On the non-$l_n^{(1)}$ and locally uniformly non-$l_n^{(1)}$ properties, and l^1 copies in Musielak—Orlicz spaces

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Abstract. It is proved that a Musielak—Orlicz space $L^\Phi(\mu)$ over a non-atomic measure is locally uniformly non-$l_n^{(1)}$ if and only if Φ satisfies the Δ_2-condition. Moreover, there are given some criteria in order that Musielak—Orlicz space be non-$l_n^{(1)}$ as well as in order that it contains an isometric copy of l^1. These results generalize the results of [1], [2], [4] and [8].

Keywords: Non-$l_n^{(1)}$ space, locally uniformly non-$l_n^{(1)}$ space, Musielak—Orlicz space, Luxemburg norm, Δ_2-condition

Classification: 46E30, 46B25

1. Introduction.

At the beginning, let us give some terminology and definitions concerning Musielak—Orlicz spaces and geometry of Banach spaces. In the whole paper, (T,Σ,μ) denotes a positive non-atomic measure space, N denotes the set of all natural numbers, R denotes the reals, R_+ denotes the non-negative reals, χ_A denotes the characteristic function of a set $A\in\Sigma$.

A function $\Phi : T\times R \to [0,+\infty]$ is said to be a Musielak—Orlicz function if $\Phi(t,\cdot)$ is even, convex, vanishing and continuous at zero, left continuous on the whole R_+ and not identically equal to the 0 function on R for μ-a.e. $t\in T$, and such that $\Phi(\cdot,u)$ is a Σ-measurable function for all $u\in R$.

A Musielak—Orlicz function Φ such that $\Phi(t_1,u)=\Phi(t_2,u)$ for all $t_1,t_2\in T$ and $u\in R$ is called an Orlicz function. For a given Musielak—Orlicz function Φ and a measure μ, the Musielak—Orlicz space $L^\Phi(\mu)$ is defined as the set of all equivalence classes of Σ-measurable functions x from T into R such that $I_\Phi(\lambda x) = \int_T \Phi(T,\lambda x)\,d\mu < +\infty$ for a certain $\lambda>0$ depending on x. If Φ is an Orlicz function, then $L^\Phi(\mu)$ is called an Orlicz space (see [11], [12], [13] and [14]). We denote by $E^\Phi(\mu)$ the subspace of $L^\Phi(\mu)$ defined as the set of all $x\in L^\Phi(\mu)$ such that $I_\Phi(\lambda x)<+\infty$ for every $\lambda>0$.

A Musielak—Orlicz space $L^\Phi(\mu)$ equipped with the Luxemburg norm

$$\|x\|_\Phi = \inf\{r>0 : I_\Phi \frac{x}{r} \leqslant 1\}$$

is a Banach space (see [11], [12] and [13]). For any Musielak—Orlicz function Φ, the function Φ^*, complementary to Φ in the sense of Young, is defined by the formula

$$\Phi^*(t,u) = \sup_{v\geqslant 0}\{|u|v - \Phi(t,v)\}$$

for all $t \in T$ and $u \in R$.

We say that a Musielak—Orlicz function Φ satisfies the Δ_2-condition if there exist a constant $k \geqslant 2$, a null set $T_0 \in \Sigma$ and a Σ-measurable non-negative function h with $\int_T \Phi(t, h(t))\, d\mu < +\infty$ such that $\Phi(t, 2\mu) \geqslant k\Phi(t, u)$ for any $t \in T \setminus T_0$ and $u \geqslant h(t)$ (see [9]).

Every Musielak—Orlicz function which satisfies the Δ_2-condition has finite values.

A normed space $(X, \|\ \ \|)$ is said to be locally uniformly non-$l_n^{(1)}$ $(n \in N, n \geqslant 2)$ if for every $x_1 \in X$ with $\|x_1\| = 1$ there exists $\delta(x_1)$ in the interval $(0,1)$ such that for all norm-one elements $x_2, \ldots, x_n$ in X, the inequality $\|x_1 \pm \cdots \pm x_n\| \leqslant n(1 - \delta(x_1))$ holds for a certain choice of signs ± 1 (see [16]).

