Marcella Palese; Ekkehart Winterroth

Some aspects of the homogeneous formalism in field theory and gauge invariance

Archivum Mathematicum, Vol. 42 (2006), No. 5, 319--327

Persistent URL: `http://dml.cz/dmlcz/108038`

ARCHIVUM MATHEMATICUM (BRNO)
Tomus 42 (2006), Supplement, 319 – 327

SOME ASPECTS OF THE HOMOGENEOUS FORMALISM IN FIELD THEORY AND GAUGE INVARIANCE

MARCELLA PALESE AND EKKEHART WINTERROTH

Abstract. We propose a suitable formulation of the Hamiltonian formalism for Field Theory in terms of Hamiltonian connections and multisymplectic forms where a composite fibered bundle, involving a line bundle, plays the role of an extended configuration bundle. This new approach can be interpreted as a suitable generalization to Field Theory of the homogeneous formalism for Hamiltonian Mechanics. As an example of application, we obtain the expression of a *formal energy* for a parametrized version of the Hilbert–Einstein Lagrangian and we show that this quantity is conserved.

1. Introduction

A geometric setting for the Hamiltonian description of Field Theory is proposed which generalizes the homogeneus Hamiltonian formalism in time-dependent Mechanics (see e.g. [27]). The aim is to provide a suitable description of the gauge character appearing in the covariant formulations of Hamiltonian *multiphase* Field Theory and their quantizations based on the seminal paper by Dedecker [4] and developed in the recent literature by many authors; see e.g. [5, 6, 13, 14, 20, 24, 25] and many references quoted therein.

One of the main features of our approach is that one can describe the polymomenta and other objects such as Hamiltonian forms in terms of differential forms with values in the vertical tangent bundle of an appropriate line bundle Θ. The introduction of the line bundle Θ, in fact, can be understood as a suitable way of describing the *gauge* character appearing in the multiphase formalism for Field Theory, essentially due to the fact that the independent variables are more than one and thus the Poincaré–Cartan invariant is defined only up to the choice of a linear connection on the basis manifold (see e.g. [9, 16, 19, 20]).

2000 *Mathematics Subject Classification.* 53C05, 58A20, 70H05, 70S05, 83C40.

Key words and phrases. jets, connections, homogeneous formalism, Hamilton equations, energy, gravity.

Both authors are supported by GNFM of INdAM, University of Torino and MIUR (PRIN 2005). The second author (E. W.) is also supported by a PhD grant of the University of Torino - *Dottorato di Ricerca in Matematica XVIII Ciclo.*

This paper is in final form and will not be submitted elsewhere.

With the aim of overcoming this ambiguities, instead of bundles over an n–dimensional base manifold X, we consider *fibrations over a line bundle* Θ *fibered over* X. We recall that a geometric formulation of the Hamiltonian formalism for Field Theory in terms of Hamiltonian connections and multisymplectic forms was developed e.g. in [22, 26, 27]. In this framework, the covariant Hamilton equations for Mechanics and Field Theory are defined in terms of multisymplectic $(n+2)$–forms, where n is the dimension of the basis manifold, together with connections on the configuration bundle. Following the analogous setting for Mechanics and for the polymomentum approach to Field Theory, we propose a new concept of event bundle, configuration bundle and Legendre bundle. Correspondingly, Hamiltonian connections, Hamiltonian forms and covariant Hamilton equations can be suitably described in this framework. This new approach takes into account the existence of more than one independent variable in Field Theory, but enables us to keep most of the features of time-dependent Hamiltonian Mechanics. In fact, the prominent role of *symplectic* structures in field theories has been stressed in [15, 16, 17, 18] and recently a symplectic approach for the study of Canonical Gravity [10] has been proposed.

