. The rest is trivial. □

The following proposition can be verified immediately.

Proposition 1. *For arbitrary equivalences P, R on s and arbitrary classes $A, B \subseteq Q^s$ the following conditions hold:*

(i) $R \subseteq \mathbf{T}[\mathbf{C}[R]]$,
(ii) $A \subseteq \mathbf{C}[\mathbf{T}[A]]$,
(iii) $P \subseteq R\ \Rightarrow\ \mathbf{C}[R] \subseteq \mathbf{C}[P]$,
(iv) $A \subseteq B\ \Rightarrow\ \mathbf{T}[B] \subseteq \mathbf{T}[A]$.

In other words, the maps $R \mapsto \mathbf{C}[R]$, $A \mapsto \mathbf{T}[A]$ define a Galois connection between the families of equivalencies on s and subclasses of Q^s.

Corollary. *For every equivalence R on s and any class $A \subseteq Q^s$ the following conditions are satisfied:*

(v) $\mathbf{C}\big[\mathbf{T}[\mathbf{C}[R]]\big] = \mathbf{C}[R]$,
(vi) $\mathbf{T}\big[\mathbf{C}[\mathbf{T}[A]]\big] = \mathbf{T}[A]$.

As it can easily be seen, for an equivalence $R \subseteq s^2$ we have $R = \mathbf{T}[\mathbf{C}[R]]$ if and only if

$$(\forall\, x, y \in s)(\exists\, f \in \mathbf{C}[R])(\langle x, y\rangle \notin R\ \Rightarrow\ f(x) \not\doteq f(y)).$$

Similarly, a class $A \subseteq Q^s$ satisfies $A = \mathbf{C}[\mathbf{T}[A]]$ if and only if

$$(\forall\, f \in Q^s)(\exists\, \langle x, y\rangle \in \mathbf{T}[R])(f \notin A\ \Rightarrow\ f(x) \not\doteq f(y)).$$

Obviously, every reflexive and symmetric relation (a symmetry for short) R on s is the intersection of all set symmetries r on s such that $R \subseteq r$. A symmetry R on s will be called *set uniform* if there is a class U of set symmetries on s such that $R = \bigcap U$ and for each $r \in U$ there is a $p \in U$ satisfying $p \circ p \subseteq r$.

Clearly, every set uniform symmetry on s is an equivalence. Moreover, an equivalence $R \subseteq s^2$ is set uniform if and only if it can be written as an intersection of a family of π-equivalences. It follows that the system of all set uniform equivalences on s is closed with respect to arbitrary intersections. As all π-equivalences and all σ-equivalences on s are set uniform for trivial reasons, all $\pi\sigma$-equivalences on s are set uniform, as well. An example of a $\sigma\pi$-equivalence which is not set uniform can be found in [M 1990].

Now, we can characterize the equivalences on s closed with respect to the described Galois connection.

Theorem 1. *Let R be an equivalence on s. Then $R = \mathbf{T}[\mathbf{C}[R]]$ if and only if R is set uniform. In general, $\mathbf{T}[\mathbf{C}[R]]$ is the least set uniform equivalence P such that $R \subseteq P$.*

Proof: If the above equality holds, then

$$R = \bigcap_{f \in \mathbf{C}[R]} \mathbf{T}[f].$$

(We write $\mathbf{T}[f]$ instead of $\mathbf{T}[\{f\}]$.) As each $\mathbf{T}[f]$ obviously is a π-equivalence, R is set uniform.

To prove the converse, take $x, y \in s$ such that $\langle x, y\rangle \notin R$. As R is set uniform, there is a π-equivalence P such that $R \subseteq P \subseteq s^2$ and $\langle x, y\rangle \notin P$. Let h be the metric for P guaranteed by Lemma 1. Then the function f defined by $f(z) = h(x, z)$ for $z \in s$ obviously belongs to $\mathbf{C}[P] \subseteq \mathbf{C}[R]$ and we have $f(x) = h(x, x) = 0 \not\doteq h(x, y) = f(y)$. This implies the desired equality for R.

The last statement is trivial. □

The problem of characterization of classes $A \subseteq Q^s$ closed with respect to the described Galois connection is far from being solved. We quote here, omitting the straightforward proofs, some necessary conditions only.

