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NON-PROVABILITY OF SOUSLIN´S HYPOTHESIS

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1. Introduction

The ordering of the real line is uniquely characterized by the following properties:

(i) it is continuous;

(ii) it has no first and no last element;

(iii) it has a denumerable dense set;

From (iii) follows

(iv) every family of non-overlapping intervals is at most denumerable.

It has been conjectured that (iii) can be replaced by (iv). This question was raised in 1920 by Souslin [13].

Souslin´s problem: Is every ordering satisfying (i),(ii), (iv) isomorphic with the ordering of the continuum?

Souslin´s continuum is such a set which satisfies (i), (ii),(iv) and is not isomorphic with the continuum.

The existence of Souslin´s continuum has been neither proved nor disproved. There are known the following results concerning Souslin´s hypothesis:

Jesenin-Vol´pin (1954) [6]: Souslin´s hypothesis (i.e. the non-existence of Souslin´s continuum) is not deducible from the axioms A, B, C of set theory (Gödel-Bernays set theory, cf. [2], without the axioms of regularity and of choice).

Hájek and Vopěnka (1965) [3]: Souslin´s hypothesis is not deducible from the axioms A, B, C, D of set theory.

The main result of the present paper is

Theorem 1. Souslin's hypothesis is not provable in set theory.

Exactly speaking, if Gödel-Bernays set theory Σ^* is consistent, then it remains consistent, if the existence of Souslin's continuum is assumed. It remains the problem, whether the assumption of non-existence of Souslin's continuum is consistent as well.

In section 2, some theorems are mentioned, which are connected with Souslin's problem. E.g., theorem 1 implies the non-provability of Kurepa's ramification hypothesis. Miller's theorem enables to transfer the problem of existence of Souslin's continuum to the problem of existence of an uncountable ramified graph of certain type. This theorem is used in construction of the model (section 4).

The intuitive intention is, to construct the required ramified graph as a limit of some countable ramified graphs ordered by inclusion. Using Vopěnka's or Cohen's methods, cf. [14-18, 1], the construction of corresponding topology of of corresponding set of sets of conditions is a matter of skill. The problem is, however, what conditions the countable graphs must satisfy, in order to converge to Souslin's continuum. It appears that the countable regular ramified graphs (2.3, 4.2) are the right graphs.

The construction is done with help of Vopěnka's method of ∇ -models, as described in [18]. It is not essential that ∇ -models are models of Σ^* in Σ^*. An analoguous construction can be done in ZF*. ∇ -models are briefly described in section 3. For the reader who knows better Cohen-type models than ∇ -models the remark 4.15 is added.

The section 5 is devoted to standard ∇ -models given by free ultrafilters. The whole section (which is not necessary for other considerations), is based on the method from [18]. The relation is shown between this method and Hajnal-Lévy-Shoenfield's modification of Gödel's function. Especially, if $(\aleph_4)^L$ (i.e. constructible $\aleph_4$) is denumerable, the non-provability of Souslin's hypothesis can be established by Lévy's adjunction of a non-constructible set.

The question is, whether an analogous model as in section 4 can be constructed for larger cardinals. In connection with [5], it would give an interesting result on measurable cardinals.

2. Equivalence theorems

In 1943, Miller proved the following theorem, cf. [15]:

2.1. Theorem. Souslin's continuum exists if and only if there exists a partially ordered set P of power $\aleph_1$ such that

a) if $Q \subseteq P$ and $\mathit{card}\ Q = \aleph_1$, then Q contains at least two comparable and at least two incomparable elements;

b) if $x, y \in P$ are incomparable, then there is $z \in P$ with $x < z$ and $y < z$.

In 1948, Sierpiński stated in [12] the following theorem (weaker than Miller's):

2.2. Theorem. Souslin's hypothesis is equivalent with the following hypothesis:

Let F be a family of sets satisfying

a) $a, b \in F \rightarrow (a \cap b = 0 \vee a \subseteq b \vee b \subseteq a)$;

b) every disjointed subfamily is at most denumerable;

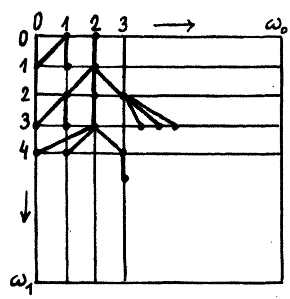
c) every monotone subfamily has the maximal element and is at most denumerable.

