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ON MINIMAL SURFACES IN E^5

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Abstract: A global characterization of minimal surfaces of E^5 which are situated in E^4 .

Key words: Minimal surface, generalized analytic function.

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We are going to prove the following

Theorem. Let $D \subset \mathbb{R}^2$ be a bounded domain and $M: D \rightarrow E^5$ a minimal surface such that $\dim T_m^2(M) = 4$ for each point m of the surface $M \equiv M(D)$, $T_m^2(M)$ being the 2-osculating space of M at m . Let $n_m \perp T_m^2(M)$ be the unit normal vector at m and $S = \{m \in M; (dn)_m = 0\}$. Then S consists of isolated points or M is situated in $E^4 \subset E^5$.

Proof. To each point $m \in M$, associate an orthonormal frame $\{m, v_1, v_2, v_3, v_4, v_5\}$ such that $T_m(M) = \{v_1, v_2\}$, $T_m^2(M) = \{v_1, v_2, v_3, v_4\}$, $n_m = v_5$. Then

$$(1) \quad \begin{aligned} dm &= \omega^1 v_1 + \omega^2 v_2, \\ dv_1 &= \omega_1^2 v_2 + \omega_1^3 v_3 + \omega_1^4 v_4, \\ dv_2 &= -\omega_1^2 v_1 + \omega_2^3 v_3 + \omega_2^4 v_4, \\ dv_3 &= -\omega_1^3 v_1 - \omega_2^3 v_2 + \omega_3^4 v_4 + \omega_{3,5}^5 v_5, \end{aligned}$$

$$\begin{aligned} dv_4 &= -\omega_1^4 v_1 - \omega_2^4 v_2 - \omega_3^4 v_3 + \omega_4^5 v_5, \\ dv_5 &= -\omega_3^5 v_3 - \omega_4^5 v_4 \end{aligned}$$

with the well known integrability conditions. From

$$(2) \quad \omega^3 = \omega^4 = \omega^5 = 0, \quad \omega_1^5 = \omega_2^5 = 0,$$

we get

$$(3) \quad \begin{aligned} \omega^1 \wedge \omega_1^3 + \omega^2 \wedge \omega_2^3 &= 0, & \omega^1 \wedge \omega_1^4 + \omega^2 \wedge \omega_2^4 &= 0, \\ \omega_1^3 \wedge \omega_3^5 + \omega_1^4 \wedge \omega_4^5 &= 0, & \omega_2^3 \wedge \omega_3^5 + \omega_2^4 \wedge \omega_4^5 &= 0 \end{aligned}$$

and the existence of functions a_1, b_1, c_∞ such that

$$(4) \quad \begin{aligned} \omega_1^3 &= a_1 \omega^1 + a_2 \omega^2, & \omega_1^4 &= b_1 \omega^1 + b_2 \omega^2, \\ \omega_2^3 &= a_2 \omega^1 + a_3 \omega^2, & \omega_2^4 &= b_2 \omega^1 + b_3 \omega^2, \end{aligned}$$

$$(5) \quad \begin{aligned} a_1 \omega_3^5 + b_1 \omega_4^5 &= c_1 \omega^1 + c_2 \omega^2, \\ a_2 \omega_3^5 + b_2 \omega_4^5 &= c_2 \omega^1 + c_3 \omega^2, \\ a_3 \omega_3^5 + b_3 \omega_4^5 &= c_3 \omega^1 + c_4 \omega^2. \end{aligned}$$

It is easy to see that

$$(6) \quad \mathbb{F} = (a_1 + a_3)v_3 + (b_1 + b_3)v_4$$

is the mean curvature vector. Our surface being minimal, we have $a_1 + a_3 = b_1 + b_3 = 0$ and (4) + (5) reduce to

$$(7) \quad \begin{aligned} \omega_1^3 &= a_1 \omega^1 + a_2 \omega^2, & \omega_1^4 &= b_1 \omega^1 + b_2 \omega^2, \\ \omega_2^3 &= a_2 \omega^1 - a_1 \omega^2, & \omega_2^4 &= b_2 \omega^1 - b_1 \omega^2, \end{aligned}$$

$$(8) \quad \begin{aligned} a_1 \omega_3^5 + b_1 \omega_4^5 &= c_1 \omega^1 + c_2 \omega^2, & a_2 \omega_3^5 + b_2 \omega_4^5 &= \\ & & &= c_2 \omega^1 - c_1 \omega^2. \end{aligned}$$