Proposition 2. *Let $A \subseteq Q^s$. If $A = \mathbf{C}[\mathbf{T}[A]]$, then the following three conditions are satisfied:*

(i) *Each constant function $s \to Q$ belongs to A.*
(ii) *For each $n \in N$ and any set sequences $\langle f_1, \ldots, f_n\rangle \in A^n$, $\langle g_1, \ldots, g_n\rangle \in A^n$, such that*

$$\sum_{i=1}^{n} |f_i(x) - f_i(y)|\,|g_i(x)| + \sum_{i=1}^{n} |f_i(y)|\,|g_i(x) - g_i(y)| \doteq 0$$

for each $\langle x, y\rangle \in \mathbf{T}[A]$, we have $\sum_{i=1}^{n} f_i g_i \in A$.
(iii) *The class A is closed in the topology of the BVS $\langle Q^s, IQ^s, BQ^s\rangle$.*

Corollary. *If $A = \mathbf{C}[\mathbf{T}[A]]$, then the following conditions hold:*

(iv) *If $f, g \in A$, then $f + g \in A$.*
(v) *If $\alpha \in BQ$, $f \in A$, then $\alpha f \in A$.*
(vi) *If $f, g \in A \cap BQ^s$, then $fg \in A$.*

(vii) *If* $n \in N$, $\langle \alpha_1, \dots, \alpha_n \rangle \in Q^n$, $\sum_{i=1}^n |\alpha_i| \in BQ$ *and* $\langle f_1, \dots, f_n \rangle \in A^n$, *then* $\sum_{i=1}^n \alpha_i f_i \in A$.

(viii) *If* $n \in N$, $\langle f_1, \dots, f_n \rangle \in A^n$, $\langle g_1, \dots, g_n \rangle \in A^n$ *and* $\sum_{i=1}^n |f_i| \in BQ^s$, $\sum_{i=1}^n |g_i| \in BQ^s$, *then* $\sum_{i=1}^n f_i g_i \in A$.

The next result is a direct consequence of Lemma 2 and the equality $\mathbf{T}[A] = \bigcap_{f \in A} \mathbf{T}[f]$ for $A \subseteq Q^s$.

Theorem 2. *Let* $A \subseteq Q^s$. *Then* $\mathbf{T}[A]$ *is an indiscernibility equivalence on* s *if and only if for each* $f \in A$ *the* π*-equivalence* $\mathbf{T}[f]$ *is compact and there is a countable class* $B \subseteq A$ *such that* $\mathbf{T}[A] = \mathbf{T}[B]$.

To complete the description of classes $A \subseteq Q^s$ for which $\mathbf{T}[A]$ is an indiscernibility equivalence, we quote without proof the following easy result.

Proposition 3. *Let* $f \in Q^s$. *Then the* π*-equivalence* $\mathbf{T}[f]$ *is compact if and only if the equivalence of infinitesimal nearness* $\doteq$ *is compact on the set* $\mathrm{rng}(f) \subseteq Q$.

Proposition 4. *Let* $f \in Q^s$ *be such that* $\mathbf{T}[f]$ *is an indiscernibility equivalence. Then there is a* $g \in BQ^s$ *such that* $\mathbf{T}[f] = \mathbf{T}[g]$.

PROOF: The class

$$R = \{\langle x, y \rangle \in s^2;\ f(x) - f(y) \in BQ\}$$

obviously is a σ-equivalence on s and $\mathbf{T}[f] \subseteq R$. Thus R is compact. By Lemma 4, R is a set and there is a finite number $z_1, \dots, z_n$ of elements of s such that $\langle z_i, z_j \rangle \notin R$ for $i \neq j$ and $s = \bigcup_{i=1}^n R\text{"}\{z_i\}$. Without loss of generality we can assume that $f(z_i) < f(z_j)$ for $i < j$ and $f(x) \leq f(z_i)$ for each i and each $x \in R\text{"}\{z_i\}$. Let $y_1, \dots, y_n \in s$ be such that $\langle y_i, z_i \rangle \in R$ and $f(y_i) \leq f(x)$ for each i and each $x \in R\text{"}\{z_i\}$. Then g can be constructed by the induction putting

$$\begin{aligned} g(x) &= f(x) - f(y_1), && \text{for } x \in R\text{"}\{z_1\}, \\ g(x) &= f(x) - f(y_{i+1}) + g(z_i) + 1, && \text{for } i < n,\ x \in R\text{"}\{z_{i+1}\}. \end{aligned}$$

□

Using Proposition 4, Theorem 2 can immediately be strengthened to

Theorem 3. *Let* $A \subseteq Q^s$. *Then* $\mathbf{T}[A]$ *is an indiscernibility equivalence if and only if there is a countable class* $B \subseteq BQ^s$ *such that* $\mathbf{T}[A] = \mathbf{T}[B]$. *If* $A = \mathbf{C}[\mathbf{T}[A]]$, *then* B *can be chosen subject to* $B \subseteq A \cap BQ^s$.

