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LIMITS OF DERIVATIVES

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(Preliminary communication)

It is well known (see [1]) that any real-valued function defined on an interval I is the limit of a sequence of Darboux functions. Moreover Mišík proved the following theorem (see [2]):

Theorem 1: Any Baire $\alpha \neq 2$ function is the limit of Darboux-Baire $< \alpha$ functions.

It can be shown that the statement of the theorem 1 is true also for the case $\alpha = 2$, i.e.,more exactly the following theorem is valid:

Theorem 2: A function f is an element of the second class Baire if and only if f is a limit of a sequence of derivatives.

A set $E \subset \langle a, b \rangle$ is called to be the stationary set for derivatives if for every derivative f that is equal zero for $x \in A$ implies that f vanishes on all interval $\langle a, b \rangle$.

A set E is called to have the Denjoy property if for every open interval J such that $J \cap E \neq \emptyset$ is $\mu(J \cap E) > 0$.

The following characterization of the stationary set for derivatives was proved by S. Marcus [3].

Theorem 3: A set $E \subset \langle a, b \rangle$ is a stationary set for derivatives if and only if for each set $A \subset \langle a, b \rangle$ such that $\mu(A) > 0$ is $E \cap A \neq \emptyset$.

The theorem 2 can be proved by the following way. If a function f is an element of B_2 then f can be written as a point limit of f_n which are suitable linear combinations of characteristic functions of sets F_σ that are at the same time G_δ. It is easy to show that there exists a sequence disjoint sets $A_n \subset \langle a, b \rangle$ such that each A_n has the Denjoy property and is dense in $\langle a, b \rangle$ Each function f_n can be changed on A_n by the following lemma.

Lemma: Let $M \subset \langle a, b \rangle$ be the set F_σ and G_δ. Let $A \subset \langle a, b \rangle$. have the Denjoy property and be dense in the interval $\langle a, b \rangle$. Then there exists a derivative φ defined on $\langle a, b \rangle$ and a countable set S such that $\{x ; \varphi(x) \neq c_n(x)\} \subset A \cup S$, where $c_n(x)$ is the characteristic function of M.

From this lemma follows that changed functions f_n converge to f except countable set which can be eliminated on the base of the theorem 3.

The complete proof of the theorem 2 and some results related to the point and uniform convergence of Darboux-Baire functions will be published later on.

R e f e r e n c e s :

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[2] L. MIŠÍK: Zu zwei Sätzen von W. Sierpiński,Rev.Math. Pures Appl.6(1967),349-360.

[3] S. MARCUS: Sur les ensembles stationaires de fonctions dérivées-finies ou infinies,Com.Acad.R.P. Romine 12(1962),399-402.

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