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NON-CONSTANT CONTINUOUS MAPS OF MODIFICATIONS OF TOPOLOGICAL SPACES

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Dedicated to Professor Miroslav Katětov on his seventieth birthday

Abstract: For every pair of monoids $M_1 \subseteq M_2$ there exists a regular T_1-space X such that all the non-constant continuous endomaps of X form a monoid isomorphic to M_1 and of its completely regular modification form a monoid isomorphic to M_2. An analogous statement is true also for compactly generated modification and sequential modification. A more general setting of simultaneous representations of small categories is investigated and stronger and more complex results are presented.

Key words: Representations, modifications.

Classification: 54H10, 54B30, 18B30

I. Introduction. In [G], J. de Groot proved that every group can be represented as the group of all homeomorphisms of a (suitable) topological space onto itself. He put the question (at the conference in Tihany in 1964, see [He]), whether every monoid (=semigroup with a unit) can be represented by all the non-constant continuous maps of a topological space into itself (i.e. whether for every monoid M there exists a topological space X such that for every non-constant continuous $f,g:X \longrightarrow X$, $g \circ f$ is non-constant again and the monoid of all these maps is isomorphic to M). This was solved in [T_1], where a metrizable space X representing a given monoid M in the above sense was constructed.

In [KR], V. Kannan and M. Rajagopalan proved that for every pair of groups $G \subseteq H$, there exists a metric space X such that all the isometries of X form a group isomorphic to G and all the homeomorphisms of X onto itself a group isomorphic to H. All the above results are strengthened in [T_3], where the following statement is proved: for every triple of monoids $M_1 \subseteq M_2 \subseteq M_3$

there exists a complete metric space X such that all the non-constant maps of X into itself which are

continuous, form a monoid $\simeq M_3$,

unif. continuous, form a monoid $\simeq M_2$,

non-expanding, form a monoid $\simeq M_1$.

Another result of this kind is presented in [T_4]: for every pair of monoids $A \subseteq B$ there exists a Tichonov space X such that all the non-constant continuous maps of X into itself form a monoid isomorphic to A and all the non-constant continuous maps of βX into itself form a monoid isomorphic to B. (However, in general it is <u>not true</u> that for every quadruple of monoids $M_1 \subseteq M_2 \subseteq$ $\subseteq M_3 = A \subseteq B$ there exists a metric space X such that both these statements are valid for X.)

The method developed in [T_3] can be used (after suitable modifications which unfortunately make it more involved) also for simultaneous representation of a pair of monoids $M_1 \subseteq M_2$ by a topological space and by its (suitable) modification. Three of these results are mentioned in the Abstract. Some further results are mentioned in the part III of this paper. A more general setting of almost full embeddings of categories, which is investigated in the present paper, admits also to obtain results of another kind than the mere representing of pairs of monoids. For example, for every cardinal number α there exists a stiff set $\mathfrak{X}$ of paracompact spaces (<u>stiff</u> in the sense that if $X, Y \in \mathfrak{X}$ and $f: X \longrightarrow Y$ is a continuous map, then either f is constant or X=Y and f is the identity) such that all the spaces from $\mathfrak{X}$ have the same compactly generated modification (and the obtained k-space is rigid).

Our notation is a standard one. If $\mathcal{K}$ is a category, then obj $\mathcal{K}$ denotes the class of all its objects and, for $a, b \in$ obj $\mathcal{K}$, $\mathcal{K}(a,b)$ denotes the set of all morphisms of $\mathcal{K}$ with the domain a and codomain b. For $a \xrightarrow{\alpha} b \xrightarrow{\beta} c$, the composition is written $\beta \circ \alpha$. The category of all sets and maps is denoted by Set. A "concrete category" means always concrete over Set. If $\mathcal{K}$ is a concrete category and $a \in$ obj $\mathcal{K}$, then $|a|$ denotes the underlying set of a (but speaking about a topological space, we often do not distinguish between the space and its underlying set if there is no danger of confusion). If $\mathcal{K}$ is a concrete category, then $a \leq b$ for $a, b \in$ obj $\mathcal{K}$ means that the identity map of $|a| = |b|$ in Set carries a morphism from $\mathcal{K}(a,b)$ (and we say that a is finer than b or b coarser than a). For concrete functors $F, G: \mathcal{K} \longrightarrow \mathcal{K}$, we denote by $F \leq G$ the fact that $Fa \leq Ga$ for all $a \in$ obj $\mathcal{K}$. In a topological space X, the closure of a set A is denoted by $\overline{A}^X$ or only by $\overline{A}$ if there is no danger of confusion.

II. Preliminaries and some negative results

II.1. Let us denote by Top the category of all topological spaces and all their continuous maps. Let us recall that a functor

$$\Phi : \mathcal{K} \longrightarrow \mathrm{Top}$$

is called an almost full embedding (see e.g. [PT]) if it is one-to-one and for every pair of objects a, b of $\mathcal{K}$

a continuous map $f: \Phi(a) \longrightarrow \Phi(b)$ is non-constant,

iff $f = \Phi(g)$ for a (unique!) $\mathcal{K}$-morphism $g: a \longrightarrow b$.

If $\mathcal{K}$ has precisely one object, say a, then the existence of an almost full embedding Φ of $\mathcal{K}$ into Top is precisely the representability of the endomorphism monoid $M = \mathcal{K}(a,a)$ as the monoid of all non-constant continuous maps of the topological space $X = \Phi(a)$ into itself. If $\mathcal{K}$ is a discrete category (i.e. it has no morphisms except the unities), then $\{\Phi(\sigma) \mid \sigma \in \mathrm{obj}\, \mathcal{K}\}$ is a stiff class of topological spaces. In $[T_1]$, it was proved that every small category can be almost fully embedded into the category of all metrizable spaces.

II.2. In what follows, we investigate the situation, when a functor

$$m: \mathrm{Top} \longrightarrow \mathrm{Top}$$

is given such that

(i) m is idempotent (i.e. $m \circ m = m$) and

(ii) m preserves the underlying sets and maps, i.e. the following diagram commutes,

$$\begin{array}{ccc} \mathrm{Top} & \xrightarrow{m} & \mathrm{Top} \\ & \searrow \quad \swarrow & \\ & \mathrm{Set} & \end{array}$$

where the unnamed arrows denote the forgetful functor. Let us call any such functor m shortly a modification.