A normed space $(X, \|\ \ \|0$ is called non-$l_n^{(1)}$ $(n \in N, n \geqslant 2)$ if for any norm-one element $x_1, \ldots, x_n$ in X, we have $\|x_1 \pm \cdots \pm x_n\| < n$ for a certain choice of signs ± 1 (see [3]).

Now, we shall give some lemmas which will be used in this paper.

Lemma 1. *(see [4]) The space l^∞ is not non-$l_n^{(1)}$.*

Lemma 2. *(see [8]) The space l^∞ contains an isometric copy of l^1.*

Theorem 3. *(see [3]) A normed space $(X, \|\ \ \|)$ is non-$l_n^{(1)}$ if and only if it does not contain any isometric copy of $l_n^{(1)}$.*

2. Results.

For a Musielak—Orlicz function Φ that has only finite values, define

$$g(t) = \sup\{u \in R_+ : \Phi(t, \cdot) \text{ is linear in the interval } [0, u]\};$$

obviously , g is a Σ-measurable function, and $g(t) = +\infty$ whenever $\Phi(t, \cdot)$ is linear on the whole R_+.

Theorem 4. *A Musielak—Orlicz space $L^\Phi(\mu)$ equipped with the Luxemburg norm is non-$l_n^{(1)}$ $(n \in N, n \geqslant 2)$ if and only if:*

a) *Φ satisfies the Δ_2-condition,*

b) *$\int_T \Phi(t, g(t))\, d\mu < n$.*

PROOF : Sufficiency. Let $\|x_1\|_\Phi = \cdots = \|x_n\|_\Phi = 1$. In virtue of the Δ_2-condition, we get $I_\Phi(x_1) = \cdots = I_\Phi(x_n) = 1$ (see [7]). Now, we shall prove that for all $u_1, \ldots, u_n \in R$ and μ-a.e. $t \in T$, we have

$$(*) \qquad \sum_{i=1}^{n} \Phi(t, u_i) > \Phi(t, g(t)) \text{ implies } \Phi\left(t, \frac{u_1 \pm \cdots \pm u_n)}{n}\right) < \frac{1}{n} \sum_{i=1}^{n} \Phi(t, u_i),$$

for a certain choice of signs ± 1. For this purpose we shall consider two cases:

I. $\max |u_i| > g(t)$. For the choice of signs ± 1 such that

$$|u_1 \pm \cdots \pm u_n| \leqslant \max_{1 \leqslant i \leqslant n} |u_i|,$$

we get

$$\Phi\left(t, \frac{u_1 \pm \cdots \pm u_n}{n}\right) \leqslant \Phi\left(t, \frac{\max|u_i|}{n}\right) < \left(\frac{1}{n}\Phi(t, \max|u_i|\right) =$$
$$= \frac{1}{n}\max\Phi(t, u_i) \leqslant \frac{1}{n}\sum_{i=1}^{n}\Phi(t, u_i).$$

II. $\max|u_i| \leqslant g(t)$. Then at least two numbers among $\Phi(t, u_i)$ $i = 1, \ldots, n$ are positive. In the opposite case, we have $\sum_{i=1}^{n}\Phi(t, u_i) = \Phi(t, u_k) \leqslant \Phi(t, g(t))$, where $1 \leqslant k \leqslant n$, which contradicts to the assumption in condition $(*)$. Thus, we get

$$\max\Phi(t, u_i)\Big/\sum_{i=1}^{n}\Phi(t, u_i) < 1.$$

For a certain choice of signs ± 1, we have $|u_1 \pm \cdots \pm u_n| \leqslant \max_{1 \leqslant i \leqslant n}|u_i|$. Therefore,

$$\Phi\left(t, \frac{u_1 \pm \cdots \pm u_n}{n}\right) \leqslant \Phi\left(t, \frac{\max|u_i|}{n}\right) = \frac{1}{n}\Phi(t, \max|u_i|) =$$
$$= \frac{1}{n}\max\Phi(t, u_i) < \frac{1}{n}\sum_{i=1}^{n}\Phi(t, u_i).$$

