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PERIPHERALLY COMPACT MAPPINGS

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Abstract: The well-known Hanai-Morita-Stone-Michael theorem characterizing peripheral compactness of closed mappings of metrizable spaces onto arbitrary topological spaces is extended to closed mappings of more general spaces and to more general mappings of metrizable spaces. In some general cases when a closed mapping f is inductively irreducible the set of the points at which f is peripherally compact is considered and described. Besides, it is established that the images of rim compact spaces under certain monotone peripherally compact mappings are rim compact.

Key words: Closed mappings, peripherally compact mappings, monotone mappings.

Classification: Primary 54C10
Secondary 54D30

All mappings below are considered to be continuous. The set of all positive integers is denoted by N.

1. A T_1-space X in which any countable discrete system of points is separated by a discrete system of their neighbourhoods is called a σ-space [L]. A space has property D, [Mo, p. 69], if any two disjoint closed subsets, one of which is countable and discrete have disjoint neighbourhoods (this is not the usual definition of property D but is equivalent to it in first countable regular spaces). Property D is weaker than pseudonormality even in the class of separable Moore

spaces (under the assumption of $P(c)$), [vD,W]. Spaces with property D will be called almost pseudonormal. It turns out that in a regular space the above mentioned properties are equivalent (K. Morita).

Proposition 1. For a space X the following two conditions are equivalent:

(i) X is a regular θ-space,

(ii) X is an almost pseudonormal T_1-space.

Proof. (i) $\Rightarrow$ (ii). Let $S = \{s_i : i \in N\}$ be a countable discrete subset of X and let F be a closed set such that $F \cap S = \emptyset$. Choose $\{O_{s_i}\}_{i \in N}$ a discrete system of neighbourhoods of s_i, $i \in N$. By regularity, for each $i \in N$ there exists an open neighbourhood V_{s_i} of s_i such that $(\mathrm{cl}V_{s_i}) \cap F = \emptyset$. If we put $W_{s_i} = O_{s_i} \cap V_{s_i}$ and $W = \bigcup_{i \in N} W_{s_i}$, then the system $\{W_{s_i}\}_{i \in N}$ is obviously discrete and hence is conservative i.e. $\mathrm{cl} \bigcup_{i \in N} W_{s_i} = \bigcup_{i \in N} \mathrm{cl}\, W_{s_i}$. Thus, W is an open neighbourhood of S such that $(\mathrm{cl}W) \cap F = \emptyset$.

(ii) $\Rightarrow$ (i). It is clear that a pseudonormal T_1-space is regular. If $\{x_i\}_{i \in N}$ is a discrete system of points, then the set $S = \{x_i : i \in N\}$ is discrete in X. We can easily find by induction a disjoint system $\{O_{x_i}\}_{i \in N}$ of open neighbourhoods of x_i, $i \in N$. It is obvious that the set $F = X \setminus \bigcup_{i \in N} O_{x_i}$ is closed in X and $F \cap D = \emptyset$. Hence, there exists an open set $U \supset D$ such that the system $\{V_{x_i}\}_{i \in N}$, where $V_{x_i} = U \cap O_{x_i}$, is discrete and this completes the proof.

Definition 1. If every compact subspace of a space X has countable character in X, then X is called a space of strongly countable type.

<u>Definition 2</u>. If every compact subspace K of a space X has a q-system i.e. a system $\{V_i\}_{i\in N}$ of open neighbourhoods such that every sequence $\{x_i\}_{i\in N}$ of distinct points $x_i \in V_i \setminus K$ has an accumulation point, then X is said to be a cq-space.

Metrizable spaces, spaces with a point-regular base, all subspaces of perfectly normal compacta, spaces of countable type with a countable network are examples of spaces of strongly countable type, [A_1], as well as their perfect images.

Spaces of strongly countable type and regular locally countably compact spaces are cq-spaces as well as perfect images of cq-spaces.