Given a modification m, we ask, for which small categories k_1, k_2 and for which functors $\Psi: k_1 \longrightarrow k_2$ there exist almost full embeddings $\Phi_1: k_1 \longrightarrow$ $\longrightarrow \mathrm{Top}$ and $\Phi_2: k_2 \longrightarrow \mathrm{Top}$ such that the square

$$\begin{array}{ccc} k_1 & \xrightarrow{\Psi} & k_2 \\ \Phi_1 \downarrow & & \downarrow \Phi_2 \\ \mathrm{Top} & \xrightarrow{m} & \mathrm{Top} \end{array}$$

commutes (i.e. $\Phi_2 \circ \Psi = m \circ \Phi_1$). In such a case, we say that Ψ has a simultaneous representation by m.

If Ψ has a simultaneous representation by m, then Ψ must be faithful. In fact, Φ_1 and m are faithful so that $m \circ \Phi_1 = \Phi_2 \circ \Psi$ is also faithful, hence Ψ must be faithful. We prove that for some modifications used in topology, the faithfulness of Ψ is also sufficient. Let us say that a modification $m:\mathrm{Top} \longrightarrow \mathrm{Top}$ is <u>comprehensive</u> if <u>every</u> faithful Ψ has a simultaneous representation by m.

Clearly, if m has to be comprehensive, then its image m(Top) must be big enough, every small category k_2 has to be almost fully embedded into m(Top). This eliminates e.g. the discrete or the indiscrete modifications. On the other hand, if m(Top) is too big, then m fails to be comprehensive again - the trivial example is the identity functor $m:\mathrm{Top} \longrightarrow \mathrm{Top}$, then only full embeddings $\Psi:k_1 \longrightarrow k_2$ have simultaneous representations by m. Below, we discuss less trivial examples of modifications which fail to be comprehensive.

II.3. First, let us recall that a modification m is called an <u>upper modification</u> (or a <u>lower modification</u>) if $mX \geq X$ (or $mX \leq X$) for all topological spaces X (in a little wider sense, these terms are used in [Č]). In Top, upper modifications coincide with the bireflections, lower modifications with all the coreflections distinct from the functor sending every space to the void space (i.e. the concrete coreflections), the corresponding bireflective (or coreflective) subcategory of Top is determined by the class $\{X \mid mX=X\}$. Let us also recall that every class $\mathscr{C}$ of topological spaces determines

<u>a lower modification</u> $m_{\mathscr{C}}$

by the rule that for every space X,

$\mathcal{O}$ is open in $m_{\mathscr{C}}X$ iff $f^{-1}(\mathcal{O})$ is open in Y for all $Y \in \mathscr{C}$ and all continuous $f:Y \longrightarrow X$

and <u>an upper modification</u> $m^{\mathscr{C}}$

by the rule that for every space X,

$\{f^{-1}(\mathcal{O}) \mid Y \in \mathscr{C},\ \mathcal{O}$ is open in Y, $f:X \longrightarrow Y$ is continuous$\}$

forms a subbasis of open sets in $m^{\mathscr{C}}X$.

(Thus, the discrete and the indiscrete modifications are $m_{\mathscr{T}}$ and $m^{\mathscr{T}}$ where $\mathscr{T}$ consists of a one-point space.) The class $b\mathscr{C} = \{X \mid m^{\mathscr{C}}X=X\}$ (or $c\mathscr{C} = \{X \mid m_{\mathscr{C}}X=X\}$) determines the bireflective hull (or the coreflective hull) of the class $\mathscr{C}$ and the modification $m^{\mathscr{C}} = m^{b\mathscr{C}}$ (or $m_{\mathscr{C}} = m_{c\mathscr{C}}$) is the corresponding bireflection (or coreflection).

II.4. Let us discuss the comprehension of the upper modifications m from the point of view of the size of m(Top). The indiscrete modification has the smallest possible image, it is the last modification in the order of concrete functors Top $\longrightarrow$ Top. The next upper modification in Top smaller than the indiscrete one in the zerodimensional modification z. The class z(Top) is still too small, no zerodimensional space X with more than one point has the property that every non-constant continuous map of X into itself is already a homeomorphism of X onto itself, hence no non-trivial group has a representation by all the non-constant continuous maps of a space in z(Top). The next smaller upper modifications are generated by continua. As it follows from our Main Theorem, some of <u>these</u> modifications are already comprehensive.

Now, we try to approximate the comprehension of upper modifications from the opposite side - when m(Top) is big (equivalently, if the functor m is close to 1_{Top}). As already mentioned, m cannot be the identity, but it cannot be either the nearest upper modification, namely the symmetric modification (X is said to be symmetric if $x \in \overline{y}$ implies $y \in \overline{x}$). We shall show that in a more general context. The class of symmetric spaces is the bireflective hull of all T_1-spaces, and the class of T_1-spaces is an extremal epireflective subcategory of Top (extremal means that the reflective maps are quotient, i.e., if X belongs to the subcategory then any finer space belongs to it, too).

Proposition 1. If $\mathcal{K}$ is the bireflective hull in Top of an extremal epireflective subcategory $\mathcal{L}$ of Top, then the reflection m onto $\mathcal{K}$ is not comprehensive.

<u>Proof.</u> Take for k_1 the trivial category with a unique morphism and for k_2 a category with a unique object a and such that $k_2(a,a)$ is an infinite group (the functor Ψ is the unique possible). Suppose that a simultaneous representation (Φ_1, Φ_2) exist, then $\Phi_2 a$ must be aT_1-space, $\Phi_2 a \in \mathcal{K}$, hence $\Phi_2 a \in \mathcal{L}$ and consequently, $\Phi_1 a \in \mathcal{L}$ (since $\Phi_1 a \leq \Phi_2 a$), which is impossible.

The last Proposition applies e.g. to the bireflective hull of Hausdorff spaces, Uryson spaces (every two points have disjoint closed neighborhoods), functionally separated spaces (the completely regular modification is Hausdorff), totally disconnected spaces, hereditarily disconnected spaces; the last two examples can be also treated similarly as zerodimensional spaces. So, these subcategories are too big as targets of upper modifications m which can be used for simultaneous representations.

II.5. In the case of lower modifications, m cannot be the least one, neither the last but one (the corresponding subcategory consists of sums of indiscrete spaces). That is very easy to show. We may prove an assertion similar to Proposition 1.

Proposition 2. If $\mathcal{C}$ is a class in Top such that $Y \in \mathcal{C}$ provided Y is coarser than a connected $X \in \mathcal{C}$, then $m_{\mathcal{C}}$ is not comprehensive.