For this choice of signs ± 1, combining the cases I and II, we get $(*)$. Define

$$A = \left\{t \in T : \sum_{i=1}^{n}\Phi(t, x_i(t)) > \Phi(t, g(t))\right\}.$$

Then, in virtue of $(*)$, we have

$$\Phi\left(t, \frac{x_1(t) \pm \cdots \pm x_n(t)}{n}\right) < \frac{1}{n}\sum_{i=1}^{n}\Phi(t, x_i(t))$$

for all $t \in A$ and a certain choice of sign ± 1. Therefore,

$$\sum_{\pm 1}\Phi\left(t, \frac{x_1(t) \pm \cdots \pm x_n(t)}{n}\right) < \frac{2^{n-1}}{n}\sum_{i=1}^{n}\Phi(t, x_i(t)).$$

Integrating this inequality on both sides over A, we get

$$\sum_{\pm 1} I_\Phi\left(\frac{(x_1 \pm \cdots \pm x_n)\chi A}{n}\right) < \frac{2n-1}{n}\sum_{i=1}^{n} I_\Phi(x_i\chi_A).$$

Hence, we obtain

$$2^{n-1}-\sum_{\pm 1} I_\Phi\left(\frac{(x_1\pm\cdots\pm x_n)}{n}\right)=\frac{2^{n-1}}{n}\sum_{i=1}^n I_\Phi(x_i)-\sum_{\pm 1} I_\Phi\left(\frac{x_1\pm\cdots\pm x_n}{n}\right)\geqslant$$
$$\geqslant\frac{2^{n-1}}{n}\sum_{i=1}^n I_\Phi(x_i\chi_A)-\sum_{\pm 1} I_\Phi\left(\frac{(x_1\pm\cdots\pm x_n)\chi_A}{n}\right).$$

Hence, in virtue of the previous inequality, we get

$$\sum_{\pm 1} I_\Phi\left(\frac{x_1\pm\cdots\pm x_n}{n}\right)<2^{n-1}.$$

Then, for a certain choice of signs ± 1, we have

$$I_\Phi\left(\frac{x_1\pm\cdots\pm x_n}{n}\right)<1.$$

Thus, in virtue of the Δ_2-condition, it follows that

$$\|\frac{x_1\pm\cdots\pm x_n}{n}\|_\Phi<1$$

for a certain choice of signs ± 1. The proof of sufficiency is finished.

Necessity. If Φ does not satisfy the Δ_2-condition, then $L^\Phi(\mu)$ contains an isometric copy of l^∞ (see [5], [6]), so $L^\Phi(\mu)$ is not non- $l_n^{(1)}$ (see Lemma 1).

Now, assume that Φ satisfies the Δ_2-condition and the condition (b) does not hold, i.e. $\int_T \Phi(t,g(t))\,d\mu\geqslant n$. In virtue of the Δ_2-condition, $\Phi(t,\cdot)$ is continuous for μ-a.e. $t\in T$. If $g(t)<+\infty$ for μ-a.e. $t\in T$, then there are pairwise disjoint sets $A_1,A_2,\ldots,A_n\in\Sigma$ such that

$$\int_{A_1}\Phi(t,g(t))\,d\mu=\cdots=\int_{A_n}\Phi(t,g(t))\,d\mu=1$$

Define $x_i=g\chi_{A\,i}$ for $i=1,2,\ldots,n$. We have $I_\Phi(x_i)=1$, and

$$I_\Phi\left(\frac{x_1\pm\cdots\pm x_n}{n}\right)=\sum_{i=1}^n I_\Phi\left(\frac{x_i}{n}\right)=\frac{1}{n}\sum_{i=1}^n I_\Phi(x_i)=1$$

for any choice of signs ± 1. Thus, we have

$$\|\frac{x_1\pm\cdots\pm x_n}{n}\|_\Phi=1$$

for any choice of signs ± 1. It means that $L^\Phi(\mu)$ is not non-$l_n^{(1)}$.

If $g(t)=+\infty$ for $t\in A$, where $A\in\Sigma$ and $\mu(A)>0$, then $\Phi(t,u)=P(t)|u|$ for every $t\in A$ and $u\in R_+$, where P is a Σ-measurable function positive on A. Define on $\Sigma\cap A$ a new non-atomic measure ν by

$$\nu(B)=\int_B P(t0\,d\mu\qquad(\forall B\in\Sigma\cap A).$$

Then $L^\Phi(\mu,A)=L^1(\nu,A)$, and therefore $L^\Phi(\mu)$ is not non-$l_n^{(1)}$ (see [4]). The proof is finished. ■

Now, we shall give a criterion in order that a Musielak—Orlicz space $L^\Phi(\mu)$ contains an isometric copy of l^1.