We recall that a space in which every closed, countably compact subspace is compact is called isocompact.

<u>Definition 3</u>. Let $f:X \to Y$ be a mapping of a topological space X onto a topological space Y. A point $y \in Y$ such that $f^{-1}(y)$ ($Fr\ f^{-1}(y)$) is a compact subspace of X is called a point at which f is compact (peripherally compact). If f is compact (peripherally compact) at each point $y \in Y$, then f is said to be compact (peripherally compact), [Va].

<u>Theorem 1</u>. If $f:X \to Y$ is a closed mapping of an isocompact Θ-space X of strongly countable type onto a space Y, then the following conditions are equivalent:

(i) Y is a space of strongly countable type,

(ii) Y is first countable,

(iii) Y is a q-space,

(iv) f is peripherally compact.

Proof. (i) $\Longrightarrow$ (ii) and (ii) $\Longrightarrow$ (iii) are obvious. (iii) $\Longrightarrow$ (iv) follows from [Mi_1, Theorem 1]. (iv) $\Longrightarrow$ (i). If f is peripherally compact, then f is inductively perfect i.e. there exists a closed set $F \subset X$ such that $f(F) = Y$ and $f|_F$ is perfect. Since F is a space of strongly countable type, so is its perfect image Y. Indeed, if K is a compact set in Y, then $(f|_F)^{-1}(K)$ is compact and hence has a countable outer base $\{U_n\}_{n\in N}$ in F. If V is an open set in Y such that $V \supset K$, then $(f|_F)^{-1}(V)$ is open in F and $(f|_F)^{-1}(V) \supset (f|_F)^{-1}(K)$, so for some $n_0 \in N$ we have that $U_{n_0} \subset (f|_F)^{-1}(V)$. Thus, $K \subset (f|_F)^{\#}(U_{n_0}) = Y \setminus f(F \setminus U_{n_0}) \subset f(U_{n_0}) \subset V$ and therefore $\{(f|_F)^{\#}(U_n)\}_{n\in N}$ form an outer base of K in Y.

Similarly we can prove the following.

Theorem 2. If $f: X \longrightarrow Y$ is a closed mapping of an isocompact θ- and cq-space X onto a space Y, then the following conditions are equivalent:

(i) Y is a cq-space,

(ii) Y is a q-space,

(iii) f is peripherally compact.

And what is more, if we consider ccq-spaces instead of cq-ones, i.e. spaces every closed countably compact subspace of which has a q-system, then we can obtain the following characterization of peripheral countable compactness of closed mappings.

Theorem 3. For every closed mapping $f: X \longrightarrow Y$ of a θ- and ccq-space X onto a space Y the following conditions are equivalent:

(i) Y is a σcq-space,

(ii) Y is a q-space,

(iii) f is peripherally countably compact.

We recall that a mapping $f:X \to Y$ of topological spaces is said to be countably discrete [T] if the image of every countable, discrete subset of X is closed in Y and f is said to be pseudoopen if f is "onto" and for each $y \in Y$ and every open $U \supset f^{-1}(y)$ we have $y \in \operatorname{Int} f(U)$ [A_2].

Since a quotient image of a sequential space is sequential, [A_2], and every countably discrete mapping onto a Hausdorff, sequential space is closed, [T], we obtain the following version of the Hanai-Morita-Stone-Michael theorem ([MH], [S], [M_1]).

<u>Theorem 4.</u> If $f:X \to Y$ is a countably discrete, quotient mapping of a metrizable space X onto a Hausdorff space Y, then the following conditions are equivalent:

(i) Y is metrizable,

(ii) Y is first countable,

(iii) Y is a q-space,

(iv) f is peripherally compact.

<u>Theorem 5.</u> If $f:X \to Y$ is a countably discrete, pseudoopen mapping with a closed graph of a metrizable space X onto a space Y, then the following conditions are equivalent:

(i) Y is metrizable,

(ii) Y is first countable,

(iii) Y is a q-space,

(iv) f is peripherally compact.

