Miroslav Doupovec

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NATURAL OPERATORS TRANSFORMING VECTOR FIELDS TO THE SECOND ORDER TANGENT BUNDLE

MIROSLAV DOUPOVEC, Brno

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Summary. We study some properties of the non-product-preserving functor T^2 of the second order tangent vectors. We determine all natural operators $T \to TT^2$ transforming vector fields to the second order tangent bundle, and all natural transformations $TT^2 \to TT^2$ over the identity of the functor T^2.

Keywords: Natural operator, natural transformation, second order tangent bundle.

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Recently, Kolář has determined all natural operators $T \to TF$ transforming every vector field on a manifold M into a vector field on FM, where F is any natural bundle corresponding to a product-preserving functor, [6]. The proof is based on the result by Kainz and Michor that every such a functor coincides with a Weil functor T^B defined by a Weil algebra B. The functor T^r of the r-th order tangent vectors is an example of a non-product-preserving functor, which has different properties.

Using a general method by Kolář, [4], we determine all natural operators transforming every vector field on a manifold M into a vector field on its second order tangent bundle T^2M. We deduce that all such operators form a 4-parameter family. In this connection we find all natural transformations $TT^2 \to TT^2$ over the identity of the second order tangent functor. — All manifolds and maps are assumed to be infinitely differentiable. The author is grateful to Prof. I. Kolář for suggesting the problem, useful discussions and valuable comments.

1. THE SECOND ORDER TANGENT FUNCTOR

Denote by $\mathcal{M}f$ the category of all manifolds and all smooth maps, by $\mathcal{FM}$ the category of fibred manifolds, by $\mathcal{VB}$ the category of differentiable vector bundles and by $\mathcal{M}f_m$ the category of m-dimensional manifolds and their local diffeomorphisms.

The space $T^{2*}M = J^2(M, R)_0$ of all 2-jets of a manifold M into reals with target zero is a vector bundle over M. The dual vector bundle

$$T^2M = (T^{2*}M)^*$$

is called the second order tangent bundle of M, [9]. Given a map $f: M \to N$, we can define a linear map $T^{2*}_{f(x)}N \to T^{2*}_x M$ by the composition of jets $V \mapsto V \circ j^2_x f$ for any $V \in T^{2*}_{f(x)}N$. The dual map $T^2_x M \to T^2_{f(x)}N$ is said to be the second order tangent map of $f: M \to N$ at x and is denoted by $T^2_x f$. We have defined the functor $T^2: \mathcal{M}f \to \mathcal{VB}$. Since any linear functional on $T^{2*}M$ can be expressed in the form

$$u^i \frac{\partial f}{\partial x^i} + u^{ij} \frac{\partial^2 f}{\partial x^i \, \partial x^j}$$

with u^{ij} symmetric in i and j, any local chart (x^i) on M induces a local chart (x^i, u^i, u^{ij}) on T^2M. Given some local coordinates (x^i) or (y^p) on M or N, the corresponding fibre coordinates on T^2M or T^2N are (x^i, u^i, u^{ij}) or (y^p, v^p, v^{pq}), respectively. Let $y^p = f^p(x^i)$ be the coordinate expression of a map $f: M \to N$, and $j^2_x f = (x^i, y^p, f^p_i, f^p_{ij})$. Then the coordinate formula for $T^2 f$ is, [3],

$$\begin{aligned} v^p &= f^p_i u^i + f^p_{ij} u^{ij} , \\ v^p_q &= f^p_r f^q_s u^{rs} . \end{aligned} \tag{1}$$

2. NATURAL OPERATORS

Let us recall the concept of a natural bundle in the sense of Nijenhuis, [7].

A natural bundle over m-manifolds is a functor $F: \mathcal{M}f_m \to \mathcal{FM}$ such that

(a) every manifold $M \in \operatorname{Ob} \mathcal{M}f_m$ is transformed into a fibred manifold $p_M: FM \to M$ over M,

(b) every local diffeomorphism $f: M \to N$ of m-manifolds is transformed into an $\mathcal{FM}$ – morphism Ff over f,

(c) for every inclusion of an open subset $i: U \to M$, we have $FU = p_M^{-1}(U)$ and Fi is the inclusion $p_M^{-1}(U) \to FM$, see also [8].