Theorem 5. *A Musielak—Orlicz space $L^\Phi(\mu)$ equipped with the Luxemburg norm contains an isometric copy of l^1 if and only if:*

c) *Φ does not satisfy the Δ_2-condition, or*

d) *$I_\Phi(g) = +\infty$, where g is the function defined before Theorem 4.*

PROOF : Sufficiency. If Φ does not satisfy the Δ_2-condition, then $L^\Phi(\mu)$ contains an isometric copy of l^∞ (see [5], [6]) and, in view of Lemma 2, it contains an isometric copy of l^1. Now, assume that Φ satisfies condition (d) and ($g(t) < +\infty$ for μ-a.e. $t \in T$. We can assume that Φ satisfies the Δ_2-condition. The measure ν_μ defined on Σ by the formula

$$\nu_\mu(A) = I_\Phi(g\chi_A)$$

is non-atomic and infinite.

Therefore, there exists a sequence $(A_k)_{k=1}^\infty$ of pairwise disjoint sets in Σ such that $I_\Phi(g\chi_{A_k}) = 1$ for every $k \in N$.Denote $a_k = g\chi_{A_k}$ and define an operator P from l^1 into $L^\Phi(\mu)$ by

$$Py = \sum_{k=1}^\infty c_k a_k \qquad (\forall y = (c_k) \in l^1).$$

P is linear and it is easily seen that $Py \in E^\Phi(\mu)$ for any $y \in l^1$. In fact, taking into account that $\Phi(t,\cdot)$ is linear on the interval $[0, g(t)]$, we get $\Phi(t, \alpha g(t) = |\alpha|\Phi(t, g(t))$ for every $|\alpha| \leqslant 1$. Given $\lambda > 0$, choose $n_0 \in N$ in such a manner that $\lambda|c_k| \leqslant 1$ for $n \geqslant n_0$. We have

$$\begin{aligned} I_\Phi(\lambda Py) &= \sum_{k=1}^{n_0-1} \int_{A_k} \Phi(t, \lambda c_k a_k(t))\, d\mu + \sum_{k=n_0}^{\infty} \int_{A_k} \Phi(t, \lambda c_k a_k(t))\, d\mu \\ &= \sum_{k=1}^{n_0-1} \int_{A_k} \Phi(t, \lambda c_k a_k(t))\, d\mu + \sum_{k=n_0}^{\infty} \lambda|c_k| \int_{A_k} \Phi(t, a_k(t))\, d\mu \\ &= \sum_{k=1}^{n_0-1} \int_{A_k} \Phi(t, \lambda c_k a_k(t))\, d\mu + \lambda \sum_{k=n_0}^{\infty} |c_k| < +\infty. \end{aligned}$$

Now, we shall prove that P is an isometry. We have

$$\begin{aligned} I_\Phi\left(\frac{Py}{\|y\|_{l^1}}\right) &= \int_T \Phi\left(t, \frac{Py}{\|y\|_{l^1}}\right) d\mu = \sum_{k=1}^\infty \int_{A_k} \Phi\left(t, \frac{c_k a_k^{(t)}}{\|y\|_{l^1}}\right) d\mu \\ &= \sum_{k=1}^\infty \frac{|c_k|}{\|y\|_{l^1}} \int_{A_k} \Phi(t, a_k)\, d\mu = \sum_{k=1}^\infty \frac{|c_k|}{\|y\|_{l^1}} = 1 \end{aligned}$$

Hence,

$$\left\|\frac{Py}{\|y\|_{l^1}}\right\|_\Phi = 1, \text{i.e. } \|Py\|_\Phi = \|y\|_{l^1}.$$

Assume now that $g(t) = +\infty$ for $t \in A$, where $A \in \Sigma$ and $\mu(A) > 0$. Then $L^{\Phi}(\mu, A) = L^1(\nu, A)$, where ν is defined as in the proof of Theorem 4. Since $L^1(\nu, A)$ contains an isometric copy of l^1 (see [8]), $L^{\Phi}(\mu, A)$ contains an isometric copy of l^1.