We point out that the extension of the Hamiltonian formalism from Mechanics to Field Theory is usually performed starting from the *non-homogeneous* formalism of Mechanics, where a gauge choice is assumed *a priori* to be performed; precisely $q^0(t) = t$, where t is the time. It is however well known − and it deserves to be noticed within our note - that Mechanics is invariant with respect to gauge choices of this kind, i.e. with respect to choices of the section $q^0(t)$; see e.g. the review in [10]. Accordingly with the just mentioned approach to Mechanics, in Hamiltonian Field Theory the configuration variables (fields) are usually assumed to depend *directly* on a number of independent variables greater than one. As an outcome, it is well known that polymomenta correspondingly defined are in a bigger number than the configurations, and that the corresponding Hamiltonians do not have a clear interpretation as physical observables. Many difficulties arises in the attempts of quantization of such a Hamiltonian Field Theory (see e.g. the detailed reviews in [11, 12, 14]).

In the present paper we generalize to Field Theory the so-called *homogeneous formalism of time-dependent Mechanics*, so that a local line coordinate τ plays the role of the 'homogeneous' local coordinate q^0, and a *formal Hamiltonian theory* is constructed where the Hamiltonian describes the *formal evolution* along the line coordinate. The latter in turn depends on the basis (independent) coordinates *when a gauge choice is performed*, i.e. a section of the line bundle is chosen. This is nearer to [3], and it keeps the advantages of a finite dimensional approach. Thus the formal Hamiltonian can be interpreted as a formal energy (beeing the conjugated momentum to the formal evolution parameter). The energy then is a gauge charge since it is related with invariance properties with respect to infinitesimal transformations of the line (vertical) coordinate.

In Section 2 we state the general framework of composite fiber bundles, their jet prolongations and composite connections. Section 3 contains *abstract Hamilton equations* and a Theorem which relates the *abstract Hamiltonian dynamics* introduced here with the standard Hamilton–De Donder equations (see [20] for a detailed review on the topic and recent developments). Proceeding in analogy with Mechanics we obtain the expression of a *'formal' energy* for an extended version of the Hilbert–Einstein Lagrangian and we show that this quantity is conserved.

The present approach is a completion of [7] where the formal aspect of the homogeneous setting was not exhaustively explicated. Some misprints and imprecisions there appearing will be also corrected.

2. Jets and connections on composite bundles

The general framework is a fibered bundle over X, $\pi : Y \to X$, with $dimX = n$ and $dimY = n + m$ and, for $r \geq 0$, its jet manifold J_rY. We recall the natural fiber bundles $\pi^r_s : J_rY \to J_sY$, $r \geq s$, $\pi^r : J_rY \to X$, and, among these, the *affine* fiber bundles π^r_{r-1}.

Greek indices $\lambda, \mu, \ldots$ run from 1 to n and they label base coordinates, while Latin indices $i, j, \ldots$ run from 1 to m and label fibre coordinates, unless otherwise specified. We denote multi–indices of dimension n by underlined Greek letters such as $\underline{\alpha} = (\alpha_1, \ldots, \alpha_n)$, with $0 \leq \alpha_\mu$, $\mu = 1, \ldots, n$; by an abuse of notation, we denote with λ the multi–index such that $\alpha_\mu = 0$, if $\mu \neq \lambda$, $\alpha_\mu = 1$, if $\mu = \lambda$. We also set $|\underline{\alpha}| \doteq \alpha_1 + \cdots + \alpha_n$. The charts induced on J_rY are denoted by $(x^\lambda, y^i_{\underline{\alpha}})$, with $0 \leq |\underline{\alpha}| \leq r$; in particular, we set $y^i_{\mathbf{0}} \equiv y^i$. The local bases of vector fields and 1–forms on J_rY induced by the coordinates above are denoted by $(\partial_\lambda, \partial^{\underline{\alpha}}_i)$ and $(d^\lambda, d^i_{\underline{\alpha}})$, respectively.