A natural bundle $F: \mathcal{M}f_m \to \mathcal{FM}$ is said to be of an order r, if, for any local diffeomorphisms $f, g: M \to N$ and any $x \in M$, the relation $j^r f(x) = j^r g(x)$ implies $Ff \mid F_x M = Fg \mid F_x M$, where $F_x M$ denotes the fibre of FM over $x \in M$. Let $C^\infty(Y \to X)$ denote the set of all smooth sections of a fibred manifold $Y \to X$. Given two fibred manifolds $Y \to X$ and $W \to Z$ such that $q: Z \to X$ is also a fibred manifold, a map $A: C^\infty(Y \to X) \to C^\infty(W \to Z)$ is called a base extending operator, [5]. We say that A is an r-th order operator, if $j^r s_1(x) = j^r s_2(x)$ implies $As_1(z) = A\, s_2(z)$ for any $s_1, s_2 \in C^\infty(Y \to X)$, any $x \in X$ and all $z \in q^{-1}(x)$. Such an operator is said to be regular, if it transforms every smoothly parametrized family of sections into a smoothly parametrized family.

Let F and G be two natural bundles on $\mathcal{M}f_n$ and let E be a natural bundle on $\mathcal{M}f_m$, $m = \dim GR^n$. A natural operator $A: F \to EG$ is defined as a system of regular base extending operators $A_M: C^\infty(FM \to M) \to C^\infty(EGM \to GM)$ for all $M \in \operatorname{Ob} \mathcal{M}f_n$ such that for every $s \in C^\infty FM$ we have $A_N(Ff \circ s \circ f^{-1}) = EGf \circ A_M s \circ (Gf)^{-1}$ for every diffeomorphism $f\colon M \to N$, and $A_U s = (A_M s) \mid GU$ for every open subset $U \subset M$. A natural operator $A\colon T \to TF$ is said to be absolute, if $A_M X = A_M O_M$ for every vector field X on the manifold M, provided O_M is the zero vector field on M.

Denote by J^r the functor which transforms every fibred manifold $Y \to X$ into its r-th jet prolongation $J^r Y \to X$ and every fibred manifold morphism $\varphi\colon Y \to \bar{Y}$ over a local diffeomorphism $\varphi_0\colon X \to \bar{X}$ into the induced map $J^r\varphi\colon J^r Y \to J^r \bar{Y}$ given by $J^r\varphi(j^r_x f) = j^r_{\varphi_0(x)}(\varphi \circ f \circ \varphi_0^{-1})$. If F is an arbitrary s-th order natural bundle, then $J^r F$ is an $(r + s)$-th order natural bundle.

Remark 1. To describe all natural operators $A\colon F \to EG$, we shall use the following assertion, [5]. Let $(J^r F)_0 = (J^r F R^m)_0$, $G_0 = (GR^m)_0$, $(EG)_0 = (EGR^m)_0$ be the standard fibres. There is a bijection between the G^s_m – equivariant maps $(J^r F)_0 \times G_0 \to (EG)_0$ over the identity of G_0 and the r-th order natural operators $F \to EG$, provided s is the maximum of the orders of the functors $J^r F$ and EG, and G^s_m means the group of all invertible s-jets from R^m into R^m with source and target 0.