Necessity. Assume that none of the conditions (c) and (d) is satisfied. This means that Φ satisfies the Δ_2-condition and $\int_T \Phi(t, g(t))\, d\mu < +\infty$. Therefore, there is $k \in N, k \geqslant 2$, such that $\int_T \Phi(t, g(t))\, d\mu \leqslant k$. In view of Theorem 4, $L^{\Phi}(\mu)$ is non-$l_n^{(1)}$ for all $n > k, n \in N$. In virtue of Theorem 3, $L^{\Phi}(\mu)$ contains no isometric copy of l^1. The proof is finished. ■

Theorem 6. *Let Φ be a Musielak—Orlicz function such that $\Phi(t, \cdot)$ is linear in no neighbourhood of 0 in R_+ for μ-a.e. $t \in T$. Then the Musielak—Orlicz space $L^{\Phi}(\mu)$ equipped with the Luxemburg norm is locally uniformly non-$l_n^{(1)}$ if and only if Φ satisfies the Δ_2-condition.*

PROOF : Sufficiency. Let $\|x_1\|_{\Phi} = \cdots = \|x_n\|_{\Phi} = 1$. Then, in virtue of the Δ_2-condition, we have $I_{\Phi}(x_1) = \cdots = I_{\Phi}(x_n) = 1$ (see [7]). Let $c > 0$ be such that the set

$$A_1 = \{t \in T : \quad c^{-1} \leqslant \Phi(t, x_1(t)) \leqslant c\}$$

satisfies the condition $I_{\Phi}(x_1 \lambda_{A_1}) \geqslant \frac{7}{8}$. Let $m > 0$ be such that $\frac{c}{m} \leqslant \frac{1}{8(n-1)}$, and define

$$A_i = \{t \in T : \quad \Phi(t, x_i(t)) \leqslant m\} \quad \text{for } i = 2, \ldots, n$$

we have

$$m\mu(T \setminus A_i) < I_{\Phi}(x_i \lambda_{T \setminus A_i}) \leqslant 1.$$

Thus,

$$\mu(T \setminus A_i) \leqslant \frac{1}{m} \quad \text{for } i = 2, \ldots, n.$$

Hence, we get

$$I_{\Phi}(x_1 \lambda_{A_1 \setminus A_i}) \leqslant c\mu(A_1 \setminus A_i) \leqslant \frac{c}{m} \leqslant \frac{1}{8(n-1)}.$$

Denoting $D = \bigcap_{i=2}^n A_i$, we have

$$\begin{aligned} \frac{7}{8} \leqslant I_{\Phi}(x_1 \lambda_{A_1}) &= I_{\Phi}(x_1 \lambda_{A_1 \setminus D}) + I_{\Phi}(x_1 \lambda_D) \\ &= I_{\Phi}(x_1 \lambda_{\bigcup_{i=2}^n (A_1 \setminus A_i)}) + I_{\Phi}(x_1 \lambda_D) \\ &\leqslant \frac{1}{8(n-1)}(n-1) + I_{\Phi}(x_1 \lambda_D), \end{aligned}$$

whence $I_{\Phi}(x_1 \lambda_D) \geqslant \frac{3}{4}$. Define

$$P(t) = \sup\left\{ \frac{n\Phi(t, \frac{u}{n})}{\Phi(t, u)} : \Phi(t, u) \in [c^{-1}, m] \right\}.$$

In virtue of the assumption that $\Phi(t,\cdot)$ is linear in no neighbourhood of 0 in R_+, we get $0 < P(t) < 1$ for μ-a.e. $t \in D$. Hence, we have $\Phi(t, \frac{u}{n}) \leqslant \frac{P(t)}{n}\Phi(t,u)$ for μ-a.e. $t \in D$, and all u satisfying $\Phi(t,u) \in [c^{-1}, m]$. Define

$$B_k = \left\{t \in D : P(t) \leqslant 1 - \frac{1}{k}\right\}.$$

By Σ-measurability of P, it follows that $B_k \in \Sigma$ for $k = 1,2,\ldots,$. There is $l \in N$ such that $I_\Phi(x_l\chi_{B_l}) \geqslant \frac{1}{2}$. Denote $\sigma = 1 - \frac{1}{l}$ and $B = B_l$. Now, we shall prove that for every $t \in B$, we have

$$(**)\qquad \sum_{\pm 1}\Phi\left(t, \frac{x_1(t) \pm \cdots \pm x_n(t)}{n}\right) \leqslant \frac{2^{n-1} - 1 + \sigma}{n}\sum_{i=1}^{n}\Phi(t, x_i(t)).$$