For $r \geq 1$, the *contact maps* on jet spaces induce the natural complementary fibered morphisms over the affine fiber bundle $J_rY \to J_{r-1}Y$

$$D_r : J_rY \times_X TX \to TJ_{r-1}Y\,, \quad \vartheta_r : J_rY \times_{J_{r-1}Y} TJ_{r-1}Y \to VJ_{r-1}Y\,, \tag{1}$$

with coordinate expressions, for $0 \leq |\underline{\alpha}| \leq r - 1$, given by $D_r = d^\lambda \otimes D_\lambda = d^\lambda \otimes (\partial_\lambda + y^j_{\underline{\alpha}+\lambda} \partial^{\underline{\alpha}}_j)$, $\vartheta_r = \vartheta^j_{\underline{\alpha}} \otimes \partial^{\underline{\alpha}}_j = (d^j_{\underline{\alpha}} - y^j_{\underline{\alpha}+\lambda} d^\lambda) \otimes \partial^{\underline{\alpha}}_j$, and the natural fibered splitting $J_rY \times_{J_{r-1}Y} T^*J_{r-1}Y = J_rY \times_{J_{r-1}Y} (T^*X \oplus V^*J_{r-1}Y)$.

Definition 1. A *connection* on the fiber bundle $Y \to X$ is defined by the (mutually dual) linear bundle morphisms over Y: $Y \times_X TX \to TY$, $V^*Y \to T^*Y$ which split the exact sequences

$$0 \to VY \hookrightarrow TY \to Y \times_X TX \to 0\,, \quad 0 \to Y \times_X T^*X \hookrightarrow T^*Y \to V^*Y \to 0\,.$$

We recall that there is a one–to–one correspondence between the connections Γ on a fiber bundle $Y \to X$ and the global sections $\Gamma : Y \to J_1Y$ of the affine jet bundle $J_1Y \to Y$ (see e.g. [22]).

In the following a relevant role is played by the composition of fiber bundles

$$Y \to \Theta \to X\,, \tag{2}$$

where $\pi_{YX} : Y \to X$, $\pi_{Y\Theta} : Y \to \Theta$ and $\pi_{\Theta X} : \Theta \to X$ are fiber bundles. The above composition was introduced under the name of *composite fiber bundle* in [21, 26] and shown to be useful for physical applications, e.g. for the description of mechanical systems with time–dependent parameters.

We shall be concerned here with the description of connections on composite fiber bundles. We will follow the notation and main results stated in [22]; see also [2].

We shall denote by $J_1\Theta$, $J_1^\Theta Y$ and $J_1 Y$, the jet manifolds of the fiber bundles $\Theta \to X$, $Y \to \Theta$ and $Y \to X$ respectively.

Let γ be a connection on the composite bundle π_{YX} projectable over a connection Γ on $\pi_{\Theta X}$, i.e. such $J_1\pi_{Y\Theta} \circ \gamma = \Gamma \circ \pi_{Y\Theta}$. Let γ_Θ be a connection on the fiber bundle $\pi_{Y\Theta}$. Given a connection Γ on $\pi_{\Theta X}$, there exists [22] a canonical morphism over Y, $\rho : J_1\Theta \times_X J_1^\Theta Y \to J_1 Y$, which sends (Γ, γ_Θ), into the *composite connection* $\gamma \doteq \gamma_\Theta \circ \Gamma$ on π_{YX}, projectable over Γ. Recall that given a composite fiber bundle (2) and a global section h of the fiber bundle $\pi_{\Theta X}$, then the restriction $Y_h \doteq h^* Y$ of the fiber bundle $\pi_{Y\Theta}$ to $h(X) \subset \Theta$ is a subbundle $i_h : Y_h \hookrightarrow Y$ of the fiber bundle $Y \to X$ [22]. Let then h be a section of $\pi_{\Theta X}$. Every connection γ_Θ induces the pull–back connection γ_h on the subbundle $Y_h \to X$. The composite connection $\gamma = \gamma_\Theta \circ \Gamma$ is reducible to γ_h if and only if h is an integral section of Γ.

We have the following exact sequences of *vector bundles over a composite bundle* Y:

(3)
$$0 \to V_\Theta Y \hookrightarrow VY \to Y \times_\Theta V\Theta \to 0\,, \qquad 0 \to Y \times_\Theta V^*\Theta \hookrightarrow V^*Y \to V^*_\Theta Y \to 0\,,$$

where $V_\Theta Y$ and $V^*_\Theta Y$ are the vertical tangent and cotangent bundles to the bundle $\pi_{Y\Theta}$.