Proof. (i) $\Longrightarrow$ (ii) and (ii) $\Longrightarrow$ (iii) are obvious. It follows from [Mi_1] that (iii) $\Longrightarrow$ (iv). (iv) $\Longrightarrow$ (i). Since f is peripherally compact, f is inductively irreducible i.e. there exists a closed set $F \subset X$ such that $f(F) = Y$ and $f|_F$ is irreducible. It is clear that $f|_F$ is countably discrete and Y is a Fréchet-Urysohn space, as a pseudo-open image of a metrizable space. Since $f|_F$ has a closed graph, it follows from [Mi_5] that $f|_F$ is pseudo-open and hence is closed. We may assume that $f|_F$ is compact and therefore is perfect. Thus, Y as a perfect image of a metrizable space is metrizable.

2. It can easily be verified that the preimage of every nowhere dense set under pseudo-open, irreducible mappings is nowhere dense as well. Thus, the image of a Baire space under a pseudo-open, irreducible mapping is also a Baire one. Making use of [Mi_2, Theorem 1] we obtain the following

Theorem 6. If $f:X \longrightarrow Y$ is a pseudo-open irreducible mapping of an almost pseudonormal, sequential p-space X which is a Baire one onto a Hausdorff space Y, then the set of all points at which f is compact is a dense G_δ-set in Y.

Corollary 1. If $f:X \longrightarrow Y$ is a pseudo-open, irreducible mapping of a Čech-complete, sequential θ-space X onto a Hausdorff space Y, then there exists a dense G_δ-set $S \subset X$ such that $f|_S$ is perfect.

We recall that a $\varkappa$-space is a space in which every non-isolated point is a limit of a sequence of distinct points.

Theorem 7. If $f:X \longrightarrow Y$ is a closed mapping of a metacompact, almost pseudonormal p- and cq-space X onto a $\varkappa$-space Y which is also a Baire space, then the set P of all points at which f is peripherally compact is a G_δ-set with a σ-discrete complement.

Proof. By Proposition 1 X is an isocompact θ-space and therefore f is inductively irreducible [Mi_3]. Let $F \subset X$ be a closed set such that $f(F) = Y$ and $f|_Y$ is irreducible. Since Y is a Baire space, there exists a dense G_δ-set $M \subset Y$ such that for each $y \in M$ $(f|_F)^{-1}(y)$ is compact and hence the set S of all points $y \in Y$ such that $(f|_F)^{-1}(y)$ is compact is dense in Y. Since F is a metacompact p-space, the set $Y \setminus S$ is σ-discrete [V] and hence S is a dense G_δ-set in Y. Since F is a cq-space, by Theorem 2 we have that S is the set of all q-points in Y, for a closed irreducible mapping is peripherally compact if and only if it is compact. Now going over to f and again making use of Theorem 2 we get S = P.

Corollary 2. If $f: X \rightarrow Y$ is a closed mapping of a metacompact, Čech-complete θ - and cq-space X onto a $\varkappa$-space Y, then the set P is a dense G_δ with a σ-discrete complement.

Note. This negligible complement can be non-void, as exhibits the factorization of $\mathbb{R}$ by contracting $\mathbb{Z}$ into a point.

Corollary 3. If $f: X \rightarrow Y$ is a closed mapping of a θ-space X with a point-regular base onto a Baire space Y, then P is a dense G_δ-set with a σ-discrete complement.

Corollary 4. If $f: X \rightarrow Y$ is a closed mapping of a Čech-complete θ-space X with a point-regular base onto a space Y, then P is a dense G_δ-set with a σ-discrete complement.

Corollary 5. If $f:X \to Y$ is a closed mapping of a metrizable space X onto a Baire space Y, then the set P coincides with the set of all points of countable character in Y and is a dense G_δ with a σ-discrete complement.