3. NATURAL OPERATORS $T \to TT^2$

Denote by $\mathcal{T}^2$ the flow operator transforming every vector field X on M into its flow prolongation $\mathcal{T}^2 X = \partial/\partial t|_0 \, (T^2(\exp tX))$, where $\exp tX$ means the flow of X. If $X^i(x)\,(\partial/\partial x^i)$ is the coordinate expression of X and $X^i_j = (\partial X^i(x)/\partial x^j)$, $X^i_{jk} = (\partial^2 X^i(x)/\partial x^j\, \partial x^k)$, then one easily evaluates the coordinate expression of $\mathcal{T}^2 X$

$$X^i \frac{\partial}{\partial x^i} + (X^i_j u^j + X^i_{jk} u^{jk}) \frac{\partial}{\partial u^i} + (X^i_k u^{kj} + X^j_k u^{ik}) \frac{\partial}{\partial u^{ij}}\,.$$

Further, the multiplication of vectors by real numbers determines the Liouville vector field $L(M)$ on $T^2 M$, the coordinate form of which is

$$u^i \frac{\partial}{\partial u^i} + u^{ij} \frac{\partial}{\partial u^{ij}}\,.$$

Clearly, $X \mapsto L(M)$, $X \in C^\infty TM$ is an absolute operator $T \to TT^2$. Moreover, given a vector field X on M and a function $f\colon M \to R$, we can iterate the derivative $X(Xf)$ of f with respect to X. In this way we obtain an operator $\tilde{D}_2\colon C^\infty(TM) \to C^\infty(T^2 M)$ with the coordinate expression

$$X^i \frac{\partial}{\partial x^i} \mapsto X^j \frac{\partial X^i}{\partial x^j} \frac{\partial f}{\partial x^i} + X^i X^j \frac{\partial^2 f}{\partial x^i\, \partial x^j}\,.$$

Analogously, using the derivative Xf of f with respect to X, we obtain the identity operator $\tilde{D}_1\colon C^\infty(TM) \to C^\infty(TM)$. Further, we have a canonical inclusion $TM \subset T^2 M$. The section $\tilde{D}_k X\colon M \to T^2 M$, $k = 1, 2$, can be extended by means of the fibre translations into a vector field constant on each fibre, so that we have constructed natural operators D_1, $D_2\colon T \to TT^2$.

Proposition 1. *All natural operators $T \to TT^2$ form the 4-parameter family*

$$k_1\mathcal{T}^2 + k_2L + k_3D_2 + k_4D_1\,, \quad k_i \in R\,. \tag{2}$$

Proof. Lemma 1 in [6] implies that the order of any natural operator $A\colon T \to TT^2$ is less than or equal to 2. By Remark 1 there is a bijective correspondence between such operators and certain G_m^3 – equivariant maps of the standard fibres. The coordinates on the standard fibre $S = T_0^2R^m$ are u^i, u^{ij}. Since T^2 is a second order functor, S is a G_m^2 – space. Denote by

$$a_j^i,\ a_{jk}^i,\ a_{jkl}^i \tag{3}$$

the canonical coordinates on G_m^3 and by tilda the coordinates of the element inverse to (3) in G_m^3. By (1), the action of G_m^2 on S is

$$\begin{aligned} \bar{u}^i &= a_j^i u^j + a_{jk}^i u^{jk}\,, \\ \bar{u}^{ij} &= a_k^i a_l^j u^{kl}\,. \end{aligned} \tag{4}$$

Let $V_m^2 = J_0^2(TR^m)$ be the space of all 2-jets of the vector fields on R^m at the origin. Using standard evaluations we find the following equations of the action of G_m^3 on V_m^2:

$$\begin{aligned} \bar{X}^i &= a_j^i X^j\,, \\ \bar{X}_j^i &= a_{kl}^i \tilde{a}_j^k X^l + a_k^i X_l^k \tilde{a}_j^l\,, \end{aligned}$$

while for X_{jk}^i we need only the action of the subgroup $a_{jk}^i = 0$:

$$\bar{X}_{jk}^i = a_{mnp}^i \tilde{a}_j^m \tilde{a}_k^n X^p + a_m^i X_{ln}^m \tilde{a}_j^l \tilde{a}_k^n\,.$$

