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REGULARLY METRIZABLE CONNECTIONS AND TENSORS OF TYPE (1,3)

(Preliminary communication)

Oldřich KOWALSKI, Praha

In a paper under the same title we deal with two problems:

a) Let ∇ be a linear connection without torsion on a manifold M. Under what conditions ∇ is locally the Levi-Civita connection of a Riemann metric on M ?

b) Let B be a tensor of type $(1,3)$ on M. Under which conditions B is locally a Riemann curvature tensor?

A Riemann metric g on M is called <u>regular at a point</u> p if the sectional curvature $\sigma(P)$ is non-zero in any 2-dimensional direction P at p .

<u>An explicit solution of both local problems is given for the regular case</u>. We state necessary and sufficient conditions for a linear connection ∇ (or for a tensor B of type $(1,3)$) to be locally induced by a regular Riemann metric g. Moreover, if the above conditions are fulfilled, the corresponding Riemann metric g can be determined, exact up to a positive constant factor, only by algebraic calculations and by the integration of an exact

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differential. (An exceptional case for the second problem arises if $\dim M = 2$.)

Also a global theorem is proved, which is related to some results by K. Nomizu and K. Yano [1], and by R. S. Kulkarni [2].

<u>Theorem</u>. Let (M, g) be a regular Riemann space of dimension $m \geq 3$. Then any curvature tensor-preserving diffeomorphism of (M, g) onto a Riemann space (M', g') is a homothety.
(M , M' are supposed to be of class C^4 and g, g' of clas C^3 .)

References

[1] NOMIZU K., YANO K.: Some results to the equivalence problem in Riemannian geometry, Math.Zeitschr. 97(1967),29-37.

[2] KULKARNI R.S.: Curvature and metric, Ann.of Math.91 (1970),311-331.

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