Taking into account [Mi_4, Corollary 6] we obtain

Corollary 6. If $f:X \to Y$ is a closed mapping of a completely metrizable space X onto a space Y, then there exists the set $S \subset X$ such that $f(S)$ is dense in Y and $f|_S$ is clopen and peripherally compact.

3. We recall that a space every point of which has a base of neighbourhoods with compact boundaries is called rim compact and a mapping of topological spaces is said to be monotone, if the preimages of all points are connected. It is known that rim compactness is preserved by clopen mappings [M], and, if the image is Hausdorff, by open monotone and quotient compact monotone mappings [K].

Theorem 8. If $f:X \to Y$ is a peripherally compact, closed, monotone mapping of a rim compact T_1-space X onto a space Y, then Y is also rim compact.

Proof. Let y be a non-isolated point in Y, then $\mathrm{Fr}\, f^{-1}(y)$ is compact, closed and non-void in X. If an open $U \ni y$, then $f^{-1}(y) \subset f^{-1}(U)$ and $f^{-1}(U)$ is open in X. Since Y is obviously a T_1-space, $f^{-1}(y)$ is closed in X. Hence $f^{-1}(y) \subset f^{-1}(y) \subset f^{-1}(U)$. For each point $x \in \mathrm{Fr}\, f^{-1}(y)$ we can choose a neighbourhood $V_x \subset f^{-1}(U)$ such that $\mathrm{Fr}\, V_x$ is compact. Let $V_{x_1}, \dots, V_{x_k}$ be a cover of $\mathrm{Fr}\, f^{-1}(y)$ and let $V = \bigcup_{i=1} V_{x_i} \cup \mathrm{Int}\, f^{-1}(y)$. Then we have $\mathrm{Fr}\, V \subset \bigcup_{i=1}^{k} \mathrm{Fr}\, V_{x_i} \cup$

$\cup \operatorname{Fr} \operatorname{Int} f^{-1}(y)$. Since $\operatorname{Fr} \operatorname{Int} f^{-1}(y) \subset \operatorname{Fr} f^{-1}(y)$, we have that $\operatorname{Fr} V$ is a closed subset of a compact space $\bigcup_{i=1}^{k} \operatorname{Fr} V_{x_i} \cup$ $\cup \operatorname{Fr} f^{-1}(y)$ and therefore it is compact. Let us consider the set $B = f^{\#}(V) = Y \setminus f(X \setminus V)$. It is clear that B is an open neighbourhood of y and $B \subset U$. We shall show that $\operatorname{Fr} B \subset f(\operatorname{Fr} V)$. Suppose the contrary, then there is a point $z \in (\operatorname{Fr} B) \setminus f(\operatorname{Fr} V)$. Hence $f^{-1}(z) \subset X \setminus \operatorname{Fr} V$. Since V is open in X, $\operatorname{Fr} V = (\operatorname{cl} V) \cap$ $\cap (X \setminus V)$ and $X \setminus \operatorname{Fr} V = (X \setminus \operatorname{cl} V) \cup V$. Since f is monotone, either $f^{-1}(z) \subset V$ or $f^{-1}(z) \subset X \setminus \operatorname{cl} V$. If $f^{-1}(z) \subset V$, then $z \notin f(X \setminus V)$, so $z \in B$. Contradiction. If $f^{-1}(z) \subset X \setminus \operatorname{cl} V$, then $z \notin f(\operatorname{cl} V) = \operatorname{cl} f(V)$ and hence the set $W = Y \setminus f(\operatorname{cl} V)$ is an open neighbourhood of z and $W \subset Y \setminus f(V) \subset Y \setminus B$. Contradiction again. Thus $\operatorname{Fr} B \subset f(\operatorname{Fr} V)$ and hence $\operatorname{Fr} B$ is compact. This completes the proof.