Then F is at most denumerable.

For the sake of completeness it is to be mentioned that in 1936 Kurepa [7] formulated his ramification hypothesis, which implies Souslin's hypothesis and is therefore not provable.

In order to avoid technical difficulties we reformulate Miller-Sierpiński equivalence theorems and introduce some notions which simplify further considerations.

2.3. Definition.



Further, the set $\omega_0 \times \omega_1$ will be considered. The elements of that set are $\langle n\alpha \rangle$ ($n \in \omega$, $\alpha < \omega_1$). The α-th row h_α is the set $\omega_0 \times \{\alpha\}$. The ramified graph is the relation r on $\omega_0 \times \omega_1$ satisfying the following conditions:

(i) r is reflexive and transitive;

(ii) if $\langle x, y \rangle \in r$ and $x \in h_\alpha$, $y \in h_\beta$ then $\alpha \leq \beta$;

(iii) if $\alpha < \beta$ and $y \in h_\beta$ then there is $x \in h_\alpha$ with $\langle x\, y \rangle \in r$;

(iv) if $x \neq y$ are from the same row, then there is no z with $\langle x\, z \rangle \in r$ and $\langle y\, z \rangle \in r$.

Two elements x, y are r-comparable, if $\langle x\, y \rangle \in r$ or $\langle y\, x \rangle \in r$ and are r-incomparable, if the converse is true. A relation $s \subseteq r$ satisfying (i) - (iv) and being a linear ordering is called a chain of a ramified graph.

A subset a of the domain $\mathcal{D}(r)$ of r is called an <u>antichain</u> of r, if it contains pairwise incomparable elements only.

The following theorem is a simple modification of Miller-Sierpiński theorem:

2.4. <u>Theorem</u>. The necessary and sufficient condition for the existence of Souslin's continuum is the existence of an uncountable ramified graph, which has no uncountable chains or antichains.

3. ∇ -models

∇ -models are parametric syntactic models of the theory $\sum^*$ of Gödel-Bernays in the theory $\sum^*$ and depend on two parameters B, z ; B being a variable for complete Boolean algebras and z being a variable for ultrafilters on B. The author of the present paper preserves the denotation both of papers [14-17] and of the later version of ∇ -models [18].

F is the sheaf over B on $C(B)$ and for every set-formula φ the value $F\ulcorner\varphi\urcorner \in B$ can be computed ($F\ulcorner\varphi \& \psi\urcorner = F\ulcorner\varphi\urcorner \wedge F\ulcorner\psi\urcorner$, $F\ulcorner\neg\varphi\urcorner = -F\ulcorner\varphi\urcorner$ etc.). The model $\nabla(B,z)$ is the model determined by the class of functions $C(B)$ and the predicate $\in^*$ ($x \in^* y \equiv F(x\ y) \in z$). For any set-formula φ, the following holds:

$$\varphi^* \equiv F\ulcorner\varphi\urcorner \in z\ .$$

To every set x there is a corresponding function $k_x \in C(B)$. It is no danger of confusion, if we identify k_x with x.

There are two important characteristics of complete Boolean algebras which enables to investigate the properties of ∇ -models.

3.1. <u>Definition</u>. $\mu(B)$ is the least $\aleph_\alpha$ such that no disjointed family $A \subseteq B$ of power $\aleph_\alpha$ exists.

3.2. <u>Definition</u>. $\sigma(B)$ is the least $\aleph_\alpha$ such that there is no basis b of B having the property that $u_0 \geq \geq u_1 \geq \ldots \geq u_\beta \geq \ldots \in b$ implies $\bigwedge_{\beta<\omega_\alpha} u_\beta \neq 0$.