The standard fibre of TT^2 is $Z = S \times R^m \times S$ with the coordinates $u^i, u^{ij}, Y^i = \mathrm{d}x^i$, $U^i = \mathrm{d}u^i$, $U^{ij} = \mathrm{d}u^{ij}$. Using (4), we deduce the transformation laws of the coordinates Y^i, U^i, U^{ij}

$$\begin{aligned} \bar{Y}^i &= a_j^i Y^j\,, \\ \bar{U}^i &= a_j^i U^j + a_{jk}^i U^{jk} + a_{jk}^i u^j Y^k + a_{jkl}^i u^{jk} Y^l\,, \\ \bar{U}^{ij} &= a_k^i a_l^j U^{kl} + \left(a_{km}^i a_l^j + a_k^i a_{lm}^j\right) u^{kl} Y^m\,. \end{aligned}$$

We have to determine all G_m^3 – equivariant maps $f\colon V_m^2 \times S \to Z$ over id_S. Let

$$Y^i = f^i(X^i, X_j^i, X_{jk}^i, u^i, u^{ij})$$

denote the first series of components of f. Consider first the equivariancy of f^i with respect to the kernel K_3 of the jet projection $G_m^3 \to G_m^1$ given by $a_j^i = \delta_j^i$, $a_{jk}^i = 0$. We obtain

$$f^i(X^i, X_j^i, X_{jk}^i, u^i, u^{ij}) = f^i(X^i, X_j^i, X_{jk}^i + a_{jkl}^i X^l, u^i, u^{ij})\,,$$

which indicates that f^i are independent of X_{jk}^i. Further, the homotheties $a_j^i = k\delta_j^i$ and the other a's vanishing give the homogeneity condition

$$kf^i = f^i(kX^i, X_j^i, ku^i, k^2u^{ij})\,.$$

Therefore

$$Y^i = g^i_j(X^k_l)\, X^j + h^i_j(X^k_l)\, u^j\,,$$

where g^i_j and h^i_j are smooth functions. The equivariancy of Y^i with respect to the kernel K_2 of the jet projection $G^2_m \to G^1_m$ characterized by $a^i_j = \delta^i_j$ means

$$g^i_j(X^k_l)\, X^j + h^i_j(X^k_l)\, u^j = g^i_j(X^k_l + a^k_{lm}X^m)\, u^j + h^i_j(X^k_l + a^k_{lm}X^m)\,(u^j + a^j_{kl}u^{kl})\,.$$

This implies $h^i_j = 0$, $g^i_j = \text{const}$. Evaluating the equivariancy of Y^i with respect to the subgroup $G \subset G^3_m$ given by arbitrary a^i_j and the other a's vanishing we find that g^i_j are G – equivariant. By the theory of invariant tensors, [1], $g^i_j = k_1\delta^i_j$, so that

(5) $$Y^i = k_1 X^i\,, \quad k_1 \in R\,.$$

Consider now the difference $A - k_1\mathcal{T}^2$, where $\mathcal{T}^2$ means the flow operator and k_1 is taken from (5). This operator transforms every vector field $X \in C^\infty(TM)$ into a vertical vector field on T^2M. We have $VT^2M = T^2M \oplus T^2M$, so that the components h^{ij} of the difference operator have the tensorial transformation law. Similarly to the case of f^i we prove that h^{ij} are independent of X^i_{jk}. The homotheties lead to the condition $k^2h^{ij} = h^{ij}(kX^i, X^i_j, ku^i, k^2u^{ij})$. Hence

$$h^{ij} = f^{ij}_{kl}(X^m_n)\, u^{kl} + g^{ij}_{kl}(X^m_n)\, u^k u^l + h^{ij}_{kl}(X^m_n)\, X^k u^l + k^{ij}_{kl}(X^m_n)\, X^k X^l\,,$$

where f^{ij}_{kl}, g^{ij}_{kl}, h^{ij}_{kl} and k^{ij}_{kl} are smooth functions. Further, taking into account the equivariancy of h^{ij} with respect to the kernel K_2 we obtain