<u>Corollary 7</u>. **A monotone perfect image of a rim compact T_1-space is rim compact.**

<u>Corollary 8</u>. If $f: X \longrightarrow Y$ is a monotone, closed mapping of a pseudonormal isocompact, rim compact T_1-space X onto a q-space Y, then Y is rim compact.

<u>Corollary 9</u>. A q-space which is a monotone closed image of a metrizable rim compact space is rim compact.

<u>Corollary 10</u>. A q-space which is a monotone closed image of a normal rim compact space with a G_δ-diagonal is rim compact.

<u>Corollary 11</u>. If $f: X \longrightarrow Y$ is a monotone, closed mapping of a rim compact T_1-space X onto a space Y and the set of all points in which f is peripherally compact is dense in Y, then Y has a π-base of open sets with compact boundaries.

Corollary 12. Every monotone closed image of a completely metrizable rim compact space has a σ-base of open sets with compact boundaries.

We recall that a mapping $f: X \longrightarrow Y$ of a topological space X onto a topological space Y is said to be bi-quotient [F], [M_2], if for every cover $\{U_\alpha\}_{\alpha \in A}$ of $f^{-1}(y)$ $(y \in Y)$ by open sets in X there exist a finite number of elements $U_{\alpha_1}, \ldots, U_{\alpha_k}$ such that $y \in \operatorname{Int} f(\bigcup_{i=1}^{k} U_{\alpha_i})$.

Lemma 1. If $f: X \longrightarrow Y$ is a monotone, bi-quotient, irreducible mapping of a Hausdorff space X onto a space Y and U is an open set in X such that $\mathrm{Fr}\, U$ is compact, then $\mathrm{Fr}\, f(U) \subset$ $\subset f(\mathrm{Fr}\, U)$.

Proof. By [Mi_5, Lemma 1] the set $f(\mathrm{Fr}\, U)$ is closed in Y and hence, if $y \in \mathrm{Fr}\, f(U) \setminus f(\mathrm{Fr}\, U)$, then $Y \setminus f(\mathrm{Fr}\, U)$ is a neighbourhood of y. Since $f^{-1}(Y) \setminus f(\mathrm{Fr}\, U) \cap f^{-1}(f(\mathrm{Fr}\, U)) = \emptyset$ and $\mathrm{Fr}\ U \subset f^{-1}(f(\mathrm{Fr}\, U))$, the sets $V_1 = U \cap f^{-1}(Y \setminus f(\mathrm{Fr}\, U))$ and $V_2 = (X \setminus \mathrm{cl}\, U) \quad f^{-1}(Y \quad f(\mathrm{Fr}\, U))$ are the preimages relative to f of some sets. Since $f^{-1}(y)$ is connected, either $f^{-1}(y) \subset V_1$ and then $y \in f(V_1) \subset f(U)$, where $f(V_1)$ is open, for f is quotient, which is impossible or $f^{-1}(y) \subset V_2$, which is also impossible, for $U \cap V_2 = \emptyset$ implies $f(U) \cap f(V_2) = \emptyset$. Thus, $\mathrm{Fr}\ f(U) \subset f(\mathrm{Fr}\, U)$.

Theorem 9. If $f: X \longrightarrow Y$ is a monotone, bi-quotient, irreducible mapping of a Hausdorff rim compact space X onto a space Y, then Y is also rim compact.

Proof. If y is a non-isolated point in Y and U is its neighbourhood, then $f^{-1}(y) \subset f^{-1}(U)$ and $f^{-1}(U)$ is open in X.

For each $x \in f^{-1}(y)$ we can choose an open neighbourhood $V_x \subset f^{-1}(U)$ such that $Fr\ V_x$ is compact. Since f is bi-quotient, we can find $V_{x_1}, \dots, V_{x_k}$ such that for the set $V = \bigcup_{i=1}^{k} V_{x_i}$ $y \in \operatorname{Int} f(V)$ and $Fr\ V \subset \bigcup_{i=1}^{k} Fr\ V_{x_i}$ is compact. By Lemma 1 $Fr\ f(V) \subset f(Fr\ V)$, but $Fr \operatorname{Int} f(V) \subset Fr\ f(V)$ and hence $Fr \operatorname{Int} f(V)$ is compact and $f(V) \subset U$.