3.3. <u>Theorem</u>. If $\omega_\alpha \leq \sigma(B)$ or $\omega_\alpha \geq \mu(B)$, then ω_α is a cardinal number of $\nabla(B, x)$.

4. The model

4.1. Now, we construct a complete Boolean algebra B in such a way that there is a Souslin's continuum in the model $\nabla(B, x)$, x being an ultrafilter on B. The Boolean algebra B is the algebra of all open regular sets of some Hausdorff topological space.

4.2. <u>Definition</u>. A ramified graph r is <u>regular</u>, if

(v) there is $\alpha \leq \omega_1$ (the <u>length</u> of the graph) and $\mathcal{D}(r)$ is the whole $\omega_0 \times \alpha$;

(vi) if $x \in h_\beta$, $\beta < \gamma < \alpha$, then there are y_1, $y_2 \in h_\gamma$, $y_1 \neq y_2$ with $\langle x y_1 \rangle \in r$, $\langle x y_2 \rangle \in r$.

4.3. <u>Definition</u>. c is the set of all regular ramified graphs of length ω_1.

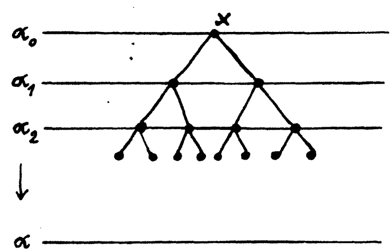
b is the set of all regular ramified graphs of countable length. For every $r \in b$, u_r is the set of all $q \in c$, $q \supset r$. (Further, the lower-case letter r is the variable for elements of b.)

4.4. <u>Lemma</u>. If $r \supseteq r'$ then $u_r \subseteq u_{r'}$.

4.5. <u>Lemma</u>. If $r, r' \in b$, then either $r \subseteq r'$ or $r \supseteq r'$ or r and r′ have no common extension; i.e. either $u_r \supseteq u_{r'}$, or $u_r \subseteq u_{r'}$ or u_r and $u_{r'}$ are disjoint.

4.6. Lemma. If $x \in \mathcal{D}(r)$ and the length of r is α, then there is a chain $s \subseteq r$ of length α containing the point x (i.e. $x \in \mathcal{D}(s)$). Moreover, if α is a limit number, then there are uncountably many such chains.

Proof. We can assume that α is a limit number. There is a sequence $\alpha_0 < \alpha_1 < \dots < \alpha_n < \dots$ confinal with α ($x \in r_{\alpha_0}$), and we can easily construct a subgraph of r of the following form:



The assertion is then obvious.

4.7. Lemma. If r is of length α, then there is $r' \supset r$ of length $\alpha + 1$.

Proof. The proposition is obvious, if α is not a limit number. Let α be a limit number. Let us enumerate all members of $\mathcal{D}(r) = \omega_0 \times \alpha$ by natural numbers: $e_0, e_1, \dots, e_n, \dots$ For every $n \in \omega$ we choose a chain s_n of length α containing the point e_n and extend this chain by adding the point $\langle n\ \alpha \rangle$ in the α-th row. The graph obtained in this way is regular ramified and has the length $\alpha + 1$.

4.8. Lemma. If $r_0 \subseteq r_1 \subseteq \dots \subseteq r_n \subseteq \dots$ are elements of b, then $r = \bigcup_{n=0}^{\infty} r_n$ belongs to b.

Proof.

The only non-trivial condition, which is to be verified is (vi) of 4.2. Let $\beta < \gamma <$ (the length of r). Then

γ is less than the length of some r_n and thus (vi) is satisfied.

4.9. <u>Lemma</u>. If $r \in b$, then there is $q \in c$ with $q \supseteq r$.

<u>Proof</u>. By induction up to ω_1.

4.10. <u>Consequence</u>. Every u_r is non-void; the intersection of a sequence $u_{r_0} \supseteq u_{r_1} \supseteq \dots \supseteq u_{r_n} \supseteq \dots$ is again an u_r

4.11. <u>Lemma</u>. The collection $\{ u_r : r \in b \}$ is a clopen basis for a Hausdorff topology t on the set c.