(6) $$\begin{aligned} f^{ij}_{kl}(X^m_n)\, u^{kl} + g^{ij}_{kl}(X^m_n)\, u^k u^l + h^{ij}_{kl}(X^m_n)\, X^k u^l + k^{ij}_{kl}(X^m_n)\, X^k X^l = \\ = f^{ij}_{kl}(\bar{X}^m_n)\, u^{kl} + g^{ij}_{kl}(\bar{X}^m_n)\,(u^k + a^k_{rs}u^{rs})\,(u^l + a^l_{tq}u^{tq}) + \\ + h^{ij}_{kl}(\bar{X}^m_n)\, X^k(u^l + a^l_{rs}u^{rs}) + k^{ij}_{kl}(\bar{X}^m_n)\, X^k X^l\,. \end{aligned}$$

This implies $g^{ij}_{kl} = 0$. Setting $u^i = 0$ and $u^{ij} = 0$ in (6), we obtain

$$k^{ij}_{kl}(X^m_n)\, X^k X^l = k^{ij}_{kl}(X^m_n + a^m_{np}X^p)\, X^k X^l\,.$$

This gives, similarly to the case of g^{ij}, $k^{ij}_{kl}(X^m_n)\, X^k X^l = k_3 X^i X^j$, $k_3 \in R$. Analogously, putting $u^{ij} = 0$ in (6) we prove that $h^{ij}_{kl}(X^m_n)\, X^k u^l = e(X^i u^j + X^j u^i)$, $e \in R$. The remaining part of (6) has the form

$$\begin{aligned} f^{ij}_{kl}(X^m_n)\, u^{kl} + e(X^i u^j + X^j u^i) = f^{ij}_{kl}(X^m_n + a^m_{np}X^p)\, u^{kl} + \\ + e[X^i(u^j + a^j_{kl}u^{kl}) + X^j(u^i + a^i_{kl}u^{kl})]\,. \end{aligned}$$

Differentiating the latter relation with respect to X^m_n we get $\partial f^{ij}_{kl}/\partial X^m_n = \text{const}$, so that $f^{ij}_{kl}(X^m_n)\, u^{kl} = (g^{ijn}_{klm}X^m_n + c^{ij}_{kl})\, u^{kl}$. Applying the theory of invariant tensors, [4], we find $f^{ij}_{kl}u^{kl} = k_2u^{ij} + fX^k_k u^{ij} + g(X^i_k u^{kj} + X^j_k u^{ik})$, $k_2, f, g \in R$. Up to now, we have deduced

(7) $$h^{ij} = k_3 X^i X^j + k_2 u^{ij} + e(X^i u^j + X^j u^i) + fX^k_k u^{ij} + g(X^i_k u^{kj} + X^j_k u^{ik})\,.$$

The equivariancy with respect to the subgroup $G_1 \subset G_m^3$ characterized by $a_j^i = \delta_j^i$ then leads to the relation

$$e(X^i a_{kl}^j u^{kl} + X^j a_{kl}^i u^{kl}) + f a_{kl}^k X^l u^{ij} + g(a_{kl}^i X^l u^{kj} + a_{kl}^j X^l u^{ik}) = 0 .$$

If the dimension m of the manifold M is greater than or equal to 2, then $e = f = g = 0$, while in the case $m = 1$ we have

(8) $$2e + 2g + f = 0 .$$

Suppose first that $m \geqq 2$. Then

(9) $$h^{ij} = k_2 u^{ij} + k_3 X^i X^j .$$

Now, we can take the difference $A - k_1 \mathscr{T}^2 - k_2 L - k_3 D_2$. Its components h^i have the tensorial transformation law. Evaluating first the equivariancy with respect to the kernel K_3 and then with respect to the homotheties we obtain

$$h^i = f_j^i(X_n^m) X^j + g_j^i(X_n^m) u^j .$$

In the same way as in the case of f^i we find

(10) $$h^i = k_4 X^i , \quad k_4 \in R .$$

Hence (5), (9) and (10) prove the proposition for $m \geqq 2$.