<u>Note</u>. If Y above is Hausdorff, then the statement of Theorem 9 remains valid for monotone, quotient, peripherally compact mappings (cf. [K]).

<u>Lemma 2</u>. Let $f: X \longrightarrow Y$ be a monotone, bi-quotient, irreducible mapping of a Hausdorff space X onto a space Y and let U be an open set in X such that Fr U is compact. If $f^{\#}(U) \neq \emptyset$, then $f^{\#}(U)$ is open in Y and fr $f^{\#}(U)$ is compact.

<u>Proof</u>. By [Mi_5, Lemma 1] $f(Fr\ U)$ is closed in Y and hence $f^{-1}(f(Fr\ U))$ is closed in X and $X \setminus f^{-1}(f(Fr\ U))$ is open in X. Let $A = f^{-1}(Y \setminus f(Fr\ U)) = X \setminus f^{-1}(f(Fr\ U))$. We shall show that $A \cap U = f^{-1}(f^{\#}(U))$. Obviously $A \cap U \supset f^{-1}(f^{\#}(U))$. If $x \in A \cap U$, then $A \cap U \cap f^{-1}(f(x)) \neq \emptyset$. Since f is monotone and $A \cap Fr\ U = \emptyset$, $A \cap U$ is the preimage of a set and hence $f^{-1}(f(x)) \subset A \cap U$. Thus, we have $f(x) \in f^{\#}(U)$ and $x \in f^{-1}(f^{\#}(U))$. The set $f^{\#}(U)$ is open in Y, for f is quotient. Since f is monotone, $f(\mathrm{cl}\ U) = f^{\#}(U) \cup f(Fr\ U)$ [P]. Now, taking into account that $\mathrm{cl}\ f^{\#}(U) = f^{\#}(U) \cup Fr\ f^{\#}(U)$, $f^{\#}(U) \cap Fr\ f^{\#}(U) = \emptyset$ and $\mathrm{cl}\ f^{\#}(U) \subset f(\mathrm{cl}\ U)$, we obtain $Fr\ f^{\#}(U) \subset f(Fr\ U)$, so $Fr\ f^{\#}(U)$ is compact.

<u>Theorem 10</u>. If $f: X \longrightarrow Y$ is a monotone, compact, pseudo-open, irreducible mapping of a Hausdorff, rim compact space X onto a space Y, then f is closed and Y is rim compact.

Proof. If on the contrary there exists a point $y \in \mathrm{cl}\ f(F) \setminus f(F)$ for some closed set $F \neq \emptyset$, then $F \cap f^{-1}(y) = \emptyset$ and $f^{-1}(y)$ is compact. Let us choose for each point $x \in f^{-1}(y)$ an open neighbourhood O_x such that $O_x \cap F = \emptyset$ and $\mathrm{Fr}\ O_x$ is compact. If $O_{x_1}, \ldots, O_{x_n}$ form a cover of $f^{-1}(y)$, then for the set $V = \bigcup_{i=1}^{k} O_{x_i}$ we have $f^{-1}(y) \subset V$, $\mathrm{Fr}\ V \subset \bigcup_{i=1}^{n} \mathrm{Fr}\ O_{x_i}$ and therefore $\mathrm{Fr}\ V$ is compact. Since a pseudo-open, compact mapping is bi-quotient and $f^{\#}(V) \neq \emptyset$, by Lemma 2 $f^{\#}(V)$ is an open neighbourhood of y and $f^{\#}(V) \cap f(F) = \emptyset$, which is impossible, for $y \in \mathrm{cl}\ f(F)$. Thus, f is closed and by Theorem 9 Y is rim compact. This completes the proof.

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