4.12. Now, B is defined as the Boolean algebra of all open regular sets of the space $\langle c\, t \rangle$. The set b can be embedded into B in such a way that every r is identified with u_r. The set is then a basis for B (i.e. for any non-zero $u \in B$ there is $r \in b$ with $r \leq u$), $r_1 \leq r_2$ is the same as $r_1 \supseteq r_2$, the meet $r_1 \wedge r_2$ is r_1 if $r_1 \supseteq r_2$, r_2 if $r_1 \subseteq r_2$ and 0 if $r_1 \cup r_2 \notin b$, and the meet $\bigwedge_{n \in \omega} r_n$ of a monotone sequence $r_0 \geqslant r_1 \geqslant \dots$ $\dots \geqslant r_n \geqslant \dots$ is $\bigcup_{n \in \omega} r_n$. Further, we choose an ultrafilter z on B.

4.13. <u>Lemma</u>. ω_1 if the first uncountable cardinal in the model $\nabla(B, z)$. Moreover, is the construction is done in the theory with the axiom of constructibility, cardinals of $\nabla(B, z)$ are absolute.

<u>Proof</u>. The assertion follows from 3.3. $\sigma(B) = \aleph_1$ according to 4.10, $\mu(B) = (2^{\aleph_0})^+$ (i.e. $\aleph_2$, if $V = L$).

4.14. Now, let us define the function $f \in C(B)$, which is to be the required ramified graph in $\nabla(B, z)$. Let x, y ~~be~~ elements of $\omega_0 \times \omega_1$

$$f(\langle x\, y\rangle) = \bigvee\{r : \langle x\, y\rangle \in r\} \ .$$

It follows that for any r of length α ,

$$F\ulcorner f \upharpoonright (\omega \times \alpha) = r\urcorner = r \ .$$

Now it is obvious, how the definition of the Boolean algebra satisfies the intuitive intention to obtain f as a "limit of countable regular ramified graphs".

4.15. Remark. If the corresponding Cohen-type model were constructed, then countable regular ramified graphs should be the sets of conditions, the symbol

$$r \leq F\ulcorner \varphi \urcorner$$

should be replaced by

$$r \Vdash^* \hat{\varphi}$$

($\hat{\varphi}$ being a formalization of the formula φ) and the equality from 4.14 should be replaced by

$$r \Vdash^* (f \upharpoonright (\omega \times \alpha) = r) \ .$$

4.16. It remains to prove that f has in ∇ the following properties:

a) f is an uncountable ramified graph;

b) every chain is countable;

c) every antichain is countable.

In the proof, we use the fact that, firstly, if $F\ulcorner\varphi\urcorner = 1$, then φ holds in ∇ , and secondly

$$u \leq F\ulcorner(\forall x)\varphi(x)\urcorner \quad \text{iff } (\forall r \leq u)(\forall x \in C(B))[r \leq F\ulcorner\varphi(x)\urcorner];$$

$$u \leq F\ulcorner(\exists x)\varphi(x)\urcorner \quad \text{iff } (\forall v \leq u)(\exists r \leq v)(\exists x \in C(B))[r \leq$$
$$\leq F\ulcorner\varphi(x)\urcorner] \ .$$

4.17. Lemma. f is an uncountable ramified graph in $\nabla(B, \varkappa)$.

Proof. The proof of the fact that f is a ramified graph is a matter of skill. As for the second part of the assertion, it follows from the fact that $F^{\ulcorner}\mathcal{D}(f) = \omega_0 \times \omega_1^{\urcorner} = 1$.