Remark 1. Every connection γ_Θ on $\pi_{Y\Theta}$ provides the dual splittings

$$VY = V_\Theta Y \oplus_Y \gamma_\Theta(Y \times_\Theta V\Theta)\,, \qquad V^*Y = Y \times_\Theta V^*\Theta \oplus_Y \gamma_\Theta(V^*_\Theta Y)\,, \tag{4}$$

of the above exact sequences. By means of these splittings one can easily construct the *vertical covariant differential* on the composite bundle π_{YX}, i.e. a first order differential operator

$$\Delta_{\gamma_\Theta} : J_1 Y \to T^*X \oplus_Y V^*_\Theta Y\,. \tag{5}$$

This operator is characterized by the property that the restriction of Δ_{γ_Θ}, induced by a section h of $\pi_{\Theta X}$, coincides with the covariant differential on Y_h relative to the pull–back connection γ_h [22].

3. Homogeneus formalism in Field Theory

We recall now that the covariant Hamiltonian Field Theory can be conveniently formulated in terms of Hamiltonian connections and Hamiltonian forms [26]. Here we shall construct a Hamiltonian formalism for Field Theory as a theory on the composite *event bundle* $Y \to \Theta \to X$, with $\pi_{\Theta X} : \Theta \to X$ a *line bundle* having local fibered coordinates (x^λ, τ).

Let us now consider the *extended homogeneous Legendre bundle* $Z_Y \doteq T^*Y \wedge (\Lambda^n T^*\Theta) \to X$. It is the trivial one-dimensional bundle $\kappa : Z_Y \to \Pi_\Theta$, where

$\Pi_\Theta \doteq V^*Y \wedge (\Lambda^n T^*\Theta) \to X$ is *extended Legendre bundle.* There exists a canonical isomorphism

$$\Pi_\Theta \simeq \Lambda^{n+1}T^*\Theta \otimes_Y V^*Y \otimes_Y T\Theta \,. \tag{6}$$

Definition 2. We call the fiber bundle $\pi_{Y\Theta} : Y \to \Theta$ the *abstract event space* of the field theory. The *configuration space* of the Field Theory is then the first order jet manifold $J^\Theta_1 Y$. The *abstract Legendre bundle* of the field theory is the fiber bundle $\Pi_\Theta \to \Theta$.

Let now γ_Θ be a connection on $\pi_{Y\Theta}$ and Γ_Θ be a connection on $\pi_{\Theta X}$. We have the following non–canonical isomorphism

$$\Pi_\Theta \simeq_{(\gamma_\Theta,\Gamma_\Theta)} \Lambda^{n+1}T^*\Theta \otimes_Y [(Y \oplus_\Theta V^*\Theta) \oplus_Y \gamma_\Theta(V^*_\Theta Y)] \otimes_Y (V\Theta \oplus_\Theta H\Theta) \,. \tag{7}$$

In this perspective, we consider the canonical bundle monomorphism over Y providing the tangent–valued Liouville form on Π_Θ, i.e.

$$\vartheta_Y : \Pi_\Theta \hookrightarrow \Lambda^{n+2}T^*Y \otimes_Y (V\Theta \oplus_\Theta H\Theta) \,, \tag{8}$$

where $H\Theta$ is the horizontal subbundle.

Let now $(x^{\hat\mu}) = (x^\mu, \tau)$, $\hat\omega = d^{\mu_1} \wedge d^{\mu_2} \wedge \ldots \wedge d^{\mu_n} \wedge d\tau$, $\partial_{\hat\mu} = (\partial_\mu, \partial_\tau)$ be, respectively, local coordinates on Θ, the induced volume form, local generators of tangent vector fields and put $\hat\omega_{\hat\lambda} \doteq \partial_{\hat\lambda} \rfloor \hat\omega$.