Proof. If Υ, k_1, k_2 are the same as in the proof of Proposition 1, then $\Phi_2 a$ must be connected, belongs to $\mathcal{C}$, thus $\Phi_1 a$ belongs to $\mathcal{C}$ as well, hence $\Phi_1 a = \Phi_2 a$.

Thus every comprehensive lower modification m is finer than a non-trivial lower modification m′ which is not comprehensive (take the coreflective hull of m(Top)$\cup$\{connected spaces\}).

II.6. Thus, we have seen that there are bounds for the comprehension of modifications, bounds both from "above" and "below". We do not know conditions necessary and sufficient for the comprehension of modifications. However, the construction presented in this paper is rather general and gives the proof of the comprehension of some current modifications.

III. The Main Theorem and its applications

III.1. Like above, the properties needed for our construction are of two kinds: one group of properties says that the image m(Top) cannot be too small, the other says that m(Top) cannot be too big.

Definition. We say that a modification m:Top $\longrightarrow$ Top is stabilized by complete metrizability if, for every space X, the following statements are fulfilled:

a) if $A \subseteq X$ is a C^*-embedded regularly closed completely metrizable subspace of X, then the topologies of X and of mX coincide on A.

b) If $A \subseteq X$ is a C^*-embedded zero set, $B \subseteq X$ is cozero set containing A and such that $\overline{B} \setminus \mathring{A}$ is completely metrizable, then mA (or mB) is a closed (or open, resp.) subspace of mX and the topologies of X and of mX coincide on $B \setminus A$.

III.2. The conditions from the preceding definition are needed in our construction (in fact, a little less is needed - see the construction in IV.2, 3, 4). By taking A=X in (a) we get that every completely metrizable space is a fixed object of every modification m stabilized by complete metrizability. If m is an upper modification, it follows that m is finer than the completely regular (=uniformizable) modification u (i.e. $u=m^{\mathcal{C}}$ where $\mathcal{C}$ is the class of all completely regular spaces, or, equivalently, $\mathcal{C}$ consists of an arc). And conversely, if an upper modification m is finer than u, then m is "almost" stabilized by complete metrizability. Indeed, the condition (a) is trivially satisfied and in (b), one can see easily that mA is a closed set and mB an open set in mX and that the topologies of X and mX coincide on $B \setminus A$ (use the fact that any $x \in B \setminus A$ has a neighborhood $U \subseteq B \setminus A$ in X which is determined by a continuous function being zero outside $B \setminus A$). So, for a given upper modification m finer than the completely regular one, it suffices to show that both mA, mB are subspaces of mX.

III.3. If m is a lower modification stabilized by complete metrizability, then m must be coarser than the sequential modification s (i.e. $s = m_{\mathcal{S}}$, where $\mathcal{S}$ consists of all finite spaces and all convergent sequences). And again, if a lower modification is coarser than s, then m is "almost" stabilized by complete metrizability: (a) is trivially satisfied and similarly all the conditions of (b) except for those conditions that mA, mB are subspaces of mX.

III.4. If m is an arbitrary modification stabilized by complete metrizability, then m must be coarser than the sequential modification s and finer than the completely regular modification u (since uX [or sX] is the coarsest [or the finest] space rendering all the continuous maps on X into a metrizable space M [or on a metrizable space M into X] continuous as a mapping $uX \longrightarrow M$ [or $M \longrightarrow sX$, resp.]). It follows from the two preceding paragraphs that for such a modification m to be stabilized by complete metrizability, it suffices and is necessary that mA, mB are subspaces of mX in the condition (b).

III.5. The conditions describing the stabilization by complete metrizability ensure that m(Top) is not too small (since m(Top) contains all metrizable spaces; Hence, by [T_1], any small category can be almost fully embedded in it). Now we add the condition ensuring that m(Top) is not too big:

Definition. We say that a modification $m:\mathrm{Top} \longrightarrow \mathrm{Top}$ is <u>essentially non-identical</u> if there exists a Hausdorff space X such that mX is Hausdorff, either $X \supsetneqq mX$ or $mX \subsetneqq X$ and neither X nor mX contains a metrizable continuum. Any such space is called a <u>distinguishing space</u> of m.

III.6. Every non-identical lower modification m is essentially non-identical. That assertion follows from the fact that Top is the coreflective hull of T_1-spaces with a unique accumulation point (such spaces are zerodimensional); if $m \lneqq 1_{\mathrm{Top}}$ then $mX \supsetneqq X$ for some of those spaces X and, hence, this space X is a distinguishing space for m.

For upper modifications m, the situation is more delicate. It is easy to show that if a totally disconnected space does not belong to m(Top) then m is essentially non-identical (such a situation occurs if m(Top) is contained in the class of regular spaces). This situation can be given a more general setting. In the case that m(Top) contains with any Hausdorff space X not containing metrizable continua all finer spaces than X, one must proceed individually for every such m .

III.7. **Main Theorem.** Every essentially non-identical modification m which is stabilized by the complete metrizability is comprehensive and all the representing spaces (i.e. all the spaces $\Phi_i(\sigma)$, $\sigma \in$ obj k_i, i=1,2, in the notation of II.2) can be always chosen to be Hausdorff spaces. If, moreover, there is a distinguishing space X of m such that both X and mX are regular (or completely regular or normal or paracompact), then all the representing spaces can be chosen with the same property.

III.8. **Remark.** The choice of the categories k_1, k_2 and Ψ in the Main Theorem is rather free. If we choose k_1 and k_2 with precisely one object, say a, $M_1=k_1(a,a)$, $M_2=k_2(a,a)$ are their endomorphism monoids, a faithful functor $\Psi:k_1 \longrightarrow k_2$ is precisely an embedding of M_1 into M_2, we obtain a representation of the pair of monoids $M_1 \subseteq M_2$ by $\Phi_1(a)$ and its modification m $\Phi_1(a)$. If we choose k_2 with precisely one object a and one morphism 1_a and k_1 is a discrete category with card obj $k_1=\alpha$, where α is a prescribed cardinal number, and Ψ sends all the objects of k_1 to the unique object of k_2, then $\mathcal{X} = \{\Phi_1(\sigma) \mid \sigma \in \text{obj } k_1\}$ is a stiff set of spaces and m $\Phi_1(\sigma)= \Phi_2(a)$, hence all the spaces $X \in \mathcal{X}$ have the same modification mX (which is a rigid space because $k_2(a,a)=\{1_a\}$).