Finally, let $m = 1$. Denote by (u_1, u_2) the coordinates on S, by (a_1, a_2, a_3) the coordinates on G_1^3, by (X, X_1, X_2) the coordinates on V_1^2 and h_1, h_2 the components of the difference $A - k_1 \mathscr{T}^2$. It follows from (7) and (8) that

$$h_2 = k_2 u_2 + k_3 X^2 + \alpha(X_1 u_2 - X u_1) , \quad \alpha \in R .$$

We easily evaluate that

(11) $$a_1 h_1(X, X_1, X_2, u_1, u_2) + a_2 k_2 u_2 + a_2 k_3 X^2 + a_2 \alpha(X_1 u_2 - X u_1) = = h_1(\bar{X}, \bar{X}_1, \bar{X}_2, \bar{u}_1, \bar{u}_2) ,$$

where $\bar{u}_1 = a_1 u_1 + a_2 u_2$, $\bar{u}_2 = a_1^2 u_2$, $\bar{X} = a_1 X$, $\bar{X}_1 = X_1 + (a_2/a_1) X$, while for X_2 we need only the action of the subgroup $a_2 = 0$: $\bar{X}_2 = (1/a_1) X_2 + (a_3/a_1^2) X$. Putting $a_1 = 1$, $a_2 = 0$ in (11) we show that h_1 does not depend on X_2. Next, the homotheties $a_1 = k$, $a_2 = 0$ imply $h_1 = f_1(X_1) X + g_1(X_1) u_1$. Further, the equivariancy of h_1 with $a_1 = 1$ leads to the relation

(12) $$f_1(X_1) X + g_1(X_1) u_1 + a_2[k_2 u_2 + k_3 X^2 + \alpha(X_1 u_2 - X u_1)] = = f_1(X_1 + a_2 X) X + g_1(X_1 + a_2 X)(u_1 + a_2 u_2) .$$

Differentiating with respect to u_2 we obtain

$$a_2[k_2 + \alpha X_1] = g_1(X_1 + a_2 X) a_2 .$$

Next, differentiating the latter relation with respect to X and setting $a_2 = 0$ we get

$\partial g_1(X_1)/\partial X_1 = 0$. This gives $g_1(X_1) = g = \text{const}$. Further, if we compare the coefficients by u_2 in (12), we find $\alpha = 0$, $g = k_2$. The relation (12) has now the form

$$f_1(X_1)\, X + a_2 k_3 X^2 = f_1(X_1 + a_2 X)\, X\,. \tag{13}$$

Differentiating with respect to X_1 we show that $\partial f_1/\partial X_1$ is constant. This yields

$$f_1 = f X_1 + k_4\,, \quad f, k_4 \in R\,. \tag{14}$$

Finally, (13) and 14) imply $f = k_3$. Thus, we have deduced

$$h_2 = k_2 u_2 + k_3 X^2\,,$$
$$h_1 = k_2 u_1 + k_3 X_1 X + k_4 X\,.$$

This completes the proof.

4. THE NATURAL TRANSFORMATIONS $TT^2 \to TT^2$

Proposition 2. *All natural transformations $TT^2 \to TT^2$ over the identity of T^2 form a 3-parameter family*

$$\bar{Y}^i = \alpha Y^i\,,$$
$$\bar{U}^i = \alpha U^i + \beta Y^i + \gamma u^i\,,$$
$$\bar{U}^{ij} = \alpha U^{ij} + \gamma u^{ij}$$

with any $\alpha, \beta, \gamma \in R$.

Proof. According to the general theory [2], the natural transformations $TT^2 \to TT^2$ over id_{T^2} are in bijection with the G^3_m – equivariant maps $f\colon Z \to Z$ of the standard fibres. The coordinate form of the map f is

$$\bar{Y}^i = f^i(u^i, u^{ij}, Y^i, U^i, U^{ij})\,,$$
$$\bar{U}^i = g^i(u^i, u^{ij}, Y^i, U^i, U^{ij})\,,$$
$$\bar{U}^{ij} = h^{ij}(u^i, u^{ij}, Y^i, U^i, U^{ij})\,.$$

Considering equivariancy with respect to the homotheties we obtain homogeneity conditions

$$k f^i = f^i(k u^i, k^2 u^{ij}, k Y^i, k U^i, k^2 U^{ij})\,,$$
$$k g^i = g^i(k u^i, k^2 u^{ij}, k Y^i, k U^i, k^2 U^{ij})\,,$$
$$k^2 g^{ij} = g^{ij}(k u^i, k^2 u^{ij}, k Y^i, k U^i, k^2 U^{ij})\,.$$