Inspired by [18] we set $\bar p_i \doteq p_i^{\hat\mu} \otimes \partial_{\hat\mu}$ and obtain

$$\vartheta_Y = \bar p_i d^i \wedge \hat\omega \,. \tag{9}$$

The polysymplectic form Ω_Y on Π_Θ is then intrinsically defined by $\Omega_Y \rfloor \psi = d(\vartheta_Y \rfloor \psi)$, where ψ is an arbitrary 1–form on Θ; its coordinate expression is given by

$$\Omega_Y = d\bar p_i \wedge d^i \wedge \hat\omega \simeq d\bar p_i \wedge d^i \wedge d\tau \,. \tag{10}$$

Let $J_1\Pi_\Theta$ be the first order jet manifold of the extended Legendre bundle $\Pi_\Theta \to X$. A connection γ on the extended Legendre bundle is then in one–to–one correspondence with a global section of the affine bundle $J_1\Pi_\Theta \to \Pi_\Theta$. Such a connection is said to be a *Hamiltonian connection* iff the exterior form $\gamma \rfloor \Omega_Y$ is closed.

A Hamiltonian $\mathcal{H}$ on Π_Θ is defined as a section $\bar p = -\mathcal{H}$ of the bundle κ. Let γ be a Hamiltonian connection on Π_Θ and U be an open subset of Π_Θ. Locally, we have $\gamma \rfloor \Omega_Y = (d\bar p_i \wedge d^i - d\mathcal{H}) \wedge \hat\omega \simeq (d\bar p_i \wedge d^i - d\mathcal{H}) \wedge d\tau \doteq dH$, where $\mathcal{H} : U \subset \Pi_\Theta \to V\Theta$ and $d = \partial_i d^i + \bar\partial^i d\bar p_i + \partial_\tau d\tau$ is the total differential on $V^\Theta \Pi_\Theta$.

The local mapping $\mathcal{H} : U \subset \Pi_\Theta \to V\Theta$ is called a *Hamiltonian.* The form H on the extended Legendre bundle Π_Θ is called a *Hamiltonian form.* Every Hamiltonian form H admits a Hamiltonian connection γ_H such that the following holds: $\gamma_H \rfloor \Omega_Y = dH$.

We define the *abstract covariant Hamilton equations* to be the kernel of the first order differential operator $\Delta_{\tilde\gamma_\Theta}$ defined as the vertical covariant differential (see Eq. 5) relative to the connection $\tilde\gamma_\Theta$ on the abstract Legendre bundle $\Pi_\Theta \to \Theta$.

In the following a 'dot' stands for $\pounds_{\partial_\tau}$, i.e. the Lie derivative along ∂_τ. In this case the Hamiltonian form H is the Poincaré–Cartan form of the *Lagrangian* $L_H = (\bar{p}_i \dot{y}^i - \mathcal{H})\, d\tau$ on $V^\Theta \Pi_\Theta$, with values in $V\Theta$. Furthermore, the *Hamilton operator* for H is defined as the Euler–Lagrange operator associated with L_H, namely $\mathcal{E}_H : V^\Theta \Pi_\Theta \to T^*\Pi_\Theta \wedge \Lambda^{n+1} T^* X$.

We state then the following.

Proposition 1. *The kernel of the Hamilton operator, i.e. the Euler–Lagrange equations for L_H, is an affine closed embedded subbundle of $V^\Theta \Pi_\Theta \to \Pi_\Theta$, locally given by the* covariant formal Hamilton equations *on the extended Legendre bundle $\Pi_\Theta \to X$*

$$\dot{y}^i = \bar{\partial}^i \mathcal{H}\,, \tag{11}$$

$$\dot{\bar{p}}_i = -\partial_i \mathcal{H}\,, \tag{12}$$

$$\dot{\mathcal{H}} = \partial_\tau \mathcal{H}\,. \tag{13}$$

These latter results could be compared with [13, Sec.4]. However, within the limits of the purpose of this note, in the following we just recall the relation with the standard polysymplectic approach (for a review of the topic see e.g. [4, 13, 15, 18, 20] and references quoted therein) and provide an example of application.