4.18. Lemma. Every chain of f is countable (in ∇).

Proof. We prove the following:

$$F^{\ulcorner}(\exists s)\varphi(s)^{\urcorner} = 0$$

where $\varphi(s)$ is the formula

$$s \subseteq f \;\&\; s \text{ is a chain of } f \;\&\; (\forall \alpha < \omega_1)(\exists x \in h_\alpha)[x \in \mathcal{D}(s)].$$

Let, on the contrary, there be $s \in C(B)$ with $F^{\ulcorner}\varphi(s)^{\urcorner} \neq 0$. There are $x_0 \in h_0$ and $r_0 \leq F^{\ulcorner}\varphi(s)^{\urcorner}$ such that

$$r_0 \leq F^{\ulcorner}x_0 \in \mathcal{D}(s)^{\urcorner} .$$

We can continue by induction. For any $\beta < \omega_1$ there are $x_\beta \in h_\beta$ and $r_\beta \supseteq \bigcup_{\gamma<\beta} r_\gamma$ such that

$$r_\beta \leq F^{\ulcorner}x_\beta \in \mathcal{D}(s)^{\urcorner} .$$

Let us denote as s_β the chain determined by the sequence $\{x_\gamma : \gamma < \beta\}$. Since $r_\beta \leq F^{\ulcorner}\varphi(s)^{\urcorner}$, it is easy to be shown that

(1) $r_\beta \leq F^{\ulcorner}s \upharpoonright (\omega \times \beta) = s_\beta^{\urcorner}$;

(2) $s_\beta \subseteq r_\beta$.

Let g be the following ordinal function from ω_1 into ω_1 :

$g(0) = 0$,

$g(\beta)$ ... the supremum of lengths of r_γ, $\gamma < \beta$.

The function g is increasing and continuous, and hence there is a limit ordinal $\alpha < \omega_1$ with $g(\alpha) = \alpha$. It follows that r_α and s_α have the same length α and that

(1) $r_\alpha \leq F^{\ulcorner}s \upharpoonright (\omega \times \alpha) = s_\alpha^{\urcorner}$;

(2) $s_\alpha \subseteq r_\alpha$

Now, we construct an extension $r' \supseteq r_\alpha$ having the property that the chain s_α cannot be extended in r' (and hence in no extension of r'). Then

$$(*) \quad r' \leq F^\ulcorner (\neg \exists x)[x \in h_\alpha \,\&\, x \in \mathcal{D}(s)]^\urcorner$$

which contradicts to $r' \leq F^\ulcorner \varphi(s)^\urcorner$. (If $(*)$ did not hold, there would be an extension $r'' \supseteq r'$ and an extension $s' \supseteq s_\alpha$ of length $\alpha + 1$ with $r'' \leq F^\ulcorner s \restriction (\omega \times (\alpha+1)) = s'^\urcorner$ and $s' \subseteq r''$.)

The extension $r' \supseteq r$ is defined in the following way: Let us enumerate all members of $\omega_0 \times \alpha$ by natural numbers: $e_0, e_1, \dots, e_n, \dots$. For every $n \in \omega_0$, we choose a chain $\overline{s_n}$ of length α different from s_α and containing the point e_n and extend this chain by adding the point $\langle n\, \alpha \rangle$ in the α-th row. The obtained graph is regular and the chain s_α cannot be extended in it.

4.19. <u>Lemma</u>. Every antichain of f is countable (in ∇).

<u>Proof</u>. Every antichain of a ramified graph can be extended to a maximal antichain (i.e. such antichain which cannot be extended). Let us suppose that there is $a \in C(B)$ with $F^\ulcorner \psi(a)^\urcorner \neq 0$ where $\psi(a)$ is the formula

$a \subseteq \omega_0 \times \omega_1 \,\&\, \operatorname{card} a = \aleph_1 \,\&\, a$ is a maximal antichain of f.

Let $y_0, y_1, \dots, y_\beta, \dots; \ \beta < \omega_1$, be all the elements of $\omega_0 \times \omega_1$ in lexicographic order (i.e. first the 0-th row, then the 1-st, etc.). There are x_0 and $r_0 \leq F^\ulcorner \psi(a)^\urcorner$ such that

$r_0 \leq F^\ulcorner x_0 \in a \,\&\, x_0$ is f-comparable with $y_0^\urcorner$.