Because of

$$\begin{aligned} dv_1 &= \omega_1^2 v_2 + \omega^1 (a_1 v_3 + b_1 v_4) + \omega^2 (a_2 v_3 + b_2 v_4) , \\ dv_2 &= -\omega_1^2 v_1 + \omega^1 (a_2 v_3 + b_2 v_4) - \omega^1 (a_1 v_3 + b_1 v_4) , \end{aligned}$$

we have $T^2(M) = \{v_1, v_2, a_1 v_3 + b_1 v_4, a_2 v_3 + b_2 v_4\}$ and

$$(9) \quad a_1 b_2 - a_2 b_1 \neq 0 .$$

From (7),

$$(10) \quad \begin{aligned} Da_1 \wedge \omega^1 + Da_2 \wedge \omega^2 &= 0 , \quad Da_2 \wedge \omega^1 - Da_1 \wedge \omega^2 = 0 , \\ Db_1 \wedge \omega^1 + Db_2 \wedge \omega^2 &= 0 , \quad Db_2 \wedge \omega^1 - Db_1 \wedge \omega^2 = 0 \end{aligned}$$

with

$$(11) \quad \begin{aligned} Da_1 &:= da_1 - 2a_2 \omega_1^2 - b_1 \omega_3^4 = \alpha_1 \omega^1 + \alpha_2 \omega^2 , \\ Da_2 &:= da_2 + 2a_1 \omega_1^2 - b_2 \omega_3^4 = \alpha_2 \omega^1 - \alpha_1 \omega^2 , \\ Db_1 &:= db_1 - 2b_2 \omega_1^2 + a_1 \omega_3^4 = \beta_1 \omega^1 + \beta_2 \omega^2 , \\ Db_2 &:= db_2 + 2b_1 \omega_1^2 + a_2 \omega_3^4 = \beta_2 \omega^1 - \beta_1 \omega^2 . \end{aligned}$$

From (8),

$$(12) \quad \begin{aligned} (dc_1 - 3c_2 \omega_1^2) \wedge \omega^1 + (dc_2 + 3c_1 \omega_1^2) \wedge \omega^2 &= \\ &= (f_1 c_1 + f_2 c_2) \omega^1 \wedge \omega^2 , \\ (dc_2 + 3c_1 \omega_1^2) \wedge \omega^1 - (dc_1 - 3c_2 \omega_1^2) \wedge \omega^2 &= \\ &= (f_2 c_1 - f_1 c_2) \omega^1 \wedge \omega^2 , \end{aligned}$$

$$(a_1 b_2 - a_2 b_1) f_1 := \alpha_1 b_1 - \alpha_2 b_2 - \beta_1 a_1 + \beta_2 a_2 ,$$

$$(a_1 b_2 - a_2 b_1) f_2 := \alpha_1 b_2 + \alpha_2 b_1 - \beta_1 a_2 - \beta_2 a_1 ,$$

and we get the existence of functions g_1, g_2 such that

$$(13) \quad \begin{aligned} dc_1 - 3c_2\omega_1^2 &= (g_1 - f_2c_1)\omega^1 + (g_2 - f_1c_1)\omega^2, \\ dc_2 + 3c_1\omega_1^2 &= (g_2 + f_2c_2)\omega^1 - (g_1 - f_1c_2)\omega^2. \end{aligned}$$

In D , consider the isothermic coordinates (u, v) such that

$$(14) \quad ds^2 = r^2(du^2 + dv^2), \quad r(u, v) > 0; \quad \omega^1 = r du, \quad \omega^2 = r dv.$$

Then

$$(15) \quad \omega_1^2 = r^{-1}(-r_v du + r_u dv)$$

because of $d\omega^1 = -\omega^2 \wedge \omega_1^2$, $d\omega^2 = \omega^1 \wedge \omega_1^2$. We get

$$(16) \quad \begin{aligned} \frac{\partial c_1}{\partial u} + 3r^{-1}r_v c_2 &= g_1 r - f_2 r c_1, \\ \frac{\partial c_1}{\partial v} - 3r^{-1}r_u c_2 &= g_2 r - f_1 r c_1, \\ \frac{\partial c_2}{\partial u} - 3r^{-1}r_v c_1 &= g_2 r + f_2 r c_2, \\ \frac{\partial c_2}{\partial v} + 3r^{-1}r_u c_1 &= -g_1 r + f_1 r c_2, \end{aligned}$$

i.e.,

$$(17) \quad \begin{aligned} \frac{\partial c_1}{\partial u} + \frac{\partial c_2}{\partial v} + r^{-1}(3r_u + f_2 r^2)c_1 + r^{-1}(3r_v - f_1 r^2)c_2 &= 0, \\ \frac{\partial c_1}{\partial v} - \frac{\partial c_2}{\partial u} + r^{-1}(3r_v + f_1 r^2)c_1 - r^{-1}(3r_u - f_2 r^2)c_2 &= 0. \end{aligned}$$

The function $w := c_1 + ic_2$ is thus a generalized analytic function [1], and the Theorem follows from the obvious fact $S = \{ (u, v); w(u, v) = 0 \}$.

R e f e r e n c e

- [1] I.N. VEKUA: Verallgemeinerte analytische Funktionen,
Berlin, Akademie-Verlag 1963. (Original edition
Moskva 1959.)

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