III.9. Let us present some examples of modifications m:Top $\longrightarrow$Top which fulfil the presumption of the Main Theorem, so that the Main Theorem can be applied on them.

a) Completely regular modification. As follows easily from III.2, the completely regular modification u is stabilized by the complete metrizability. The classical example of a regular T_1-space which is not completely regular [E], is a distinguishing space of u such that both X and uX are regular T_1. Hence every faithful $\Psi:k_1 \longrightarrow k_2$ has a simultaneous representation by u such that all the representing spaces are regular T_1-spaces.

b) Sequential modification. The sequential modification $s=m_\varphi$ (see

III.3) is stabilized by the complete metrizability. (In fact, sA is a closed subspace of sX for every closed $A \subseteq X$; sB is an open subspace of sX for every cozero set $B \subseteq X$; the proof of (a) and of the other requirements in (b) in III.1 is trivial, see III.3). It has a distinguishing space X such that both X and sX are paracompact, see III.6. Hence every faithful $\mathcal{K}: k_1 \longrightarrow k_2$ has a simultaneous representation by s such that all the representing spaces are paracompact.

c) Further lower modifications. If $\mathcal{C}$ is a class of topological spaces which is closed with respect to continuous images, then for every space X

$A \subseteq X$ is closed in $m_{\mathcal{C}}X$ iff $A \cap K$ is closed in K for each subspace K of X which belongs to $\mathcal{C}$.

This description implies easily that $m_{\mathcal{C}} A$ is a closed subspace of $m_{\mathcal{C}} X$ for every closed $A \subseteq X$ and $m_{\mathcal{C}} B$ is an open subspace of $m_{\mathcal{C}} X$ for every cozero set $B \subseteq X$ provided that $\mathcal{C}$ is closed also with respect to closed subspaces. Thus, if $\mathcal{S} \subseteq \mathcal{C}$ and $\mathcal{C}$ is closed with respect to continuous images and closed subspaces, then $m_{\mathcal{C}}$ is stabilized by the complete metrizability, see III.3. Moreover, if $m_{\mathcal{C}}$ is not identical, then it has a distinguishing space X such that both X and $m_{\mathcal{C}} X$ are paracompact, see III.6. Hence the Main Theorem can be applied e.g. on the following classes $\mathcal{C}$:

$\mathcal{C}$ = all compact spaces, i.e. $m_{\mathcal{C}}$ is a compactly generated modification;

$\mathcal{C}$ = all the spaces of the cardinality $\leq \alpha$, where α is a given infinite cardinal, i.e. $m_{\mathcal{C}}$ is the coreflection on the subcategory of all the spaces with the tightness $\leq \alpha$;

$\mathcal{C}$ = all the compact spaces of the cardinality $\leq \alpha$.

For any class $\mathcal{C}$ containing $\mathcal{S}$, we can form its closure with respect to closed subspaces and then with respect to continuous images. If the obtained class $\mathcal{D}$ is still not so large that $m_{\mathcal{D}}$ is already the identity, then $m_{\mathcal{D}}$ is comprehensive and all the representing spaces can be chosen to be paracompact.

d) Composition of modifications. If m is an upper modification and m' a lower one, then both $m \circ m'$ and $m' \circ m$ are modifications again, but it is neither an upper nor a lower modification in general. For example, the modification $s \circ u$ sends the space $Y = Z \amalg T$, where Z is a non-compact Lindelöf space with a unique non-isolated point and T is the real line, the open subbasis of which is formed by all open intervals and the set of all irrational numbers (and $\amalg$ denotes the coproduct, i.e. the sum), on the space $s(uY) = D \amalg R$, where D is discrete and R is the real line with the usual topology, so that neither $s \circ uY \leq Y$ nor $Y \leq s \circ uY$. Let us notice that still $s \circ u$ is stabilized by the complete metrizability and the space Z is a distinguishing space for $s \circ u$. However, since the distinguishing space X of a modification m has to satisfy either $X \leqq mX$ or $mX \leqq X$ (and this is necessary for the construction in the proof of the Main Theorem), all the representing spaces fulfil the same inequality for any faithful Ψ (this can be seen from the construction), so that we work "in essence" only with lower or upper modifications.

IV. The proof of the Main Theorem

IV.1. Let us denote by **G** the category of all directed connected graphs without loops (i.e. the objects of **G** are all (V,R), where V is a set and $R \subseteq V \times V$ such that never $(v,v) \in R$ and for every $v, v' \in V$ [not necessarily distinct] there exist $v_0 = v, v_1, \dots, v_n = v'$ in V with $(v_{i-1}, v_i) \in R \cup R^{-1}$; $h:(V,R) \longrightarrow (V',R')$ is a morphism of **G** iff it is an RR'-compatible map, i.e. it maps V into V' such that $(v,v') \in R \Longrightarrow (h(v), h(v')) \in R'$). Let **H** be a category, the objects of which are all triples (V,R,S), where (V,R) is an object of **G** and $S \subseteq R$; $h: :(V,R,S) \longrightarrow (V',R',S')$ is a morphism of **H** iff it is both RR'-compatible and SS'- compatible. There is a natural forgetful functor

$$\Gamma : \mathbf{H} \longrightarrow \mathbf{G}$$

which forgets the second relation, i.e. $\Gamma(V,R,S) = (V,R)$, $\Gamma(h) = h$. In [T_3], for any faithful functor $\Psi : k_1 \longrightarrow k_2$, where k_1 and k_2 are small categories, full embeddings (= <u>full</u> one-to-one functors) $\Lambda_1 : k_1 \longrightarrow \mathbf{H}$, $\Lambda_2 : k_2 \longrightarrow \mathbf{G}$ are constructed such that the square

$$\begin{array}{ccc} k_1 & \xrightarrow{\Psi} & k_2 \\ {\scriptstyle \Lambda_1} \downarrow & & \downarrow {\scriptstyle \Lambda_2} \\ \mathbf{H} & \xrightarrow{\Gamma} & \mathbf{G} \end{array}$$

commutes. Hence to prove our Main Theorem, it is sufficient to construct, for a given essentially non-identical modification m stabilized by the complete metrizability, almost full embeddings $\Phi_1 : \mathbf{H} \longrightarrow \mathrm{Top}$, $\Phi_2 : \mathbf{G} \longrightarrow \mathrm{Top}$ such that the square

$$\begin{array}{ccc} \mathbf{H} & \xrightarrow{\Gamma} & \mathbf{G} \\ {\scriptstyle \Phi_1} \downarrow & & \downarrow {\scriptstyle \Phi_2} \\ \mathrm{Top} & \xrightarrow{m} & \mathrm{Top} \end{array}$$

commutes. Then $\Phi_1 \circ \Lambda_1$ and $\Phi_2 \circ \Lambda_2$ give a simultaneous representation of Ψ by m.

