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COMPACTNESS IN SPACES OF UNIFORM MEASURES

(Preliminary Communication)

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The theory of uniform measures was developed by Berezanskij [1], LeCam [4] and Frolík [2],[3]. For topics on free uniform measures, see [5].

In the paper with the title announced above I offer a generalization of the classical theorem on compactness in the space ℓ^1. Viz. I prove that every weakly compact subset in $\mathfrak{M}_u(X)$ is compact.

Moreover, the following results are in force (as proved in the paper):

Theorem 1. Let us be given a uniform space X and a set $M \subset \mathfrak{M}_u(X)$. The following conditions are equivalent:

(a) M is relatively U.E.B.-compact (in $\mathfrak{M}_u(X)$);

(b) M is relatively weakly compact;

(c) M is relatively weakly countably compact;

(d) M is relatively U.E.B.-countably compact;

(e) if S is any U.E.B.-set endowed with the simple topology then M is equicontinuous on S .

Theorem 2. Let us be given a uniform space X and a set $M \subset \mathfrak{M}_{\mathcal{U}}(X)$. Then M is relatively sequentially compact (in the U.E.B.-topology) if and only if it is relatively weakly sequentially compact.

Theorem 3. For any uniform space X , the space $\mathfrak{M}_{\mathcal{U}}(X)$ is weakly sequentially complete.

Further, to introduce vector-valued uniform measures, a vector-valued analogue of Grothendieck's completion theorem is proved. Theorems analogous to those above hold for vector-valued uniform measures.

All these results are also proved for free uniform measures.

Theorems 1 - 3 are shown to contain (mostly well-known) results on σ-additive and separable measures on topological spaces, σ-additive set functions on σ-algebras and cylindrical measures on locally convex spaces.

Part of announced results is contained in the collection of mimeographed notes of Zdeněk Frolík Seminar Abstract Analysis (Prague 1974/75).

The paper is submitted to Fundamenta Mathematicae.

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