Now, it would be possible to rewrite the classical proof of the Stone-Weierstrass theorem into our setting to obtain a partial answer to the posed question. We limit ourselves to the formulation of the result.

Theorem 4. *Let $A \subseteq BQ^s$ be such that $\mathbf{T}[A]$ is an indiscernibility equivalence. Then $A = \mathbf{C}[\mathbf{T}[A]] \cap BQ^s$ if and only if the following conditions hold:*

(i) *Each constant function $s \to BQ$ belongs to A.*
(ii) *For all $f, g \in A$ we have $f + g \in A$ and $fg \in A$.*
(iii) *A is closed in the topology of the BVS $\langle Q^s, IQ^s, BQ^s \rangle$.*

Concerning the notion of dimensional compactness in BVS', we record without proof the following two trivial observations.

Lemma 5. *Let $\langle W, M, G \rangle$ be a BVS. Then for every class $A \subseteq W$ the following conditions hold:*

(i) *If A is dimensionally compact and $B \subseteq A$, then B also is dimensionally compact.*
(ii) *If the π-equivalence $\doteq_M$ is compact on the class A, then A is dimensionally compact.*

Proposition 5. *Let $\langle W, M, G \rangle$ be a trim BVS and $A \subseteq W$ be a balanced class. Then A is dimensionally compact if and only if $A \cap G$ is dimensionally compact.*

Proof: It is enough to show that A is dimensionally compact provided $A \cap G$ is. Assume that $w \subseteq A$ is an infinite independent set. Then by Theorem 6.2 from [Sm–Z 1991], there is a $q \in HR$, $q \geq 1$, and a $(1, q)$-valuation $\Phi : W \to HR$ inducing the biequivalence structure on W. By the set choice lemma for π-relations (see [Sm–Z 1991, Lemma 1.1]), there is a set function $\eta : w \to Q$ such that

$$1 \doteq \frac{\eta(f)}{\Phi(f)} \leq 1$$

for each $f \in w$. Then the infinite set $\{\eta(f) \cdot f;\ f \in w\} \subseteq A \cap G$ is independent, contradicting the dimensional compactness of $A \cap G$. □

Our last result relates the property of dimensional compactness of the class $\mathbf{C}[R] \subseteq Q^s$ of all continuous set functions in the BVS $\langle Q^s, IQ^s, BQ^s \rangle$ and the property of compactness of the respective π-equivalence $R \subseteq s^2$.

Theorem 5. *Let R be π-equivalence on s. Then the following four conditions are equivalent:*

(a) *R is compact on s.*
(b) *The π-equivalence $\doteq^s$ is compact on the class $\mathbf{C}[R] \cap BQ^s$.*
(c) *The class $\mathbf{C}[R] \cap BQ^s$ is dimensionally compact in $\langle Q^s, IQ^s, BQ^s \rangle$.*
(d) *The class $\mathbf{C}[R]$ is dimensionally compact in $\langle Q^s, IQ^s, BQ^s \rangle$.*

Proof: The implication (a) ⇒ (b) is in fact a restating of Lemma 3 only, (b) ⇒ (c) follows from the condition (ii) of Lemma 5 and so does (c) ⇒ (d) from Proposition 5. Thus it suffices to prove (d) ⇒ (a). Assume that R is not compact. Let $u \subseteq s$ be an infinite set, such that $\langle x, y \rangle \notin R$ for all distinct $x, y \in u$. Let h be the metric for R guaranteed by Lemma 1. For all $x \in u$, $t \in s$ we put

$$d(x, t) = \min \{h(t, y);\ x \neq y \in u\}.$$

Then for all $x \in u$, $t_1, t_2 \in s$ there are $y_1, y_2 \in u$ such that $d(x, t_1) = h(x, y_1)$, $d(x, t_2) = h(x, y_2)$ and

$$h(t_1, y_1) \leq h(t_1, y_2) \leq h(t_1, t_2) + h(t_2, y_2),$$
$$h(t_2, y_2) \leq h(t_2, y_1) \leq h(t_2, t_1) + h(t_1, y_1).$$