IV.2. We will construct the functors Φ_1, Φ_2 in IV.3 - 4 below. First, let us introduce some notation and show some auxiliary statements. If P is a space metrized by a complete metric ρ, $A \subseteq P$ its subspace with $\rho(a,a') \geq 1$ for all distinct $a,a' \in A$ and X is a space with $|X|=|A|$, we denote by

$$P_A X$$

the space on $|P|$ with the topology sup(t,d), where t is the topology given by ρ and d is the topology of X extended on $|P|$ such that any point $x \in |P| \setminus |X|$ is isolated. Hence X is a zero set and C^*-embedded subspace of $P_A X$ and $P_A X \setminus X$ is metrizable (by a complete metric).

Observation. If m is a modification stabilized by the complete metrizability, then mX is a closed and $P \setminus A$ an open subspace of $mP_A X$. Moreover, if $X \leq mX$ (or $X \gtrsim mX$), then $P_A X \leq mP_A X$ (or $P_A X \gtrsim mP_A X$).

IV.3. In the rest of IV, we suppose that an essentially nonidentical modification m stabilized by the complete metrizability is given and X is its distinguishing space.

Let P be a space metrized by a complete metric ρ, A its subspace with $\rho(a,a') \geq 1$ for all distinct $a,a' \in P$ and $|A|=|X|$, p_1, p_2, p_3 are three distinguished points of P such that $\rho(p_i,p_j) \geq 1$ and $\rho(p_i,A) \geq 1$ for $i,j \in \{1,2,3\}$, $i \neq j$. Depending on it, we construct the functor $\Phi_2 : \mathbf{G} \longrightarrow \mathrm{Top}$. The construction of Φ_2 is just the arrow construction, described in a general setting e.g. in [PT]: each arrow $r \in R$ in a connected graph $(V,R) \in \mathrm{obj}\,\mathbf{G}$ is replaced by a copy of $mP_A X$. More in detail, we take a copy $(mP_A X)_r$ of the space $mP_A X$ (all the points, subspaces, ... of $(mP_A X)_r$ are denoted as in $mP_A X$, only the letter r is added) for each $r \in R$ and, in the coproduct $\coprod_{r \in R} (mP_A X)_r$, we identify, for each $r=(v_1,v_2) \in R$,

$p_{1,r}$ with $p_{1,r'}$ iff $r'=(v_1,v_2') \in R$, we denote the obtained point by v_1,
$p_{1,r}$ with $p_{2,r'}$ iff $r'=(v_1',v_1) \in R$, " " " " " " v_1,
$p_{2,r}$ with $p_{2,r'}$ iff $r'=(v_1',v_2) \in R$, " " " " " " v_2,
$p_{3,r}$ with $p_{3,r'}$ for all $r' \in R$, " " " " " " c,

taking all the sets $\bigcup_{r \in R} \{y_r \mid \rho_r(y_r,p_{3,r}) < \varepsilon\}$ with $\varepsilon > 0$ as a local basis of c and all the sets $\bigcup_{r \in R_1} \{y_r \mid \rho_r(y_r,p_{1,r}) < \varepsilon\} \cup \bigcup_{r \in R_2} \{y_r \mid \rho_r(y_r,p_{2,r}) < \varepsilon\}$

with $\varepsilon > 0$ as a local basis of v, where R_i is the set of all $r \in R$ such that v is its i-th member. Hence the obtained space $\Phi_2(V,R)$ contains V (as its

C^{*}-embedded discrete zero set) and $\Phi_2(V,R) \setminus \bigcup_{r \in R} (mX)_r$ can be metrized by a complete metric.

Observation. $m\Phi_2(V,R) = \Phi_2(V,R)$.
If $h:(V,R) \to (V',R')$ is a morphism of **G**, we define $f = \Phi_2(h)$ such that it maps each $(mP_AX)_r$ onto $(mP_AX)_{r'}$ as the identity, for all $r=(v_1,v_2) \in R$ and $r' = (h(v_1),h(v_2)) \in R'$, i.e., in our convention, $f(x_r)=x_{r'}$.

Observation. $\Phi_2:\mathbf{G} \to \mathrm{Top}$ is a correctly defined one-to-one functor. Every $\Phi(V,R)$ is regular or ... or paracompact whenever mX has this property.

IV.4. The functor $\Phi_1:\mathbf{H} \to \mathrm{Top}$ is also constructed by the arrow-construction; given $(V,R,S) \in \mathrm{obj}\,\mathbf{H}$, then,

if $X \lneq mX$, the arrows in S are replaced by copies of mP_AX and the arrows in $R \setminus S$ are replaced by copies of P_AX;

if $mX \lneq X$, the arrows in S are replaced by copies of P_AX and the arrows in $R \setminus S$ are replaced by copies of mP_AX.

We do not describe the arrow-construction with all details as in IV.3 because the identifications are as in IV.3 and the local basis of the glueing-points is also as in IV.3.

Observation. $m\Phi_1(V,R,S) = \Phi_2(V,R)$; $\Phi_1(V,R,S)$ is regular or ... or paracompact whenever both X and mX have this property.

If $h:(V,R,S) \to (V',R',S')$ is a morphism of **H**, we define $g = \Phi_1(h)$ similarly as in IV.3, i.e. g maps the space replacing an arrow $r=(v_1,v_2)$ onto the space replacing the arrow $r'=(h(v_1),h(v_2))$ as the identity. Here, we have to mention that if $r \in S$, then $r' \in S'$ so that r and r' both are replaced by copies of mP_AX (if $X \lneq mX$) or both are replaced by copies of P_AX (if $mX \lneq X$), hence the identity map is continuous; if $r \in R \setminus S$, then either $r' \in R' \setminus S'$ and then r and r' are replaced by copies of the same space again, or $r' \in S'$; in this last case, the identity map is continuous again, being a map $(P_AX)_r \to (mP_AX)_{r'}$, if $X \lneq mX$ or $(mP_AX)_r \to (P_AX)_{r'}$ if $mX \lneq X$.

Observation. $\Phi_1:\mathbf{H} \to \mathrm{Top}$ is a correctly defined one-to-one functor and $m \circ \Phi_1 = \Phi_2 \circ \Gamma$.