This implies

$$f^i = \alpha_1 u^i + \beta_1 Y^i + \gamma_1 U^i\,, \tag{15}$$
$$g^i = a_1 u^i + b_1 U^i + c Y^i\,,$$
$$g^{ij} = a_2 u^{ij} + b_2 U^{ij} + h^{ij}(u^i, Y^i, U^i)\,,$$

where h^{ij} are certain polynomials. Consider now the equivariancy of f^i with respect to the kernel K_2. We obtain

$$\alpha_1 u^i + \beta_1 Y^i + \gamma_1 U^i = \alpha_1(u^i + a^i_{jk}u^{jk}) + \beta_1 Y^i + \\ + \gamma_1(U^i + a^i_{jk}U^{jk} + a^i_{jk}u^j Y^k + a^i_{jkl}u^{jk}Y^l).$$

Then we have $\alpha_1 = 0$, $\gamma_1 = 0$, and β_1 is arbitrary, so that the function f^i in (15) has the form

$$f^i = \beta_1 Y^i . \tag{16}$$

Analogously, using the equivariancy of g^i with respect to the kernel K_2 we find

$$a_2 = a_1 , \quad b_1 = b_2 = \beta_1 , \quad h^{jk}(u^i, Y^i, U^i) = 0 . \tag{17}$$

Substituting (16) and (17) to (15) we complete the proof.

Remark 2. For a Weil functor T^B, all natural operators $T \to TT^B$ can be constructed from the flow operator $\mathcal{T}^B$ by applying all natural transformations H of TT^B into TT^B over the identity of T^B, [6]. This is not true for the non-product-preserving functor T^2. In this case all natural operators $T \to TT^2$ form a 4-parameter family, while all natural transformations $H: TT^2 \to TT^2$ over id_{T^2} form a 3-parameter family. Hence the composition $H \circ \mathcal{T}^2$ forms a 3-parameter family only, in which the operator D_2 is not included.

Remark 3. In the case of a Weil functor T^B, Theorem 1 from [6] implies that the difference between a natural operator $T \to TT^B$ and its associated absolute operator is a linear operator. This is not true for the non-product-preserving functors, the operator D_2 being the simpliest counter-example.

Remark 4. The operators $\mathcal{T}^2$, L and D_1 transform every vector field on a manifold M into a vector field on T^2M tangent to the subbundle $TM \subset T^2M$, but D_2 does not. With a little surprise we can express it by saying that the natural operator D_2: $T \to TT^2$ is not compatible with the natural inclusion $TM \subset T^2M$.

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Souhrn

PŘIROZENÉ OPERÁTORY TRANSFORMUJÍCÍ VEKTOROVÁ POLE NA TEČNÝ BANDL DRUHÉHO ŘÁDU

MIROSLAV DOUPOVEC

V článku jsou určeny všechny přirozené operátory převádějící libovolné vektorové pole na varietě M na vektorové pole na tečném bandlu druhého řádu T^2M. V této souvislosti jsou nalezeny všechny přirozené transformace $TT^2 \to TT^2$ nad identickým zobrazením funktoru T^2.

Резюме

ЕСТЕСТВЕННЫЕ ОПЕРАТОРЫ, ПРЕОБРАЗУЮЩИЕ ВЕКТОРНЫЕ ПОЛЯ В КАСАТЕЛЬНОЕ РАССЛОЕНИЕ ВТОРОЙ СТЕПЕНИ

MIROSLAV DOUPOVEC

Определяются все естественные операторы, преобразующие любое векторное поле на многообразии M в векторное поле на касательном расслоении второй степени T^2M. В связи с тем определяются все естественные преобразования $TT^2 \to TT^2$ над тождественным отображением функтора T^2.

Author's address: Katedra matematiky a deskriptivní geometrie VUT Brno, fakulta strojní, Technická 2, 616 69 Brno.