Jan Malý

Fine localization in potential theory [Abstract of thesis]

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Das nächste Kapitel behandelt die Lösung des Problems der minimalen kritischen Menge des Treibstoffes im Kernreaktor. Es ist nötig, eine solche Verteilung des Spaltmaterials im Reaktor zu bestimmen, dass dieser Reaktor im kritischen Zustand ist und dass dabei die betreffende kritische Menge des Treibstoffes minimal wird. Diese Aufgabe hat höchstens eine Lösung, und ähnlich ist es auch bei der Diskretisierung. Auf einer geeigneten Funktionsmenge werden die notwendigen und ausreichenden Bedingungen des Extrems eines gewissen Funktionals, der von der Funktion der Treibstoffverteilung abhängt und der die Menge des Treibstoffes charakterisiert, untersucht.

Die approximative Lösung derOptimalisationsaufgabe gründet sich auf die Approximation der Lösung des elliptischen Systems der Differentialgleichungen für die Komponenten des Neutronstroms. Diese Approximation wird auf der Basis der Galerkinmethode vorgeschlagen. Die Ergebnisse der formalen Diskretisierung dieses Gleichungssystems werden dann auf eine konkrete Methode der Endelemente angewandt.

MOMENT PROBLEM AND ITS APPLICATION

V. BENEŠ, St. výzkumný ústav pro stavbu strojů, Praha 9-Běchovice, Czechoslovakia
(14.4. 1986, supervisor J. Štěpán)

The thesis deals with the theory of the general moment problem and its engineering applications.

Simplicial measures - the extreme points in the marginal moment problem on a product of two spaces are investigated by means of so-called A-sets. A characterization of the A-sets is given and the relation between the support of a simplicial measure and an A-set is studied. The whole theory essentially generalizes the known results.

The optimization moment problem theory makes it possible to construct optimal conservative estimators of the expectation of a function of random parameters with a partially known distribution. This model is suitable in engineering for the evaluation of the fatigue-life of machine components. The presented method was used when testing the working reliability of a compressor during surging.

FINE LOCALIZATION IN POTENTIAL THEORY

J. MALÝ, Faculty of Mathematics and Physics, Charles University, Sokolovská 83, 18600 Praha 8, Czechoslovakia
(7.5. 1986, supervisor J. Lukeš)

The dissertation is devoted to the fine Dirichlet problem and fine hyperharmonicity in the axiomatic potential theory. It is shown that the structure of standard H-cones (see [1]) is rich enough to develop the potential theory on open and finely open subsets of the underlying space including the Dirichlet problem.

The substantial part of the dissertation treats the capacity with values in the cone of all positive superharmonic functions on a finely open set and related quasi-topological notions.

The main results, in a very rough formulation, are as follows:

Theorem A. Let U be a finely open set. For each boundary function f which is integrable with respect to the harmonic measure there exists a unique finely harmonic function h on U which is quasi-continuous up to the boundary extended by the values of f. This function h coincides with the "Perron solution" of the considered Dirichlet problem.

Theorem B. Let U be a finely open set. Let u be a quasi--l.s.c. and finely l.s.c. function on U. Suppose that for every $x \in U$ there is a fundamental system of fine neighborhoods V of x with the property $\varepsilon_x^{CV}(u) \leq u(x)$. Then u is finely hyperharmonic on U.

The results of the dissertation are published in [2].

References:

[1] N. BOBOC, Gh. BUCUR, A. CORNEA: Order and Convexity in Potential Theory: H-Cones. Lecture Notes in Mathematics 853, Springer-Verlag, Berlin-Heidelberg-New York 1981.

[2] J. LUKEŠ, J. MALÝ, L. ZAJÍČEK: Fine Topology Methods in Real Analysis and Potential Theory. Lecture Notes in Mathematics 1189, Springer-Verlag, Berlin-Heidelberg-New York-London-Paris-Tokyo 1986.

SOME ASPECTS OF CONVEX ANALYSIS AND THE THEORY OF ASPLUND SPACES

L.JOKL, Faculty of Civil Engineering, The Czech Technical Univ., Prague, Thakurova 7, 16629 Praha 6-Dejvice, Czechoslovakia
(7.5. 1986, supervisor J. Kolomý)

Theorem 1.28 and Corollary 2.3 in [1] form a mechanism in which Fréchet differentiability works. We show, using methods of convex analysis, that this differentiability can be replaced by any $\mathcal{A}$-differentiability having the property (m) defined below. For instance, Gâteaux differentiability on separable Banach spaces can be included in this mechanism.

We say that a family $\mathcal{A}$ of bounded subsets of a Banach space X is a generating system if (i) $A \in \mathcal{A}$ implies $-A \in \mathcal{A}$ and (ii) the span of the set $\bigcup\{A : A \in \mathcal{A}\}$ is dense in X. A function $f: X \longrightarrow R$ is said to be $\mathcal{A}$-differentiable at a point $x \in X$ if there exists an element x^* (called an $\mathcal{A}$-derivative of f at x and denoted by $\mathcal{A}$-df(x)) in the dual Banach space X^* such that the relation

$$\lim_{t \downarrow 0} \sup_{h \in A} |t^{-1}(f(x+th)-f(x)) - \langle h, x^* \rangle| = 0$$

is satisfied for all A in $\mathcal{A}$. We denote by $\mathcal{T}_{\mathcal{A}}$ the topology of uniform convergence on members of $\mathcal{A}$ for the set X^*. We say that $\mathcal{A}$ has the property (m) if the topology $\mathcal{T}_{\mathcal{A}}|M$ is metrizable for each set $M \subset X^*$.

Theorem 1. Let $\mathcal{A}$ be a generating system having the property (m). Then the following statements (a) and (b) are equivalent.

(a) $\{x \in X : \mathcal{A}\text{-df}(x) \text{ exists}\}$ is a dense G_δ subset of X for every continuous convex function $f: X \longrightarrow R$.

(b) For every pair [M,V], where $M \subset X^*$ is bounded and nonempty and V is a $\mathcal{T}_{\mathcal{A}}$-neighbourhood of the point $0 \in X^*$, there exists a weak* open set $W \subset X^*$ such that $M \cap W \neq \emptyset$ and $M \cap W - M \cap W \subset V$.

We say that X is an almost Asplund space if there exists a generating system $\mathcal{A}$ having the property (m) so that (a) or