IV.5. The parts IV.6 - 9 are devoted to the construction of such space P, its subspace A and the distinguished points p_1, p_2, p_3, that the functors Φ_1 and Φ_2, constructed from them as described in IV.3 - 4, are almost full. First, let us show that it is sufficient to construct them such that the following statements a), b) are true:

a) if $(V,R) \in \text{obj}\ \mathbf{G}$ and $\ell: mP_A R \longrightarrow \Phi_2(V,R)$ is continuous, then either ℓ is constant or there exists $r \in R$ such that ℓ is the identity map of $mP_A R$ onto its r-th copy in $\Phi_2(V,R)$, i.e. $\ell(x)=x_r$ for all $x \in mP_A R$;

b) if $(V,R,S) \in \text{obj}\ \mathbf{H}$ and ℓ is a continuous map of $mP_A R$ (or $P_A R$) into $\Phi_1(V,R,S)$, then either ℓ is constant or there exists $r \in R$ such that $\ell(x)=x_r$ for all $x \in mP_A R$ (or for all $x \in P_A R$).

Thus, let us suppose that a) is valid and that $f: \Phi_2(V,R) \longrightarrow \Phi_2(V',R')$ is a non-constant continuous map. For each $r \in R$, we investigate the domain-restriction $(mP_A X)_r \longrightarrow \Phi_2(V',R')$ of f, analogously as in [PT], pp. 105-6. If one of these restrictions is constant, say the r-th one, necessarily $f(c)=f(p_{1,r})=f(p_{2,r})$, hence all these restrictions must be constant, so that f must be constant, which is a contradiction. Thus, by a), for every $r \in R$ there exists $r' \in R'$ such that $f(x_r)=x_{r'}$ for all $x \in mP_A X$. Since (V,R) is connected, for every $v \in V$ there exists $r \in R$ and $i \in \{1,2\}$ such that $v=p_{i,r}$. Then $h(v)=p_{i,r'}$ give a $\mathbf{G}$-morphism $h:(V,R) \longrightarrow (V',R')$ such that $f=\Phi_2(h)$.

Let us suppose that b) is satisfied and that $g: \Phi_1(V,R,S) \longrightarrow \Phi_1(V',R',S')$ is a continuous non-constant map. We find $h:(V,R) \longrightarrow (V',R')$ similarly as in the previous case. However, b) implies that if $r=(v_1,v_2) \in S$, then $r'=(h(v_1),h(v_2))$ must be in S' because $X \neq mX$, so that the identity map

$mP_A R \longrightarrow P_A R$ is not continuous whenever $mX \geq X$ and
$P_A R \longrightarrow mP_A R$ is not continuous whenever $X \geq mX$.

Thus, the constructed h is also SS'-compatible, hence it is a $\mathbf{H}$-morphism and $g=\Phi_1(h)$.

IV.6. The construction of P, A and p_1, p_2, p_3 such that a), b) in IV.5 are fulfilled, heavily depends on the existence of a Cook continuum. Let us recall that a Cook continuum is a non-degenerate metrizable continuum Q such that

if K is a subcontinuum of Q and $f:K \longrightarrow Q$ is a continuous map, then either f is constant or $f(x)=x$ for all $x \in K$.

A continuum with these properties was constructed by H. Cook in [C]. A more detailed version of the construction is contained in Appendix A in [PT]. We use its non-degenerate subcontinua (in what follows, continuum always means a non-degenerate continuum).
We choose pairwise disjoint subcontinua of Q, denoted by A_o,B_o,C_o, A_{o1},A_1, $A_{1,o}$, $B_{o1},B_1,B_{1,o},\dots$ (there is a countable collection of them); we will call them building blocks. We metrize them such that the diameter of A_o, B_o, C_o is 1, that of A_1,B_1,C_1 is $\frac{1}{2}$ etc., in general the space with single index i has

diameter 2^{-i} and spaces with indices i, j have diameter $2^{-\min(i,j)}$. Furthermore, in each of these subcontinua we choose two points of distance equal to

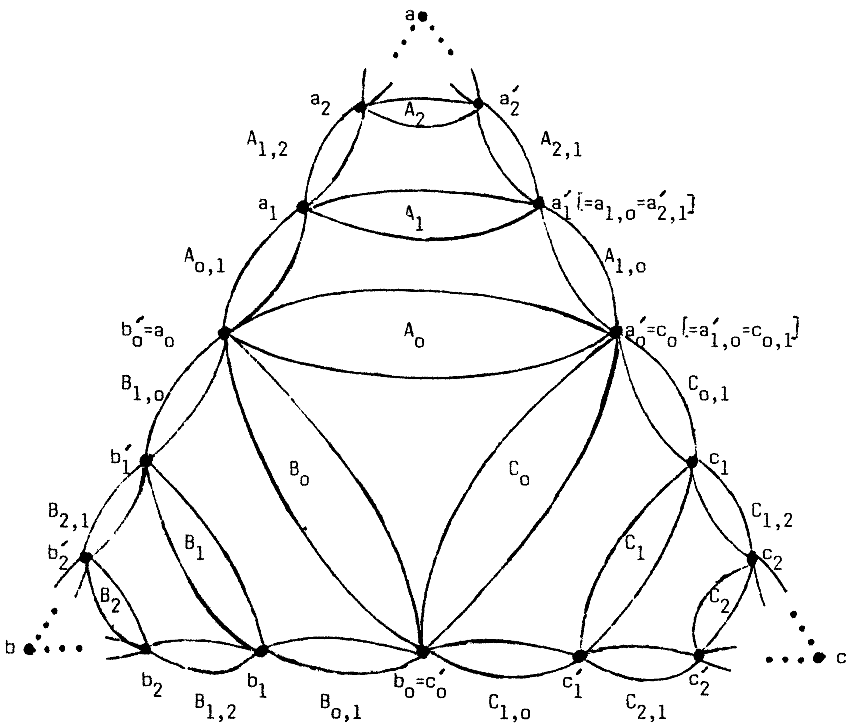

Figure 1

the diameter of the subcontinuum; we call the points in A_o, a_o and a_o', in B_o, b_o and b_o', in $A_{o,1}$, $a_{o,1}$ and $a_{o,1}'$, etc. We now glue the spaces into a connected metric as indicated by Figure 1. Finally, we form the completion of the resulting metric space by adding the points a, b and c as indicated (i.e. $a=\lim a_n$). The resulting space will be called a <u>triangle space</u> (the construction of the triangle space is also, in another notation, in [PT], pp. 223-4).