Lemma 1. *Let γ_H be a Hamiltonian connection on $\Pi_\Theta \to X$. Let $\tilde{\gamma}_\Theta$ and Γ be connections on $\Pi_\Theta \to Y$ and $\Theta \to X$, respectively. Let σ and h be sections of the bundles $\pi_{Y\Theta}$ and $\pi_{\Theta X}$, respectively.*

Then the standard Hamiltonian connection on $\Pi_\Theta \to X$ turns out to be the pull–back connection $\tilde{\gamma}_\phi$ induced on the subbundle $\Pi_{\Theta\,\phi} \hookrightarrow \Pi_\Theta \to X$ by the section $\phi = h \circ \sigma$ of $Y \to X$.

Proof. The abstract Legendre bundle is in fact a composite bundle $\Pi_\Theta \to Y \to \Theta$. Our claim then follows for any section ϕ of the composite bundle $Y \to \Theta \to X$ of the type $\phi = h \circ \sigma$, since the extended Legendre bundle $\Pi_\Theta \to X$ can be also seen as the composite bundle $\Pi_\Theta \to Y \to X$. □

As a straightforward consequence we can state the following [7]

Proposition 2. *Let $\Delta_{\tilde{\gamma},\phi}$ be the covariant differential on the subbundle $\Pi_{\Theta\,\phi} \hookrightarrow \Pi_\Theta \to X$ relative to the pull–back connection $\tilde{\gamma}_\phi$. The kernel of $\Delta_{\tilde{\gamma},\phi}$ coincides with the Hamilton–De Donder equations of the standard polysymplectic approach to field theories.*

Example 1 (**Formal gravitational energy).** Let us now specify the above formalism for an extended version of the Hilbert–Einstein Lagrangian, i.e. essentially the Hilbert–Einstein Lagrangian for a metric g parametrized by the line coordinate τ.

Let then $dim X = 4$ and X be orientable. Consider the configuration composite bundle $Lor(X)_\Theta \to \Theta \to X$ coordinated by $(g^{\mu\nu}, \tau, x^\lambda)$, where (τ, x^λ) are coordinates of the line bundle Θ and $g^{\mu\nu}$ are Lorentzian metrics on X (provided that they exist), i.e. sections of $Lor(X) \to X$. We call $Lor(X)_\Theta$ the bundle of Lorentzian metrics (on X) *parametrized by* τ. The bundle $Lor(X)_\Theta \to \Theta$ is not necessarily trivial; it is characterized as follows. Every section h of the line bundle Θ defines the restriction $h^* Lor(X)_\Theta$ of $Lor(X)_\Theta \to \Theta$ to $h(X) \subset \Theta$, which is a subbundle of $Lor(X)_\Theta \to X$. One can think of $h^* Lor(X)_\Theta \to X$ as being the bundle $Lor(X)$ of Lorentzian metrics on X with the background parameter function $h(x^\mu)$ (similar considerations can be found in parametrized Mechanics, see e.g. [22]). However in what follows we will not fix such a section.

The *extended* Hilbert–Einstein Lagrangian is the form $\lambda_{HE} = L_{HE}\,\hat{\omega}$, were $L_{HE} = r\sqrt{\underline{g}}$. Here $r : J_2^\Theta(Lor(X)_\Theta) \to I\!R$ is the function such that, for any parametrized Lorentz metric g, we have $r \circ j_2^\Theta g = s$, being s the scalar curvature associated with g, and $\underline{g}$ is the determinant of g.

Put $\pi^{\mu\nu} = \sqrt{\underline{g}} g^{\mu\nu}$ and $\phi_{\mu\nu}^{\hat{\rho}\hat{\lambda}} \doteq \partial L_{HE}/\partial \dot{\pi}^{\mu\nu}_{\hat{\rho}\hat{\lambda}}$.