For at least one choice of signs ± 1, such that $|x_1(t) \pm \cdots \pm x_n(t)| \leqslant \max_{1\leqslant i\leqslant n}|x_i(t)|$, we have

$$(1)\quad \Phi\left(t, \frac{x_1(t) \pm \cdots \pm x_n(t)}{n}\right) \leqslant \Phi\left(\frac{\max|x_i(t)|}{n}\right) \leqslant$$
$$\frac{\sigma}{n}\Phi(t, \max|x_i(t)|) = \frac{\sigma}{n}\max\Phi(t, x_i(t)) \leqslant \frac{\sigma}{n}\sum_{i=1}^{n}\Phi(t, x_i, (t)),$$

for every $t \in B$. For the remaining $2^{n-1} - 1$ choice of signs ± 1,by the convexity of Φ, we have

$$(2)\qquad \Phi\left(t, \frac{x_1(t) \pm \cdots \pm x_n(t)}{n}\right) \leqslant \frac{1}{n}\sum_{i=1}^{n}\Phi(t, x_i(t)),\ \text{for every } t \in B.$$

Combining (1) and (2), we get $(**)$. Integrating the inequality $(**)$ both-sides over B, we get

$$\sum_{\pm 1} I_\Phi\left(\frac{(x_1 \pm \cdots \pm x_n)\chi_B}{n}\right) \leqslant \frac{2^{n-1} - 1 + \sigma}{n}\sum_{i=1}^{n} I_\Phi(x_i\chi_B).$$

Hence, we obtain

$$\begin{aligned}
2^{n-1} - &\sum_{\pm 1} I_\Phi\left(\frac{x_1 \pm \cdots \pm x_n}{n}\right) \\
&= \frac{2^{n-1}}{n}\sum_{i=1}^{n} I_\Phi(x_i) - \sum_{\pm 1} I_\Phi\left(\frac{x_1 \pm \cdots \pm x_n}{n}\right) \\
&\geqslant \frac{2^{n-1}}{n}\sum_{i=1}^{n} I_\Phi(x_i\chi_B) - \sum_{\pm 1} I_\Phi\left(\frac{(x_1 \pm \cdots \pm x_n)\chi_B}{n}\right) \\
&\geqslant \frac{2^{n-1}}{n}\sum_{i=1}^{n} I_\Phi(x_i\chi_B) - \frac{2^{n-1} - 1 + \sigma}{n}\sum_{i=1}^{n} I_\Phi(x_i\chi_B) \\
&= \frac{1-\sigma}{n}\sum_{i=1}^{n} I_\Phi(x_i\chi_B) \geqslant \frac{1-\sigma}{n} I_\Phi(x_i\chi_B) \\
&\geqslant \frac{1-\sigma}{2n} = \eta.
\end{aligned}$$

Thus, we have

$$\sum_{\pm 1} I_\Phi\left(\frac{(x_1 \pm \cdots \pm x_n)}{n}\right) \leqslant 2^{n-1} - \eta = 2^{n-1}(1-q),$$

where $q = \eta/2^{n-1}$ and it depends only on x_1. Therefore, for a certain choice of signs ± 1, we get

$$I_\Phi\left(\frac{x_1 \pm \cdots \pm x_n}{n}\right) \leqslant 1 - q.$$

In virtue of the Δ_2-condition, we have

$$\left\|\frac{x_1 \pm \cdots \pm x_n}{n}\right\|_\Phi \leqslant 1 - \beta(q)$$

for a certain choice of signs ± 1, where β is a function from $(0,1)$ into $(0,1)$ such that $\|x\| \leqslant 1 - \beta(q)$, whenever $I_\Phi(x) \leqslant 1 - q$ (see [**1**]).

Necessity. If Φ does not satisfy the Δ_2-condition, then $L^\Phi(\mu)$ contains an isometric copy of l^∞ (see [**5**], [**6**]). Therefore, in view of Lemma 1, $L^\Phi(\mu)$ is not locally uniformly non-$l_n^{(1)}$. The proof of the Theorem 6 is finished. ∎

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