IV.7. We need four triangle spaces, say T_1, T_2, T_3, T_4, T_i is created from a collection $Z_i=\{A_o^{(i)},B_o^{(i)},C_o^{(i)},A_{1,o}^{(i)},\ldots\}$ of subcontinua of Q as in IV.6

such that the collection $Z = \bigcup_{i=1}^{4} Z_i$ is stiff (any pairwise disjoint collection of subcontinua of Q is stiff). The subspaces $A_0^{(i)}, B_0^{(i)}, \ldots$ are called <u>building blocks</u> of T_i. The copies of building blocks of T_i are called also building blocks of Y whenever Y is created from the triangle spaces (or also from $C_{2,x}$) as described below. Denote by S_i the subset of all the glueing points of T_i, i.e. $S_i = \{a_0^{(i)}, b_0^{(i)}, c_0^{(i)}, a_1^{(i)}, a_1^{\prime(i)}, \ldots\}$ and by $a^{(i)}$, $b^{(i)}$, $c^{(i)}$ the points added in the forming of the completion.

IV.8. The rôle of T_3 and T_4 is to obtain sufficiently large stiff collection of spaces of a special form. For this reason, we choose a rigid collection

$$\mathcal{V} = \{(V_x, R_x) \mid x \in X\}$$

of objects of **G**, where X <u>is the distinguishing space of</u> m (we recall that each (V_x, R_x) has no non-identical endomorphism and if $x \neq x'$, there is no morphism $(V_x, R_x) \longrightarrow (V_{x'}, R_{x'})$, such a collection does exist, see [PT]). In $T_3 \amalg T_4$, we identify $a^{(3)}$ with $a^{(4)}$ and $b^{(3)}$ with $b^{(4)}$, the obtained space is denoted by T, the obtained points by a and b. We use the arrow construction again: in (V_x, R_x), we replace each $r \in R_x$ by a copy of T; more in detail, in the coproduct $\coprod_{r \in R_x} (T)_r$ (where $(T)_r$ are copies of the space T; we use the convention of IV.3 that points, subspaces,... of $(T)_r$ are denoted as in T, only the index r is added) we make the following identifications for each $r = (v_1, v_2) \in R_x$:

a_r with $a_{r'}$ iff $r' = (v_1, v_2') \in R_x$, we denote the obtained point by v_1,

a_r with $b_{r'}$ iff $r' = (v_1', v_1) \in R_x$, we denote the obtained point by v_1,

b_r with $b_{r'}$ iff $r' = (v_1', v_2) \in R_x$, we denote the obtained point by v_2,

$c_r^{(3)}$ with $c_{r'}^{(3)}$ for all $r' \in R_x$, we denote the obtained point by c_x,

$c_r^{(4)}$ with $c_{r'}^{(4)}$ for all $r' \in R_x$, we denote the obtained point by c_x' ;

the local basis of the glued points is defined similarly as in IV.3, so that we obtain a complete metric space; it is denoted by C_x.

IV.9. For every $x \in X$, we form the space $T_{2,x}$ such that we replace the subcontinuum $C_0^{(2)}$ in the triangle space T_2 by the space C_x. More in detail: in $C_x \amalg (T_2 \setminus (C_0^{(2)} \setminus \{c_0^{(2)}, c_0^{\prime(2)}\}))$, we identify c_x with $c_0^{(2)}$ and c_x' with $c_0^{\prime(2)}$. Let us denote points, subspaces etc. of $T_{2,x}$ which are outside C_x, as in T_2, only the index x is added. Our desired space P is obtained from

$$T_1 \amalg \coprod_{x \in X} T_{2,x}$$

by the identification of

$a^{(1)}$ with $a_x^{(2)}$ for all $x \in X$, the obtained point is p_1,

$b^{(1)}$ with $b_x^{(2)}$ for all $x \in X$, the obtained point is p_2;

the local basis of the glued points is defined as in IV.3, so that P is really metrizable by a complete metric. Its points p_1, p_2 are already defined, we put $p_3=c^{(1)}$. Finally, the C^*-embedded discrete zero set A with the same underlying set as X is formed by all $c_x^{(2)}$, $x \in X$.

IV.10. It remains to prove that P, p_1, p_2, p_3 and A satisfy the statements a), b) in IV.5. First, we prove several auxiliary lemmas.

Lemma. Let $\mathcal{Z}$ be a collection of all building blocks of all $T_1,\dots,T_4$. Let Y be a space containing $Z \in \mathcal{Z}$ such that the boundary of Z in Y consists of two points z_1, z_2. Let $Z' \subseteq Z$ and let $f: Z' \longrightarrow Y$ be a continuous non-constant map. Then either $Z'=Z$ and f is the inclusion (i.e. $f(z)=z$ for all $z \in Z$) or $f(Z') \cap Z \subseteq \{z_1,z_2\}$.

Proof. Put $\mathcal{O}=f^{-1}(Z \setminus \{z_1,z_2\})$. Suppose that $\mathcal{O} \neq \emptyset$. If $Z' \setminus \mathcal{O}=\emptyset$, then $f(Z') \subseteq Z \setminus \{z_1,z_2\}$, hence f must be constant, which is a contradiction. Hence $Z' \setminus \mathcal{O} \neq \emptyset$. Choose $z \in \mathcal{O}$ and denote by K the component of z in $\mathcal{O}$. Since $Z' \setminus \mathcal{O} \neq \emptyset$, $\overline{K}$ intersects the boundary of $\mathcal{O}$ (see [K], § 42,III); hence $f(\overline{K})$ is a subcontinuum of Z (it is non-degenerate because $f(\overline{K}) \cap \{z_1,z_2\} \neq \emptyset$ and $f(\overline{K}) \cap (Z \setminus \{z_1,z_2\}) \neq \emptyset$). Since a subcontinuum of Z' is mapped continuously onto a subcontinuum of Z, necessarily $Z'=Z$ and $f(y)=y$ for all $y \in \overline{K}$. Hence $f(z)=z$; but z was an arbitrarily chosen point of $\mathcal{O}$, so that $f(z)=z$ for all $z \in \mathcal{O}$. Consequently $\mathcal{O}=Z \setminus \{z_1,z_2\}$, hence $f(z)=z$ for all $z \in Z$.

IV.11. Let $T_1,\dots,T_4$ be as in IV.7.