Now, consider that

$$L_{HE} = \pi^{\mu\nu}\hat{R}_{\mu\gamma} = \bar{\phi}_{\mu\nu}\dot{\pi}^{\mu\nu} - \mathcal{H}\,, \tag{14}$$

where $\hat{R}_{\mu\gamma} = \hat{R}^\lambda_{\mu\lambda\gamma}$ denotes the components of the Ricci tensor of the Lie-dragged metric and $\bar{\phi}_{\mu\nu} \equiv \phi_{\mu\nu}^{\hat{\rho}\hat{\lambda}} \otimes \partial_{\hat{\rho}\hat{\lambda}}$. Hence the *formal Hamiltonian* turns out to be

$$\mathcal{H} = (-\pi^{\mu\gamma}\hat{R}_{\mu\gamma} + \bar{\phi}_{\mu\nu}\dot{\pi}^{\mu\nu})\,. \tag{15}$$

Notice that the formal Hamiltonian does not depend explicitly on τ. From the covariant Hamilton equations, in particular from Eq. (13), we have $\dot{\mathcal{H}} = 0$; thus the formal Hamiltonian turns out to be a conserved quantity. In fact we can interprete it as a conserved formal energy for the gravitational field (compare with [10] where an analogous approach is followed by defining the Cauchy data on a three-dimensional submanifold of space-time).

We stress that, as far as a section $h(x^\mu)$ of $\Theta \to X$ has not been fixed *a priori*, our approach provides an appropriate covariant Hamiltonian description of gravitation, which does not require neither a $(3+1)$ splitting of space-time - as it is done in the ADM-like formalisms - nor the fixing of a background connection - as it is done whitin the Palatini-like approaches. Both of the latter approaches we mentioned, in fact, can provide Hamiltonian descriptions of gravitation, which however loose the required genuine covariance.

We finally notice that this formal approach stresses the underlying algebraic structure of Field Theory, which was shown to be related with a new K–theory for vector bundles carrying the same kind of 'special' multisymplectic structure [29] (related multisymplectic 3-forms on manifolds have been also studied e.g. in [1, 23]).

References

[1] Bureš, J., Vanžura, J., *Unified treatment of multisymplectic 3-forms in dimension 6*, preprint arXiv:math.DG/0405101.

[2] Cabras, A., Kolář, I., *Connections on some functional bundles*, Czechoslovak Math. J. **45** (120) (1995), 529–548.

[3] Crnković, C., Witten, E., *Covariant description of canonical formalism in geometrical theories*; *Three hundred years of gravitation*, Cambridge Univ. Press, Cambridge (1987), 676–684.

[4] Dedecker, P., *On the generalization of symplectic geometry to multiple integrals in the calculus of variations*, Differential geometrical methods in mathematical physics (Proc. Sympos., Univ. Bonn, Bonn, 1975), Lecture Notes in Math. **570**, Springer, Berlin, 1977, 395–456.

[5] De León, M., Marrero, J. C., Martin de Diego, D., *A new geometric setting for classical field theories*, Classical and quantum integrability (Warsaw, 2001); Banach Center Publ. **59**, Polish Acad. Sci., Warsaw, (2003), 189–209.

[6] Echeverria-Enriquez, A., De León, M., Munoz-Lecanda, M. C., Roman-Roy, N., *Hamiltonian systems in multisymplectic field theories*, preprint arXiv:math-ph/0506003;

[7] Francaviglia, M., Palese, M., Winterroth, E., *A new geometric proposal for the Hamiltonian description of classical field theories*, Proc. VIII Int. Conf. Differential Geom. Appl., O. Kowalski et al. eds., 2001 Silesian University in Opava, 415–424; — *A general geometric setting for the energy of the gravitational field*, Inst. Phys. Conf. Ser. **176**, I. Ciufolini et al. eds., Taylor & Francis 2005, 391-395.

[8] Giachetta, G., Mangiarotti, L., Sardanashvili, G., *New Lagrangian and Hamiltonian Methods in Field Theory*, World Scientific, Singapore, 1997.