We can continue by induction. For any $\beta < \omega_1$ there are x_β

and $r_\beta \supseteq \bigcup_{\gamma<\beta} r_\gamma$ such that

$r_\beta \leq F\ulcorner x_\beta \in a$ & x_β is f-comparable with $y_\beta\urcorner$.

Using the same argument as in 4.18 we can find a limit ordinal α having the following property:

(1) r_α has the length α ;

(2) $\{\psi_\beta : \beta < \alpha\} = \omega_0 \times \alpha$;

(3) $\{x_\beta : \beta < \alpha\} \subset \omega_0 \times \alpha$.

Let us denote as a_α the set $\{x_\beta : \beta < \alpha\}$. Since $r_\alpha \leq F\ulcorner\psi(a)\urcorner$ it is easy to be shown that

(4) a_α is a maximal antichain in r_α ;

(5) $r_\alpha \leq F\ulcorner a \cap (\omega_0 \times \alpha) = a_\alpha\urcorner$.

Now, we construct an extension $r' \supseteq r_\alpha$ of length $\alpha + 1$ having the property that a_α is a maximal antichain in r' (and hence in every extension of r'). Then

(*) $r' \leq F\ulcorner a = a_\alpha\urcorner$

which contradicts to $r' \leq F\ulcorner\psi(a)\urcorner$. (If (*) did not hold, there would be an extension $\bar{r} \supseteq r'$ and $x \notin a_\alpha$ with $\bar{r} \leq F\ulcorner x \in a\urcorner$. But in this case we could find an extension $\tilde{r} \supseteq \bar{r}$ with $x \in \mathcal{D}(\tilde{r})$ and hence $a_\alpha \cup \{x\}$ would be an antichain in $\tilde{r}$.)

The extension $r' \supseteq r$ is defined in the following way: Let us enumerate all the members of $\omega_0 \times \alpha$ by natural numbers: $e_0, e_1, \dots, e_n, \dots$. For every $n \in \omega$ there is $\bar{e}_n \in a_\alpha$ comparable with e_n . Hence we can choose a chain s_n of length α containing the points $e_n, \bar{e}_n$ and extend this chain by adding the point $\langle n\ \alpha \rangle$ in the α-th row. Obviously, a_α is a maximal antichain in this regular extension.

5. Remark on free ultrafilters

5.1. This chapter is devoted to construction of standard ∇ -models with help of free ultrafilters. The reader is supposed to be familiar with sections 8, 9 of [18] and the denotation from it is used without reference. The significance of free ultrafilters is, that if B is a complete Boolean algebra in a model-class P and z is a free ultrafilter, then $\nabla(B, z)$ is standard model (isomorphic with the model-class W_z^B). The existence of free ultrafilters is guaranteed, if ω_1 is sufficiently large cardinal of $\Delta(P)$.

5.2. If P is assumed to be the class L of all constructible sets, then we can easily prove the following

Theorem. $W_z^B = L_z$ (L_z being the class of z-constructible sets, cf. [4, 8, 11, 9].

Proof. 1. $L_z \subseteq W_z^B$. It suffices to prove $z \in W_z^B$. It holds that $z \subseteq B \in L$ and the identity function I on B is a (B,B)-function such that $z = \{x : I(x) \in z\}$.

2. $W_z^B \subseteq L_z$. This follows from the fact that every $w(fz)$ can be obtained by Gödel's operations from z and constructible sets.

5.3. Using the result from [18] and computing the power of the Boolean algebra defined in section 4, we can prove the following

Theorem. If $\aleph_1 > (\aleph_4)^L$, then there is a set z such that Souslin's hypothesis does not hold in $\Delta(L_z)$.

6. Acknowledgement. The author is greatly indebted to express his gratitude to members of Prague Seminar in Set theory for the valuable discussion on the present paper.

7. Remark. As P. Vopěnka informed the author, A. Hajnal from Budapest heard during his stay in the U.S.A. in 1964 that S. Tennenbaum obtained a similar result as the present (by Cohen's methods). However, the author does not know either of any published paper concerning this problem, or of any abstract announcing this result.

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