Thus $d(x, t_1) \doteq d(x, t_2)$ whenever $\langle t_1, t_2 \rangle \in R$. As $d(x, x) \not\doteq 0$ for any $x \in u$, it follows that by the assignment

$$g_x(t) = \frac{d(x, t)}{d(x, x)},$$

for $t \in s$, a set of functions $w = \{g_x;\ x \in u\} \subseteq \mathbf{C}[R]$ is defined. Moreover, for all $x, y \in u$ we have $g_x(x) = 1$ and $g_x(y) = 0$ if $x \neq y$. Hence w obviously is infinite and independent, contradicting the dimensional compactness of $\mathbf{C}[R]$. □

As the class $\mathbf{C}[R]$, contains, e.g., all constant functions $s \to Q$, the π-equivalence $\doteq^s$ obviously is not compact on $\mathbf{C}[R]$. Thus the property of dimensional compactness, in contrast to the compactness property, is preserved by the passage from the class $\mathbf{C}[R] \cap BQ^s$ of all bounded continuous functions to the class $\mathbf{C}[R]$ of all continuous functions. For the class $\mathbf{C}[R] \cap BQ^s$ the properties of compactness and dimensional compactness coincide provided R is a π-equivalence. The question whether there exist dimensionally compact classes $A \subseteq G$ in a (trim) BVS $\langle W, M, G \rangle$, such that the π-equivalence $\doteq_M$ is *not* compact on A, remains open even for the BVS $\langle Q^s, IQ^s, BQ^s \rangle$.

There are, however, some indirect indications that the answer to the question is affirmative. Modifying the notion of independence, a class $A \subseteq W$ will be called strongly independent in a BVS $\langle W, M, G \rangle$ if $A \cap M = \emptyset$ and for any two nonempty disjoint subsets $v, w \subseteq A$ the following condition holds:

$$[v] \cap ([w] + M) \subseteq M.$$

Obviously, every strongly independent class is independent. Now, a class $A \subseteq W$ will be called weakly dimensionally compact (in $\langle W, M, G \rangle$) if each strongly independent set $w \subseteq A$ is finite. As it can easily be seen, dimensional compactness implies weak dimensional compactness, and Lemma 5 and Proposition 5 remain true when replacing the notion of dimensional compactness through its new weak version, as well. Inspecting the proof of Theorem 5, one can find that even weak dimensional compactness of the class $\mathbf{C}[R]$ implies compactness of the π-equivalence R. Hence each of the conditions (a) — (d) of Theorem 5 is equivalent to any one of the following two:

(e) *The class $\mathbf{C}[R] \cap BQ^s$ is weakly dimensionally compact in $\langle Q^s, IQ^s, BQ^s \rangle$.*
(f) *The class $\mathbf{C}[R]$ is weakly dimensionally compact in $\langle Q^s, IQ^s, BQ^s \rangle$.*

However, as proved by M. Šmíd [Sm 1987], the infinite set of functions $w = \{f_x;\ x \in s\}$, defined by $f_x(x) = 0$ and $f_x(y) = 1$ for $y \in s$, $y \neq x$, considered in the BVS $\langle Q^s, IQ^s, BQ^s\rangle$, does not contain any infinite strongly independent subset. Hence w is weakly dimensionally compact in $\langle Q^s, IQ^s, BQ^s\rangle$. On the other hand, $w \subseteq BQ^s$ and $f_x \not\doteq^s f_y$ for $x \neq y$ are trivial. Thus $\doteq^s$ is not compact on w.

To conclude, we will formulate more precisely the two open problems mentioned in the text.

Problem 1. Characterize classes $A \subseteq Q^s$, satisfying $A = \mathbf{C}[\mathbf{T}[A]]$, in terms of closedness with respect to some explicitly defined algebraic operations (with perhaps infinite arities from N) on Q^s and closedness with respect to some "natural" topology (topologies) on Q^s (e.g. induced by some BVS structure(s) on Q^s).

Problem 2. Find an example of an infinite set $w \subseteq BQ^s$ such that $f \not\doteq^s g$ for all distinct $f, g \in w$ and w is dimensionally compact in $\langle Q^s, IQ^s, BQ^s\rangle$, or prove that such a w does not exist. Decide the same question for other BVS structures on Q^s (e.g. for the spaces $\mathcal{L}_p(n)$) and for general BVS'.

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