[9] Goldschmidt, H., Sternberg, S., *The Hamilton–Cartan formalism in the calculus of variations*, Ann. Inst. Fourier, Grenoble **23** (1) (1973), 203–267.

[10] Grabowska, K., Kijowski, J., *Canonical gravity and canonical energy*, Proc. VIII Int. Conf. Differential Geometry and Appl. (Opava, Czech Republic August 27–31, 2001) O. Kowalski, D. Krupka and J. Slovak eds., 2001 Silesian University in Opava, 261–274.

[11] Hélein, F., Kouneiher, J., *Covariant Hamiltonian formalism for the calculus of variations with several variables: Lepage-Dedecker versus De Donder-Weyl*, Adv. Theor. Math. Phys. **8** (3) (2004), 565–601.

[12] Hélein, F., Kouneiher, J., *The notion of observable in the covariant Hamiltonian formalism for the calculus of variations with several variables*, Adv. Theor. Math. Phys. **8** (4) (2004), 735–777.

[13] Kanatchikov, I. V., *Canonical structure of classical field theory in the polymomentum phase space*, Rep. Math. Phys. **41** (1) (1998), 49–90.

[14] Kanatchikov, I. V., *Precanonical quantum gravity: quantization without the space-time decomposition*, Internat. J. Theoret. Phys. **40** (6) (2001), 1121–1149.

[15] Kijowski, J., *A finite-dimensional canonical formalism in the classical field theory*, Comm. Math. Phys. **30** (1973), 99–128.

[16] Kijowski, J., *Multiphase spaces and gauge in the calculus of variations*, Bull. Acad. Polon. Sciences, Math. Astr. Phys. **XXII** (12) (1974), 1219–1225.

[17] Kijowski, J., Szczyrba, W., *A canonical structure for classical field theories*, Comm. Math. Phys. **46** (2) (1976), 183–206.

[18] Kijowski, J., Tulczyjew, W. M., *A symplectic framework for field theories*, Lecture Notes in Phys. **107**, Springer-Verlag, Berlin-New York, 1979.

[19] Kolář, I., *A geometrical version of the higher order Hamilton formalism in fibred manifolds*, J. Geom. Phys. **1** (2) (1984), 127–137.

[20] Krupková, O., *Hamiltonian field theory*, J. Geom. Phys. **43** (2002), 93–132.

[21] Mangiarotti, L., Sardanashvily, G., *Gauge Mechanics*, World Scientific, Singapore, 1998.

[22] Mangiarotti, L., Sardanashvily, G., *Connections in classical and quantum field theory*, World Scientific, Singapore, 2000.

[23] Panák, M., Vanžura, J., *3-forms and almost complex structures on 6-dimensional manifolds*, preprint arXiv:math.DG/0305312.

[24] Rey, A. M., Roman-Roy, N., Salgado, M., *Gunther's formalism* (*k-symplectic formalism*) *in classical field theory: Skinner-Rusk approach and the evolution operator*, J. Math. Phys. **46** (2005), 052901, 24pp.

[25] Rey, A. M., Roman-Roy, N., Salgado, M., *k-cosymplectic formalism in classical field theory: the Skinner–Rusk approach*, preprint arXiv:math-ph/0602038.

[26] Sardanashvily, G., *Generalized Hamiltonian formalism for field theory. Constraint systems*, World Scientific, Singapore, 1995.

[27] Sardanashvily, G., *Hamiltonian time–dependent mechanics*, J. Math. Phys. **39** (5) (1998), 2714–2729.

[28] Saunders, D. J., *The geometry of jet bundles*, Cambridge University Press, Cambridge, 1989.

[29] Winterroth, E., *A K–theory for certain multisymplectic vector bundles*, Proc. VIII Int. Conf. Differential Geometry and Appl., O. Kowalski et al. eds., 2001 Silesian University in Opava, 153–162.

Department of Mathematics, University of Torino
via C. Alberto 10, 10123 Torino, Italy
E-mail: marcella.palese@unito.it
ekkehart@dm.unito.it