Lemma. Let $i,j \in \{1,\dots,4\}$, let Y be a space containing T_i such that the boundary of T_i in Y consists of $a^{(i)}$, $b^{(i)}$, $c^{(i)}$. Let $f: T_j \longrightarrow Y$ be a non-constant continuous map. Then either $i=j$ and $f(x)=x$ for all $x \in T_j$ or $f(T_j) \cap T_i \subseteq$ $\subseteq \{a^{(i)},b^{(i)},c^{(i)}\}$.

Proof. Put $\mathcal{U}=T_i \setminus \{a^{(i)},b^{(i)},c^{(i)}\}$, $\mathcal{O}=f^{-1}(\mathcal{U})$.

α) Let $j \neq i$: If there is a building block Z of T_j with $Z \cap \mathcal{O} \neq \emptyset$, f/Z must be constant, by IV.10; hence its image is a point $y \in \mathcal{U}$. Then f maps all the building blocks, intersecting Z, on y again; hence it maps all the building blocks, intersecting them, on y again. We conclude f maps the whole T_j on y, which is a contradiction. Hence $f(T_j) \cap \mathcal{U}=\emptyset$.

β) Let i=j: Let there be a building block Z of T_i with $Z \cap \mathcal{O} \neq \emptyset$. Then either $Z \subseteq \mathcal{O}$ and f(z)=z for all $z \in Z$ or f/Z is constant. Let us suppose that f/Z is constant for a building block Z with $Z \cap \mathcal{O} \neq \emptyset$, f maps Z on a point $y \in \mathcal{U}$. If y is an interior point of a building block of T_i, we can repeat the argument of α) and conclude that f must be constant on T_i. Thus, let us suppose that y is in S_i (see IV.7). However, every building block Z' of T_i can be joined with Z by a finite sequence $Z=Z_0, Z_1, \dots, Z_n=Z'$ of building blocks such that $Z_{i-1} \cap Z_i = \{z_i\}$ and none of the points $z_1, \dots, z_{n-1}$ is equal to y, so that all $Z_1, \dots, Z_n=Z'$ must be mapped on y again. We conclude that if Z is a building block of T_i such that $Z \cap \mathcal{O} \neq \emptyset$, then necessarily $Z \subseteq \mathcal{O}$ and f(z)=z for all $z \in Z$. But if $\mathcal{O} \neq \emptyset$, it contains at least one building block of T_i, hence it contains all the building blocks which intersect it, etc. Thus, in this case, f(z)=z for all $z \in T_i$.

IV.12. Let $T_{2,x}$, $x \in X$, be as in IV.9.

Lemma. Let $x, x' \in X$, let Y be a space containing $T_{2,x}$ such that the boundary of $T_{2,x}$ in Y consists of $a_x^{(2)}$, $b_x^{(2)}$, $c_x^{(2)}$. Let $f: T_{2,x'} \longrightarrow Y$ be a non-constant continuous map. Then either x=x and f(z)=z for all $z \in T_{2,x'}$ or $f(T_{2,x'}) \cap T_{2,x} \subseteq \{a_x^{(2)}, b_x^{(2)}, c_x^{(2)}\}$.

Proof. Put $\mathcal{U} = Y \setminus \{a_x^{(2)}, b_x^{(2)}, c_x^{(2)}\}$, $\mathcal{O} = f^{-1}(\mathcal{U})$. By IV.11, any copy of T_3 or T_4 in $T_{2,x'}$ which intersects $\mathcal{O}$, is mapped by f either onto some of its copies in $T_{2,x}$ "as the identity" or f is constant on it. However, if it is constant on it, it must be constant on the whole $C_{x'}$ (the graph $(V_{x'}, R_{x'})$ is connected!). Then it must be constant on the whole $T_{2,x'}$ - the proof is analogous as in IV.11. Let us suppose that f maps any copy of T_3 and of T_4 which intersects $\mathcal{O}$, onto some of its copies in $\mathcal{U}$ as the identity and that there is a copy of T_3 or T_4 which intersects $\mathcal{O}$. Then every copy of T_3 and T_4 is in $\mathcal{O}$ (and f maps them on some of their copies as the identity). Then necessarily there is a morphism $h: (V_{x'}, R_{x'}) \longrightarrow (V_x, R_x)$ such that f maps the r-th copy of T_3 (or T_4) onto its r'-th copy in $\mathcal{U}$, where $r=(v_1, v_2) \in R_{x'}$ and $r' = (h(v_1), h(v_2))$. Since $\mathcal{G}$ is a rigid collection of graphs, then necessarily $x=x'$ and h is the identity, i.e. f maps C_x onto itself as the identity. Then necessarily f(z)=z for all $z \in T_{2,x}$ - the rest of the proof is analogous as in IV.11.

IV.13. **Lemma.** Let $(V,R) \in$ obj **G** (or $(V,R,S) \in$ obj **H**), let $Y = \Phi_2(V,R)$ (or $Y = \Phi_1(V,R,S)$). Let $f: P \longrightarrow Y$ be a non-constant continuous map. Then there exists $r \in R$ such that $f(z) = z_r$ for all $z \in P$.

Proof. Any building block of Y has the boundary in Y consisting of two points. Hence, by IV.10, f maps any building block Z of P on some of its copies in Y as the identity or f is constant on Z or f maps Z into $Y \setminus \mathrm{Int}\, \tilde{Z}$ for any copy $\tilde{Z}$ of Z in Y. In the last case, f/Z must be constant again. In fact, if we subtract from Y all the interiors of all the building blocks of Y, the remaining subspace consists of components homeomorphic to the components of mX (or of X and mX) and one-point components. Since neither X nor mX contains a [non-degenerate] metrizable continuum and both X and mX are Hausdorff spaces (see II.4), f(Z) must be a one-point set. Hence f maps any building block Z of P either onto some of its copies in Y as the identity or f/Z is constant. If f/Z is constant for some building block Z, then f/T is constant for the triangle space T (or the copy $T_{2,x}$) containing Z, by IV.11 (or by IV.12). Hence it is constant on the whole P. If f maps every building block Z of P on some of its copies in Y as the identity, then f maps any triangle space on some of its copies as the identity, by IV.11, and it maps any $T_{2,x}$ in P on some of its copies as the identity, by IV.12. Consequently there exists $r \in R$ such that $f(z)=z_r$ for all $z \in P$.

Corollary. The statements a) and b) in IV.5 follow immediately from IV.13 because the identity maps $P \longrightarrow P_A X$ and $P \longrightarrow mP_A X$ are continuous.

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